Proceedings

The 7th International Congress of Serbian Society of Mechanics

Sremski Karlovci, June 24-26, 2019

Edited by:

Mihailo Lazarević
Srboljub Simić
Damir Madjarević
Ivana Atanasovska
Andjelka Hedrih
Bojan Jeremić

The 7th International Congress of Serbian Society of Mechanics

Editors:
Mihailo P.Lazarević
Srboljub Simić
Damir Madjarević
Ivana Atanasovska
Anđelka Hedrih
Bojan Jeremić

Circulation
140 copies

Published by
Serbian Society of Mechanics,Belgrade,2019

Printed by
Planeta Print, Belgrade, 2019

CIP - Каталогизација у публикацији - Народна библиотека Србије, Београд

531/534(082)(0.034.2)

SRPSKO društvo za mehaniku. Međunarodni kongres (9 ; 2019 ; Sremski Karlovci)
Proceedings [Elektronski izvor] / The 7th International Congress of Serbian Society of Mechanics, Sremski Karlovci, June 24-26, 2019 ; edited by Mihailo Lazarević ... [et al.]. - Belgrade : Serbian Society of Mechanics, 2019 (Belgrade : Planeta Print). - 1 USB fleš memorija ; 9 x 5 cm (u obliku kartice)

Sistemski zahtevi: Nisu navedeni. - Nasl. sa naslovne strane dokumenta. - Tiraž 140. - Bibliografija uz svaki rad.

ISBN 978-86-909973-7-4

а) Механика - Зборници

COBISS.SR-ID 277232652

7th International Congress of Serbian Society of Mechanics
Sremski Karlovci, Serbia, June 24-26, 2019

FINITE ELEMENT MODEL OF IMPERFECT PLATE IN THERMAL ENVIRONMENT

M. Cetkovic[1]

[1] Department of Engineering Mechanics and Theory of Structures
University of Belgrade, Bul. Kralja Aleksandra 73, 11000 Belgrade, Serbia
e-mail: marina@grf.bg.ac.rs

Abstract

In this paper a finite element model for thermal buckling of imperfect plates using Layer Wise (LW) plate mode [1] is presented. The model assumes layerwise variation of in-plane displacements and constant transverse displacement through the thickness of the plate, non-linear strain-displacement relations (in von Karman sense) and linear thermo mechanical material properties. The Koiter model for imperfection is adopted. The Principle of virtual displacements (PVD) is used to derive the weak form of linearized buckling problem. The weak form is discretized using Lagrangian nine-node isoparametric finite element. The original MATLAB program is coded for finite element solution. The effects of imperfection amplitude, imperfection form and plate aspect ratio a/h on critical temperature are analyzed. The accuracy of the numerical model is verified by comparison with the available results from the literature.

Key words: thermal buckling, imperfect plate, layer-wise model, finite element

1. Introduction

Plates are finding wide engineering applications in aerospace, automotive, marine and civil engineering structures. During manufacturing process or in service life many defects may arise which may influence the overall load bearing capacity of the structure. These defects or imperfections can be classified into two broad categories: initial geometrical imperfections and material or constructional imperfections. The initial geometrical imperfections include imperfections in structural configuration, such as small initial curvature in a flat plate as well as imperfections in the loading mechanisms, such as eccentricities. The material or constructional imperfections consist of cracks in general and are important for safe design of structures.

The initial geometric imperfections are in most cases randomly distributed in real structures. Therefore, introducing the geometric imperfections into the mathematical model is not an easy task, since their real shape is not known in advance. Even more it is not possible to determine the most unfavorable shape of imperfection, as concluded by Schneider et al. [2], since there is a dependency between the structural response (such as deflections, stresses, natural frequency or buckling loads) and the imperfection form and amplitude. Most of the studies, reported up to date were based on simplified assumption that the initial geometric imperfection has a similar form to the deformed shape of the plate. However, the limited number of investigations was dealing with general form of imperfections, the reason why in this paper a general geometric imperfection is studied.

Mathematical models of plate structures with geometric imperfections were mostly formulated for bending, buckling, post buckling, linear or nonlinear vibrations of plate and shell structures in both mechanical and thermal environment. The models are based on Classical Plate Theory (CPT), First-order Shear Deformation Theory (FSDT) or Higher-order Shear Deformation Theory (HSDT), implemented on both isotropic, laminated composite or Functionally Graded Plates (FGP) plates and shells. All these models may be divided into two broad categories, in which the first are able to include only sinusoidal mode of imperfection and the second, which are able to include the general form of imperfection into a mathematical model.

The sinusoidal form of imperfection was analyzed by Yamaki [3] and Timoshenko and Gere [4]. They developed analytical solutions for bending and post buckling of plates with symmetric initial imperfection. The effect of geometric imperfections on the post-buckling behavior of imperfect thin laminated plates under axial compression was studied by Hui [5]. The initial geometric imperfection was taken to be of the same form as the buckling mode. Chen at all. [6, 7] analyzed nonlinear vibration of isotropic and FGM plates having initial sinusoidal imperfection using CLPT. Girish and Ramachandra [8] analyzed thermal post buckling of composite plates with sinusoidal form of imperfections using HSDT and Galerkin procedure for the solution of governing equations. Mossavarali at all. [9, 10] used CLPT and HSDT to analyze thermal buckling of plates with sinusoidal imperfection. Shariat and Eslami [11, 12] also analyzed thermal buckling of FGMP plates using FSDT and HSDT utilizing only sinusoidal imperfection form. Tung and Duc [13, 14] investigated buckling of GFP plates with sinusoidal form of imperfection under in-plane compressive, thermal and combined loads using HSDT and Galerkin method of solution.

The general form of imperfection was analyzed by Nanda and Pradyumna [15]. They studied nonlinear free vibration and transient response of laminated shells, while using imperfection function capable of modeling variety of sine type, global type and localized type of imperfection. Kitipornchai at all. [16] Showed that vibration frequencies are very much dependent on the imperfection mode and its magnitude. Yang and Kitipornchai [17] investigated the sensitivity of post-buckling behavior of FGM plates to initial geometrical imperfections in general form. Yang and Huang [18] studied nonlinear transient response of simply supported imperfect FGP plates in thermal environment. An imperfection function is used to model general initial geometric imperfections including sin type, global type and localized type. Rafiee and Liew [19] analyzed nonlinear dynamic stability of initially imperfect piezoelectric FGM nanotube under combined thermal and electrical loadings with general form of imperfection. Gupta and Talha [20] investigated nonlinear flexural and vibration response of FGP plates with initial geometrical imperfections. The initial geometric imperfection has been incorporated using generic imperfection function.

From the previously cited references it may be concluded that there have not been reported any papers in which thermal buckling of plates with general form of imperfection have been studied using layer wise concept or Layer-Wise (LW) Theory of Reddy [1]. Moreover, most of the mentioned studies present the analytical solutions for imperfect plates, the reason why in this paper a finite element solution is presented. After establishing the accuracy of the present layer-wise model for linear and geometrically nonlinear bending, vibration and buckling analysis of laminated composite and sandwich plates subjected to mechanical load in the authors previous papers [24, 25, 26] as well as for thermal bending and buckling of laminated composite and sandwich plates [21, 22, 23], in this paper a thermal buckling analysis of imperfect plates is further investigated. The mathematical model assumes layer-wise variation of in-plane displacements and constant transverse displacement through the thickness of the plate, non-linear strain-displacement relations (in von Karman sense) and linear thermo mechanical material properties. The Koiter's model for imperfection is adopted. The Principle of virtual displacements (PVD) is used to derive the weak form of linearized buckling problem. The weak form is discretized using Lagrangian nine-node isoparametric finite element. The original MATLAB program is coded for finite element solution. The effects of imperfection amplitude, imperfection form, temperature distribution, side to thickness ratio and aspect ratio on critical temperature are analyzed. The accuracy of the numerical model is verified by comparison with the results from the author's previous papers and available results from the literature.

2. Theoretical Formulation

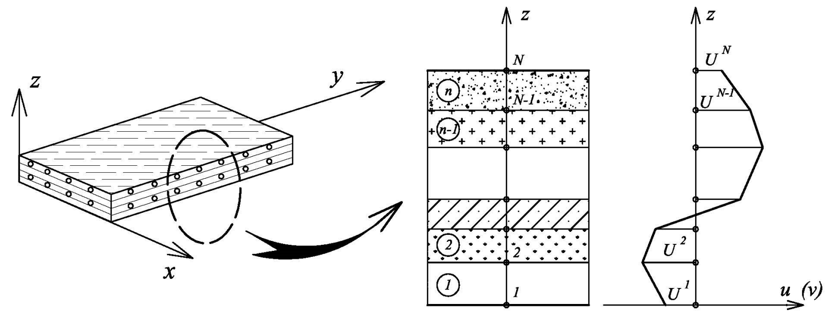

Fig. 1 Plate model and displacement field

A plate model is composed of n orthotropic layers. It is assumed that 1) layers are perfectly bonded together, 2) material of each layer is linearly elastic and has three planes of material symmetry (i.e., orthotropic), 3) strains are small, 4) each layer is of uniform thickness, 5) inextensibility of normal is imposed.

2.1 Displacement field

The displacements components (u_1, u_2, u_3) at a point (x, y, z) of plate are expressed as:

$$u_1(x,y,z) = u(x,y) + \sum_{I=1}^{N} U^I(x,y)\cdot \Phi^I(z)$$
$$u_2(x,y,z) = v(x,y) + \sum_{I=1}^{N} V^I(x,y)\cdot \Phi^I(z), \tag{1}$$
$$u_3(x,y,z) = w(x,y)$$

where (u, v, w) are displacements of a point $(x, y, 0)$ on the reference plane of the laminate, functions $\Phi^I(z)$ are one-dimensional linear Lagrange interpolation functions of thickness coordinates and (U^I, V^I) are the values of (u_1, u_2) at the I-th plane, Figure 1.

2.2 Strain-displacement relations

The strains associated with the displacement field (1) are computed using von Karman's non-linear strain-displacement relation and Koiter imperfection model:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \sum_{I=1}^{N} \frac{\partial U^I}{\partial x}\Phi^I + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial w_o}{\partial x},$$

$$\varepsilon_{yy} = \frac{\partial u_2}{\partial y}\frac{1}{2}\left(\frac{\partial u_3}{\partial y}\right)^2 = \frac{\partial v}{\partial y} + \sum_{I=1}^{N} \frac{\partial V^I}{\partial y}\Phi^I + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{\partial w}{\partial y}\frac{\partial w_o}{\partial y}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \sum_{I=1}^{N}\left(\frac{\partial U^I}{\partial y} + \frac{\partial V^I}{\partial x}\right)\Phi^I + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w_o}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial w_o}{\partial x}, \qquad (2)$$

$$\gamma_{xz} = \sum_{I=1}^{N} U^I \frac{d\Phi^I}{dz} + \frac{\partial w}{\partial x}$$

$$\gamma_{yz} = \sum_{I=1}^{N} V^I \frac{d\Phi^I}{dz} + \frac{\partial w}{\partial y}$$

where w_o is initial imperfection function, assumed in the following form:

$$w_o = \mu \cdot h \cdot sech\left[\delta_1\left(\frac{x}{a} - \psi_1\right)\right] \cdot cos\left[\mu_1\pi\left(\frac{x}{a} - \psi_1\right)\right] \cdot sech\left[\delta_2\left(\frac{y}{b} - \psi_2\right)\right] \cdot cos\left[\mu_2\pi\left(\frac{y}{b} - \psi_2\right)\right] \qquad (3)$$

where μ represents the amplitude of imperfection and varies between 0 and 1, δ_1 and δ_2 are the constants defining the localization degree of the imperfection symmetric about $x/a = \psi_1$ and $y/b = \psi_2$, μ_1 and μ_2 are the half-wave numbers of the imperfection in the x and y axis, respectively, while h is plate thickness. A variety of imperfection modes such as the sine type, the global type, and the localized type, can be described by this expression by varying different coefficients, listed in table 1. Figure 1 shows some typical imperfection shapes considered in this article for the analysis.

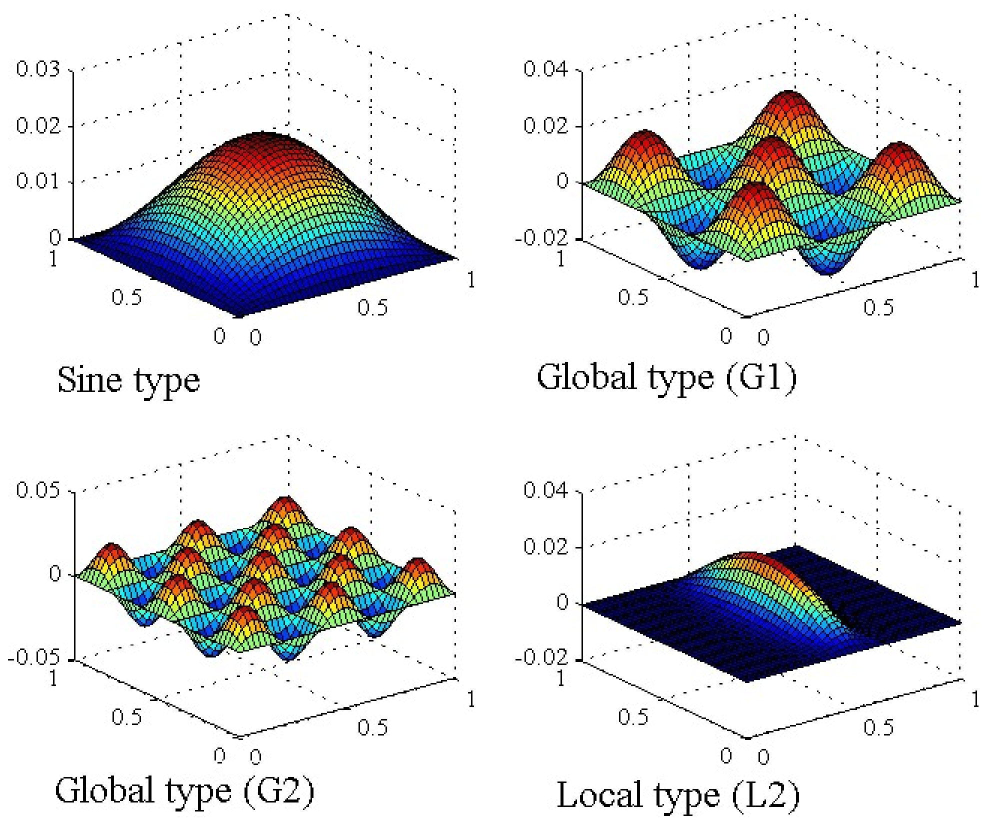

Fig. 2. Geometric imperfection modes

Types		Imperfection parameters
Sine type		$\delta_1 = \delta_2 = 0, \mu_1 = \mu_2 = 1, \psi_1 = \psi_2 = 0.5$
Global type	G1	$\delta_1 = \delta_2 = 0, \mu_1 = \mu_2 = 3, \psi_1 = \psi_2 = 0.5$
	G2	$\delta_1 = \delta_2 = 0, \mu_1 = \mu_2 = 5, \psi_1 = \psi_2 = 0.5$
	G3	$\delta_1 = \delta_2 = 0, \mu_1 = \mu_2 = 7, \psi_1 = \psi_2 = 0.5$
Local type	L1	$\delta_1 = 15, \delta_2 = 0, \mu_1 = 2, \mu_2 = 1, \psi_1 = 0.25, \psi_2 = 0.5$
	L2	$\delta_1 = 15, \delta_2 = 0, \mu_1 = 2, \mu_2 = 1, \psi_1 = 0.5, \psi_2 = 0.5$
	L3	$\delta_1 = 15, \delta_2 = 0, \mu_1 = 2, \mu_2 = 3, \psi_1 = 0.5, \psi_2 = 0.5$
	L4	$\delta_1 = 15, \delta_2 = 0, \mu_1 = 2, \mu_2 = 5, \psi_1 = 0.5, \psi_2 = 0.5$
	L5	$\delta_1 = 15, \delta_2 = 0, \mu_1 = 2, \mu_2 = 7, \psi_1 = 0.5, \psi_2 = 0.5$

Table 1. Geometric imperfection modes

2.3 Constitutive equations

An orthotropic linear Hook's material is assumed for each layer, to formulate constitutive equations as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & Q_{45} \\ 0 & 0 & 0 & Q_{45} & Q_{55} \end{bmatrix}^{(k)} \times \left(\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^{(k)} - \begin{Bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ \alpha_{xy} \\ 0 \\ 0 \end{Bmatrix}^{(k)} \cdot \Delta T \right) \tag{4}$$

where $\boldsymbol{\sigma}^{(k)} = \{\sigma_{xx} \ \sigma_{yy} \ \tau_{xy} \ \tau_{xz} \ \tau_{yz}\}^{(k)T}$ and $\boldsymbol{\varepsilon}^{(k)} = \{\varepsilon_{xx} \ \varepsilon_{yy} \ \gamma_{xy} \ \gamma_{xz} \ \gamma_{yz}\}^{(k)T}$ are stress and strain components respectively, $\boldsymbol{Q}_{ij}^{(k)}$ and $\alpha^{(k)} = \{\alpha_{xx} \ \alpha_{yy} \ \alpha_{xy} \ 0 \ 0\}^{(k)T}$ are transformed reduced elastic stiffness [25] and coefficients of thermal expansion in global coordinates, while ΔT is temperature rise.

2.4 Temperature rise

The temperature rise for thermal buckling problem analyzed in this paper, includes uniform temperature rise and linear temperature rise. The uniform temperature rise assumes that the plate initial temperature is T_i. The temperature is uniformly raised to a final value T_f in which the plate buckles. The temperature change is then $\Delta T = T_f - T_i$. The linear temperature rise through the thickness is assumed as:

$$\Delta T(z) = \frac{\Delta T}{h}\left(z + \frac{h}{2}\right) + T_{bott} \tag{5}$$

where z is the coordinate variable in the thickness direction measured from the middle plane of the plate, and temperature difference is $\Delta T = T^{top} - T^{bott}$.

2.5 Governing equations and boundary conditions

The governing equations of the present LW theory are derived using the Principle of virtual displacements (PVD). After performing the integration in the thickness direction the internal work and external work due to in-plane thermal forces become:

$$\delta V+\delta U=\int_{\Omega}\left[\left\{\delta\varepsilon_{0L}\right\}^{T}\cdot\{N\}+\left\{\delta\varepsilon^{I}\right\}^{T}\cdot\left\{N^{I}\right\}+\left\{\delta\varepsilon_{0NL}\right\}^{T}\cdot\left\{N_{T}^{0}\right\}\right]d\Omega=0 \tag{6}$$

where $\{N\}=\{N_{xx}\ N_{yy}\ N_{xy}\ Q_{xz}\ Q_{yz}\}^{T}$, $\{N^{I}\}=\{N_{xx}^{I}\ N_{yy}^{I}\ N_{xy}^{I}\ Q_{xz}^{I}\ Q_{yz}^{I}\}^{T}$ an $\{N_{T}^{0}\}=\{N_{Txx}^{0}\ N_{Tyy}^{0}\ N_{Txy}^{0}\ 0\ 0\}^{T}$ are the stress resultants in middle and I-th plane, respectively.

3. Finite element solution

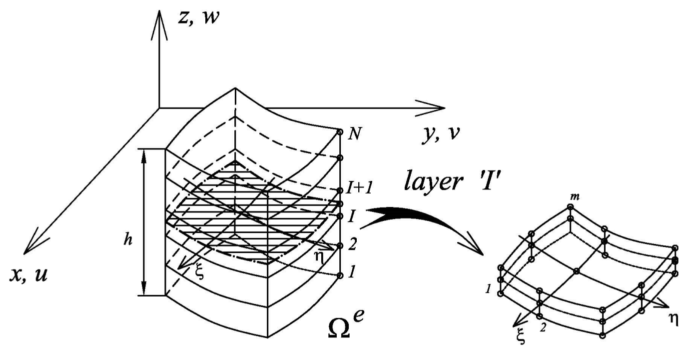

Fig. 3. Finite element model

3.1 Displacement field

The GLPT finite element consists of middle surface plane and I=1, N planes through the thickness of the plate, Figure 2. The element requires only the C^0 continuity of major unknowns, thus in each node only a displacement components are adopted, that are (u,v,w) in the middle surface element nodes and (U^I,V^I) in the I-th plane element nodes. The generalized displacements over finite element Ω^e are expressed as:

$$\begin{Bmatrix}u\\v\\w\end{Bmatrix}^{e}=\begin{Bmatrix}\sum_{j=1}^{m}u_j\Psi_j\\\sum_{j=1}^{m}v_j\Psi_j\\\sum_{j=1}^{m}w_j\Psi_j\end{Bmatrix}^{e}=\sum_{j=1}^{m}\left[\mathbf{\Psi}_j\right]^e\left\{\mathbf{d}_j\right\}^e,\quad \begin{Bmatrix}U^I\\V^I\end{Bmatrix}^{e}=\begin{Bmatrix}\sum_{j=1}^{m}U_j^I\,\Psi_j\\\sum_{j=1}^{m}V_j^I\,\Psi_j\end{Bmatrix}^{e}=\sum_{j=1}^{m}\left[\overline{\mathbf{\Psi}}_j\right]^e\left\{\mathbf{d}_j^I\right\}^e. \tag{7}$$

where $\{\mathbf{d}_j\}^e=\{u_j^e\ \ v_j^e\ \ w_j^e\}^T$, $\{\mathbf{d}_j^I\}^e=\{U_j^I\ \ V_j^I\}^T$ are displacement vectors in the middle plane and I-th plane, respectively, and Ψ_j^e are interpolation functions, for the j-th node of the element Ω^e, while $[\Psi_j]^e$ and $[\overline{\Psi}_j]^e$ are given in [25].

3.2 Geometry of the element

The geometry of the element is interpolated with the same interpolation functions over the element Ω^e, as the generalized displacements, thus isoparametric finite element formulation is adopted.

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}^e = \begin{Bmatrix} \sum_{j=1}^{m} x_j \Psi_j \\ \sum_{j=1}^{m} y_j \Psi_j \\ \sum_{j=1}^{m} z_j \Psi_j \end{Bmatrix}^e . \tag{8}$$

where $\{x_j \; y_j \; z_j\}$ are $\{x \; y \; z\}$ coordinates of j-th node of the element Ω^e.

4. Numerical results and discussion

Using previously derived finite element solution, an original computer program was coded using MATLAB programming language, for thermal buckling of imperfect plates. The parametric effect side to thickness ratio a/h, imperfection amplitude and imperfection form on critical temperature are analyzed.

Example 1: Plate is simply supported ($a/b = 1$) and made of material with following material constants: $E = 1\,GPa,\ \nu = 0.21,\ \alpha = 1\cdot 10^{-6}\, 1/{}^{0}C,\ \Delta\overline{T}_{cr} = \Delta T_{cr}\cdot\alpha$

μ	$a/h = 2$		$a/h = 10$		$a/h = 20$		$a/h = 100$	
	CFS[21]	FEM	CFS[21]	FEM	CFS[21]	FEM	CFS[21]	FEM
0	0.1523	0.1665	$0.1295\,10^{-1}$	$0.1306\,10^{-1}$	$0.3356\,10^{-2}$	$0.3371\,10^{-2}$	$0.1359\,10^{-3}$	$0.1367\,10^{-3}$
0.1	0.1715	0.1695	$0.1356\,10^{-1}$	$0.1318\,10^{-1}$	$0.3458\,10^{-2}$	$0.3400\,10^{-2}$	$0.1391\,10^{-3}$	$0.1379\,10^{-3}$
0.2	0.1908	0.1779	$0.1417\,10^{-1}$	$0.1354\,10^{-1}$	$0.3560\,10^{-2}$	$0.3490\,10^{-2}$	$0.1423\,10^{-3}$	$0.1415\,10^{-3}$
0.3	0.2102	0.1907	$0.1479\,10^{-1}$	$0.1411\,10^{-1}$	$0.3661\,10^{-2}$	$0.3638\,10^{-2}$	$0.1456\,10^{-3}$	$0.1474\,10^{-3}$
0.4	0.2295	0.2056	$0.1540\,10^{-1}$	$0.1495\,10^{-1}$	$0.3762\,10^{-2}$	$0.3843\,10^{-2}$	$0.1488\,10^{-3}$	$0.1556\,10^{-3}$
0.5	0.2488	0.2198	$0.1602\,10^{-1}$	$0.1598\,10^{-1}$	$0.3864\,10^{-2}$	$0.4104\,10^{-2}$	$0.1521\,10^{-3}$	$0.1661\,10^{-3}$

Table. 2.Critical buckling temperature under uniform temperature rise versus imperfection size

Table 2 shows variation of critical temperature ΔT_{cr} of square (a/b=1) simply supported plate under uniform temperature rise as a function of imperfection size μ and plate thickness ratio a/h. It is shown that ΔT_{cr} increases with the increase on imperfection size and is greater for thicker compared to thin plates, as expected. The results obtained with the present finite element model are in good agreement with closed form solution [21].

Example 2: Plate is simply supported ($a / h = 100, a / b = 1$) and made of material with following material constants: $E = 2\ GPa, \nu = 0.3, \alpha = 2 \cdot 10^{-6} 1 / {}^{0}C, \Delta\overline{T}_{cr} = \Delta T_{cr}$

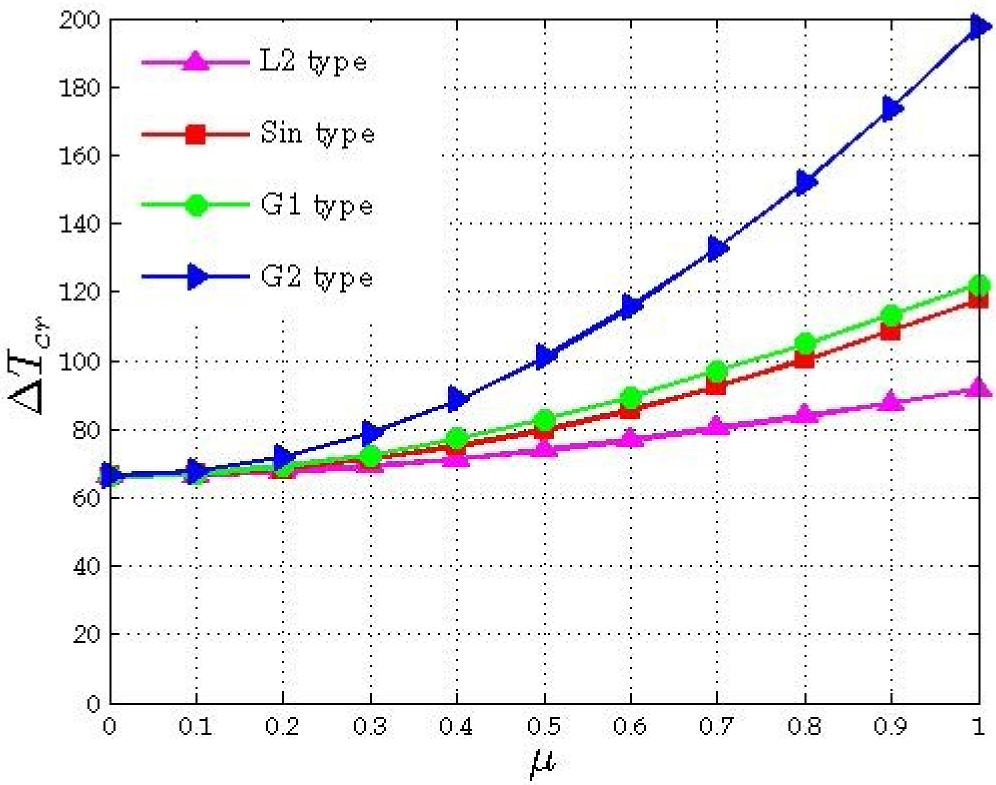

Fig. 4. Critical buckling temperature under uniform rise versus a) b/a and b) b/h ratio

Figure 4 shows variation of critical temperature ΔT_{cr} of thin (a/h=100) simply supported plate under uniform temperature rise, as a function of imperfection size μ, for different imperfection modes (sin type, local type and global type). It may be observed that critical temperature is most sensitive to global G2 imperfection mode, and less sensitive to local L2 imperfection mode. As reported from the previous example, critical temperature increases with increase of imperfection size μ.

5. Conclusion

The present study extends the previous works on linear and nonlinear thermo mechanical bending, buckling and vibrations of perfect plates [23, 24, 25, 26], to thermal buckling of imperfect plates, using Koiter's model of imperfection. The finite element solution is derived for thermal buckling using LW theory of Reddy [1]. The buckling temperature obtained with the present LW solution is compared with the closed form solutions of LW [21] model from the previous paper. It may be concluded that present model gives acceptable critical temperatures for thick and thin, perfect and imperfect plates and may serve as a benchmark for further investigations.

Acknowledgment The author is thankful for the financial support received by Ministry of Education and Science of the Republic of Serbia, Project TP 36048.

References

[1] J.N. Reddy, E.J. Barbero, J.L. Teply, *A plate bending element based on a generalized laminated plate theory,* International Journal for Numerical Methods in Engineering, Vol. 28, 2275-2292, 1989.

[2] W. Schneider, I. Timmel, K. Höhn, *The conception of quasi-collapse-affine imperfections: a new approach to unfavorable imperfections of thin-walled shell structures*, Thin-Wall Struct. Vol 43(8), 1202–24, 2005.

[3] N. Yamaki, *Post buckling behavior of rectangular plates with small initial curvature loaded in edge compression*, J Appl Mech Trans ASME, Vol. 26, 407–14, 1959.

[4] S.P. Timoshenko, J.M. Gere, *Theory of elastic stability*, 2nd ed, New York, McGraw-Hill, 1961.

[5] D. Hui*, Imperfection sensitivity of axially compressed laminated flat plates due to bending-stretching coupling*, International Journal of Solids and Structures, Vol. 22(1), 13-22, 1986.

[6] C. Chena, W. Chenga, A. Tan, *Non-linear vibration of initially stressed plates with initial imperfections*, Thin-Walled Structures, Vol. 43, 33–45, 2005.

[7] C. Chena, A. Tan, *Imperfection sensitivity in the nonlinear vibration of initially stresses functionally graded plates*, Composite Structures, Vol. 78, 529–536, 2007.

[8] J. Girish, L.S. Ramachandra, *Thermo mechanical post buckling analysis of symmetric and antisymmetric composite plates with imperfections*, Composite Structures**,** Vol. 67(4), 453-460, 2005.

[9] A. Mossavarali, G.P. Saheli, M.R. Eslami. *Thermo elastic buckling of isotropic and orthotropic plates with imperfections*, J Thermal Stress, Vol. 23, 853-72, 2002.

[10] A. Mossavarali, M. R. Eslami*, Thermo elastic buckling of plates with imperfections based on higher order displacement field*, J Thermal Stress, Vol. 25(8), 745-71, 2002.

[11] B. A.S. Shariat, M.R. Eslami, *Effect of initial imperfection on thermal buckling of functionally graded plates*, J Thermal Stress, Vol. 28, 1183-98, 2005.

[12] B. A. S. Shariat, M.R. Eslami, *Thermal buckling of imperfect functionally graded plates*, Int. J. Solids Struct., Vol. 43, 4082-4096, 2006.

[13] H. Tung, N. D. Duc, *Thermo elastic Stability of Thick Imperfect Functionally Graded Plates*, Vietnam Journal of Mechanics, VAST, Vol. 32(1) ,47 – 58, 2010.

[14] H. Tung, N. D. Duc., *Nonlinear analysis of stability for functionally graded plates under mechanical and thermal loads*, Composite Structures, Vol. 92, 1184–1191, 2010.

[15] N. Nanda, S. Pradyumna, *Nonlinear dynamic response of laminated shells with imperfections in hydrothermal environments*, Journal of Composite Materials, Vol. 45(20), 2103–2112, 2011.

[16] S. Kitipornchai,, J. Yang, K.M. Liew, *Semi-analytical solution for nonlinear vibration of laminated FGM plates with geometric imperfections*, International Journal of Solids and Structures, Vol 41, 2235–2257, 2004.

[17] J. Yang, K.M. Liew, S. Kitipornchai, *Imperfection sensitivity of the post-buckling behavior of higher-order shear deformable functionally graded plates*, International Journal of Solids and Structures, Vol. 43, 5247–5266, 2006.

[18] J. Yang, X. Huang, *Nonlinear transient response of functionally graded plates with general imperfections in thermal environments*, Comput. Methods Appl. Mech. Engrg, Vol. 196, 2619–2630, 2007.

[19] M. Rafiee , X.Q.He , K.M. Liew, *Non-linear dynamic stability of piezoelectric functionally graded carbon nanotube-reinforced composite plates with initial geometric imperfection*, International JournalofNon-LinearMechanics, Vol. 59, 37–51, 2014.

[20] A. Gupta, M. Talha, *Nonlinear flexural and vibration response of geometrically imperfect gradient plates using hyperbolic higher-order shear and normal deformation theory*, Composites Part B, Vol. 123, 241-261, 2017.

[21] M. Cetkovic, *Imperfection Sensitivity of Plates in Thermal Environment*, 6th International Congress of Serbian Society of Mechanics Mountain Tara, Serbia, June 19-21, 2017.
[22] M. Cetkovic, *Thermal Buckling of Laminated Plates Using Layerwise Displacement Model*. Composite Structures, Vol. 142, 238-253, 2016.
[23] M. Cetkovic, *Thermo-Mechanical Bending of Laminated Composite and Sandwich Plates Using Layerwise Displacement Model*. Composite Structures, Vol. 125, 388–399, 2015.
[24] M. Cetkovic, Vuksanovic Đ. *Large Deflection Analysis of Laminated Composite Plates using Layerwise Displacement Model*, Structural Engineering and Mechanics, Vol. 40(2), 257-277, 2011.
[25] M. Cetkovic, *Nonlinear Behavior of Laminated Composite Plates*. In: PhD Thesis (in serbian), Faculty of Civil Engineering in Belgrade, Serbia, 2011.
[26] M. Cetkovic, Vuksanovic Đ. *Bending, Free Vibrations and Buckling of Laminated Composite and Sandwich Plates Using a Layerwise Displacement Model*, Composite Structures, Vol. 88(2), 219-227, 2009.