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INVESTIGATING COMPOSITE POLYHEDRAL FORMS OBTAINED BY COMBINING CONCAVE CUPOLAE OF II SORT WITH ARCHIMEDEAN SOLIDS

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Abstract

Concave cupolae of II sort, combined with the other concave or convex polyhedra with at least one matching side, provide many possibilities for the formation of various composite polyhedra. The paper presents research on regular –faced polyhedral structures obtained by joining bases of some concave cupolae of II sort, with the appropriate sides of Archimedean solids: truncated cube, truncated dodecahedron and great rhombicosidodecahedron.

Key words: concave cupola, Archimedean solids, polyhedron, cluster

1. INTRODUCTION

The structural efficiency, attractiveness of the architectural form and the entire appearance of buildings is becoming an important area of modern engineering, especially in virtue of the wider availability of computer tools. The attainment of very complex forms are becoming feasible, which could not be achieved by traditional means. This paper describes the origination of certain structural forms that use a distinctive geometric patterns based on some of uniform polyhedra (Archimedes' solids) and the geometry of the concave cupolae of II sort [7] [8] [9]. These polyhedra share a common property of being regular-faced, so congruent faces can be joined, creating complex polyhedra and cluster polyhedra.

Most of the buildings we see today, contain some of geometry of the polyhedral structures, whether a simple forms (prism, cube) or complex forms of spatial grids, using the geometry of the Platonic solids. Architectural use of these patterns and their impact on our environment is of great importance. They can define the general shape of the building as well as its internal configuration.

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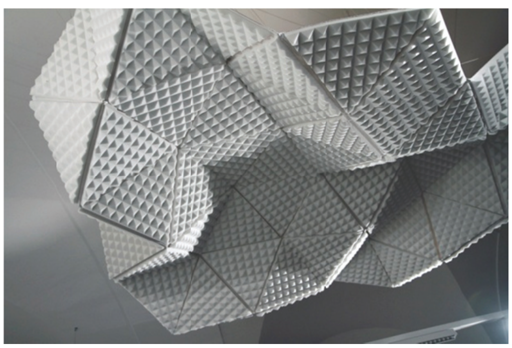
Polyhedral shapes are also often combined to form conglomerates, such as the spatial grid.

In some previous research of using polyhedral shapes in construction [3], [4], [5], [6], [10], [12], [14] the priority was mainly given to the convex polyhedral forms, which are for many reasons a natural choice for the building structures, due to its stability, efficiency and rationality (lower ratio of surface area / volume). However, we are witnessing even the growing application of concave polyhedral forms in recent years. Fig. 1 provides a few illustrations of the above, through the examples of the application of these forms in so-called “acoustical clouds” [14], or as carrier of the entire structure form [16], [17], [10]. The first two examples show the use of concave polyhedral surface obtained by folding the isosceles triangular net, which is to be used as a basic principle of forming the concave cupola of II sort.

If we take into consideration the group of polyhedra with tested static rigidity (such as concave cupolae of II sort [7] - [9]) it gives a wider and safer field for research in this area.



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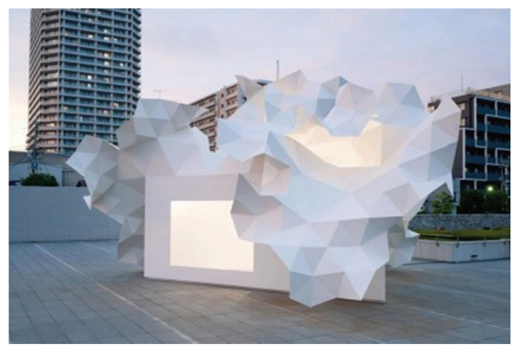

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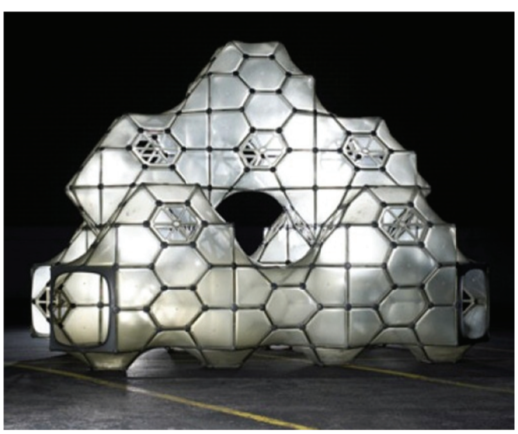

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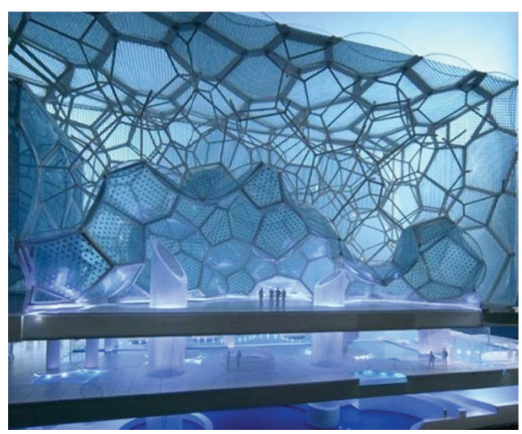

d)

Figure 1: The Examples of concave polyhedral forms application in architecture: a) acoustical cloud [14], b) Bloomberg Pavilion [16], c) curved space system [17], d) swimming pool interior [10]

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We shall not exempt neither an aesthetic effect that the unusual, faceted, dynamic, crystalline forms make to the viewer. Actually, among these heterogeneous and fragmented forms of sharp angles and pointed shapes, we recognize some of the characteristics of Gothic architecture, which has made some of the world's most beautiful buildings of architectural heritage.

1.1. Composite polyhedra

In this paper we elaborate the formation of composite regular–faced concave polyhedra, because the possible types of convex, regular-faced polyhedra (Platonic solids, Archimedean solids, family of prisms, antiprisms, Johnson's solids, and variations of Johnson's solids) are already examined [14], [18], [19], [20]. The final list of all convex regular-faced polyhedra are published by A. V. Timofeenko [13] 2009th.

He gave a strict definition of composite polyhedra:

If a convex polhedron with regular faces can be divided by somea plane into two polyhedra with regular faces, then it is said to be composite,

where it is clear that this term applies only to convex polyhedra, while P. Huybers [4], [5] and D.G. Emmerich [2] named as composite polyhedra even those that originate from the augmentation of uniform polyhedra by (Johnson's) cupolae, which are not convex. Thus, we accepted the second stand.

Concave polyhedra that originate in an analogous manner as the composite convex polyhedra, with congruent matching faces, are commonly referred to as: cluster polyhedra, polyhedral arrangements, close-packs or augmented polyhedra. The cluster polyhedra will be also discussed in this paper.

AUGMENTATIONS

Augmented polyhedron is a uniform polyhedron with one or more other solids adjoined [21].

At the regular faces of uniform polyhedron, we can subjoin other figures with identical bases to the respective face of the polyhedron. Most often the augmentation involves adding pyramids (regular-faced, though not necessarily) (P1, J1, J2), cupolae (J3, J4, J5) or pentagonal rotunda (J6), [18], [19], [20].

In this paper, instead of these solids, we will add concave cupolae of II sort that share the same bases (faces) with uniform polyhedra - Archimedean solids:

a) truncated cube (U9), to which we add a quadratic concave cupola II (CC II-4)

b) truncated dodecahedron (U26) to which we add pentagonal concave cupola II (CC II-5)

c) great rhombicosidodecahedron (U28) to which we add pentagonal concave cupola II (CC II-5).[10]

[10] We deliberately omitted Great Rhombicuboctahedron - (U 11) due to the limited scope of the paper, and a great similarity to the presented cases.

1.2. Truncated cube and square concave cupola of II sort

Archimedean solids are uniform polyhedra with regular polygons as their faces, while their vertices are identical. This implies that these solids have a high degree of planar and central symmetry. There are a total of 13 solids (Walsh 1972, Ball and Coxeter 1987.), some of them are derived from Platonic solids (by truncations, expansions, moving and twisting of faces), and two Archimedean solids are originated by expansion of other Archimedean solids [22].

Truncated cube (Fig. 2) is a solid that belongs to octahedral symmetry group. It originates by such a truncation of cube which gives a 3D figure consisted of regular octagons and triangles. Octagonal sides, as for the cube, are set in the orthogonal disposition, which gives a certain familiarity of this form with Cartesian geometry of space in which we are accustomed. These faces are joined by eight equilateral triangles. We will use just the octagonal faces of truncated cube, in order to form a composite, augmented polyhedron by joining it the octagonal bases of square concave cupola of II sort.

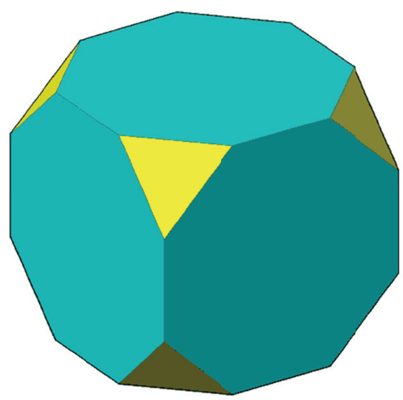

Figure 2: Truncated cube

Square concave cupola of II sort (Fig. 3) is a polyhedron formed by connecting two regular polygons, n-gon and 2n-gon (adopted bases of the solid) in parallel planes, by two-row of equilateral triangles which make lateral surface. The initial n-sided polygon is square in this case, and its parallel and co-axial 2n-sided polygon is octagon.

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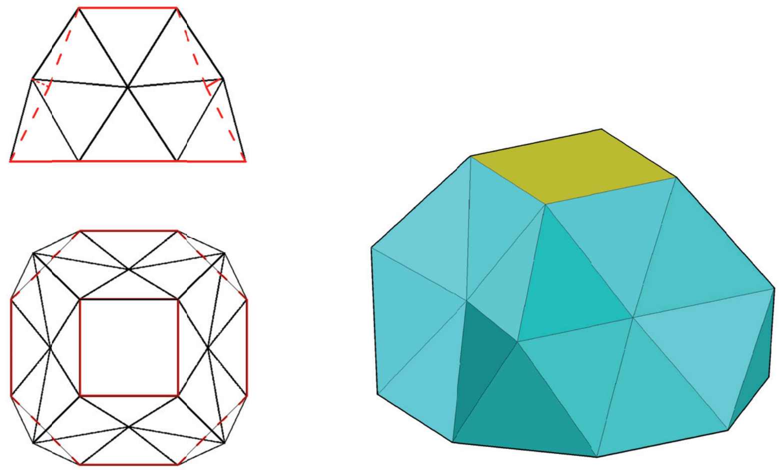
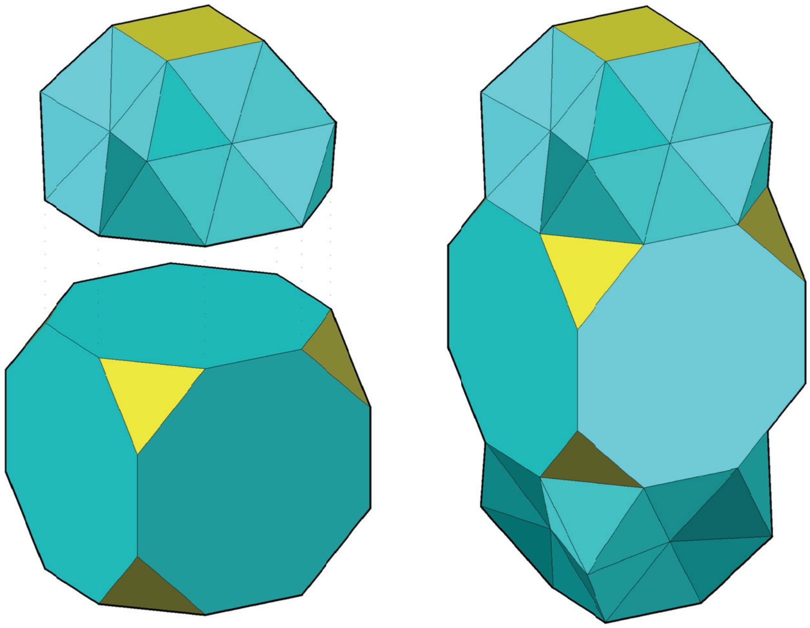


a) b)

Figure 3: Square concave cupola of II cort: a) front and top view b) axonometric view

The plane net of the lateral surface of these polyhedra is a segment of a equilateral triangular net, the folding and creasing of which provide an array of n (= 4) spatial concave equilateral hexa-triangular cells (CEHTC) - Flexible, mechanical, triangular units, by which polar array the concave cupola of II sort (CC II-4 in further text) is formed. These cells are connected by equilateral triangles, just like the octagonal faces of truncated cube (TC in further text).

Combining the identical octagonal faces of these two solids (Fig. 4a), a composite polyhedron arises, which we call the *concaugmented* Truncated cube. If we add another CC II-4 to the opposite (parallel) face of the same Archimedean solid, we get parabi-concaugmented TC (Fig. 4b). Rotating CC II-4 for the angle ($2\pi / 2n = 2\pi / 8 = 45°$), the triangular faces which were attached to the octagonal faces of TC, now will be attached to the triangular and vice versa, and this solid can be called, modeled on [13], [18] [19], [20]: gyrate concaugmented TC.



a) b)

Figure 4: a) Joining the congruent faces of polyhedra, b) parabi–concaugmented truncated cube

Accordingly, augmentations of TC by CC II-4 may be named as shown in **Table 1.** Thus, we have 9 basic types of composite polyhedra obtained by adding CC II-4 and a total of 56 different composite polyhedron. If we include in the calculus another type of CC II-4 with a counter-folded edges, which gives a minor height of the lateral shell, this number of augmented composite polyhedra would be not only doubled, but with cross combination of these two types of CC II it would raise to a much larger number of polyhedra.

	Number of CC II augmen ted	**Term (basic type)**	**Variations of the type**	**Total of type's cases**
1	1	**Concaugmented TC**	*gyrate*	2
2	2	On parallel faces: **parabi–concaugmented TC**	*gyrate+ bigyrate (+2)*	3
3	2	On adjecent faces: **metabi–concaugmented TC**	*gyrate+ bigyrate (+2)*	3
4	3	With axes in the same plane: **parametatri-concaugmented TC**	*gyrate (E, M)) + parabigyrate + metabigyrate + trigyrate* (+5)	6

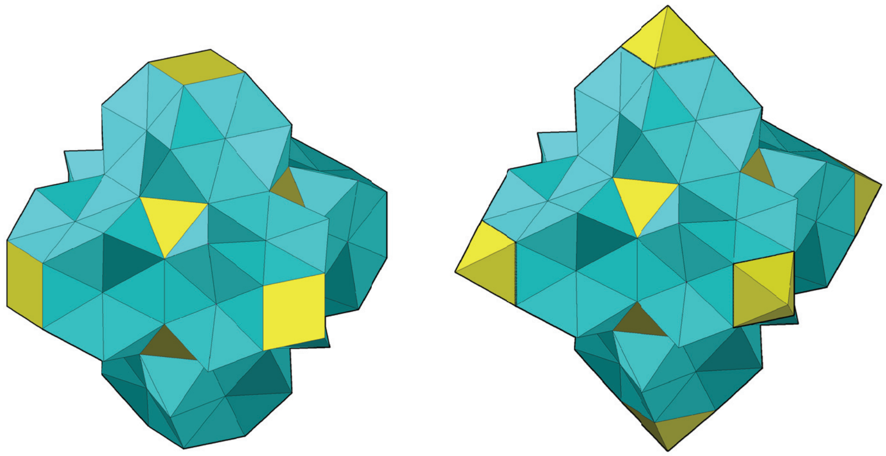
5	3	With axes in orthogonal trihedron disposition: **orthometatri-concaugmented TC** (L and R)	*gyrate* + *bigyrate* + *trigyrate* (+3)	4
6	4	With co-planar perpendicular axes : **metaparaquadri-concaugmented TC**	*gyrate* + *parabigyrate* + *metabigyrate* + *trigyrate* + *quadragyrate* (+5)	6
7	4	With 3 co-planar and 1 orthogonal axis: **metatripara-concaugmented TC**	*gyrate (E, M)* + *parabigyrate* + *egzometabigyrate (L i R)* + *mezometabigyrate* + *trigyrate* + *ortotigyrate (L, R)* + *quadragyrate* (+10)	11
8	5	With 4 co-planar and 1 orthogonal axis: **quint-concaugmented TC**	*gyrate (E, M)* + *parabigyrate* + *egzometabigyrate* + *mezometabigyrate* + *trigyrate* + *ortotigyrate* + *quadragyrate* + *ortoquadragyrate* + *quintagyrate* (+10)	11
9	6	**hexa-concaugmented TC**	*gyrate* + *parabigyrate* + *metabigyrate* + *trigyrate* + *ortotigyrate* + *quadragyrate* + *ortoquadragyrate* + *quintagyrate* + *hexagyrate* (+9)	10
		total		**56**

Table 1: Overview of the basic types of TC augmentations of and their variations

This paper includes also a providing suggestions of the combining possibilities of CC II and Archimedean solids, in order to examine the potential of applying some specific cases in construction and architecture.

Consider the example of parabi-concaugmented TC given in Fig. 4b. The solid in such a disposition is rested on the square base of CC II-4, the lower unit acts as of ground-level entrance area - the foyer, the basic premise is the very volume of the truncated cube, while the higher CC II-4 acts as a dome/ cupola in the architectural sense of the word. Given that the horizontal square face of the cupola covers the top of the polyhedron space, it is also possible for this polygon to be augmented by quadrilateral pyramid, which (along with other slope faces of CC II-4) assumes a role of drainage, such as roof planes. The dome area and the foyer area are possible to be divided into two floors, depending on the edge size (hypothetical edge of a triangle a = 5m), while the main space of TC can be divided into three floors. Thus, this structure would contain a total of 7 floors (without tower). Due to the orthogonality of the solids geometry, Archimedes' TC, this facility can lean with its faces on the neighboring buildings in the line, on the corner, as it can stand alone.

Another variant of modeling polyhedral structure of the proposed two solids is given in Fig. 5.



a) b)

Figure 5: a) Hexa-concaugmented TC and b) its new augmentation by pyramids

The Fig. 5 shows the hexa-concaugmented TC, with the same spatial units as above, except for the four new CC II-4 that are now added on the lateral faces of the TC. These new volumes, set with the axes in a horizontal position, will form a central area of the building, which branches into four distinct units – choncal halls. The edges of the basic squares of CC II-4 correspond to the altitudes of octagon's diagonals, obtained by dividing it into three segments, so the four new cupolae can be divided accordingly. The lower thirds of newly-added units, due to the inclined planes that make this "hall", can be used to accommodate installations, etc., while middle and upper thirds can be assembled in one spatial entity, the purpose of which may be different.

Also, these "choncal halls" can be individually used as multi-purpose halls, meeting rooms, auditoriums and amphitheaters, where the inclined lower surfaces can be used as an advantage - sloping floor surface suitable for seats.

Square faces of the lateral cupolae can be left as window panes, or again, augmented by quadrilateral pyramids. In this manner, the whole structure could be composed of the same elements - equilateral triangles (Fig. 5b), which is of a great importance in the unification of the constituent units, i.e. for application of prefabricated triangular elements.

1.2.1. Cluster polyhedra

Such complex polyhedra, as in the previous examples, can be combined further on to a form of polyhedral cluster or conglomerate. An example of such an assemblage is shown in Fig. 6 (orthogonal projection) and in Fig. 7 (axonometric view).

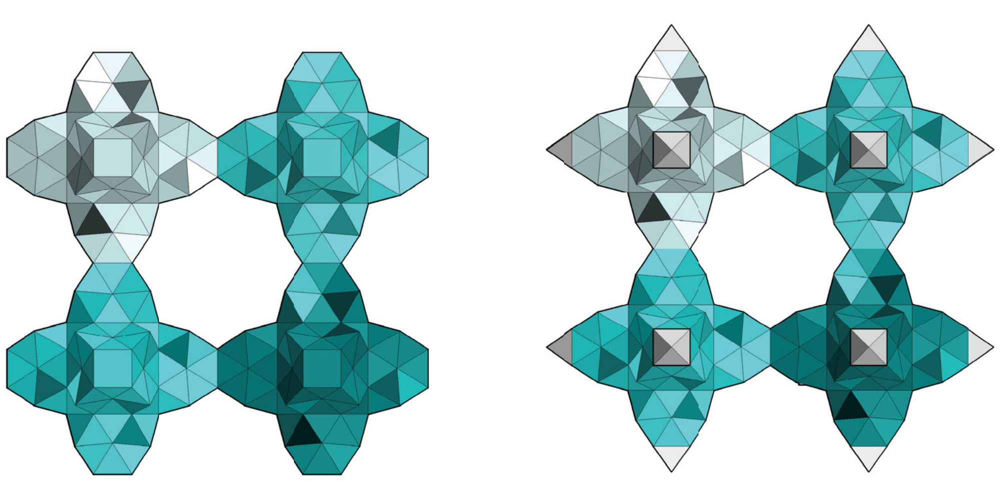


Figure 6: The example of cluster polyhedron, formed by joining eight composite polyhedra from Fig. 5

Eight of these composite polyhedra are connected with its square faces in the orthogonal disposition. This composition is multi-symmetric. It has three planes of symmetry, and is also centrally symmetric. Since this is not a close-packing, in the interspaces there will be uncovered, open passage – seen in the top view as a non-convex polygon - octagram.

From the viewpoint of applications in architectural and urban practice, this open tract also plays an outstanding role in the possibility of supporting the organization of space, green areas, squares, fountains and the like.

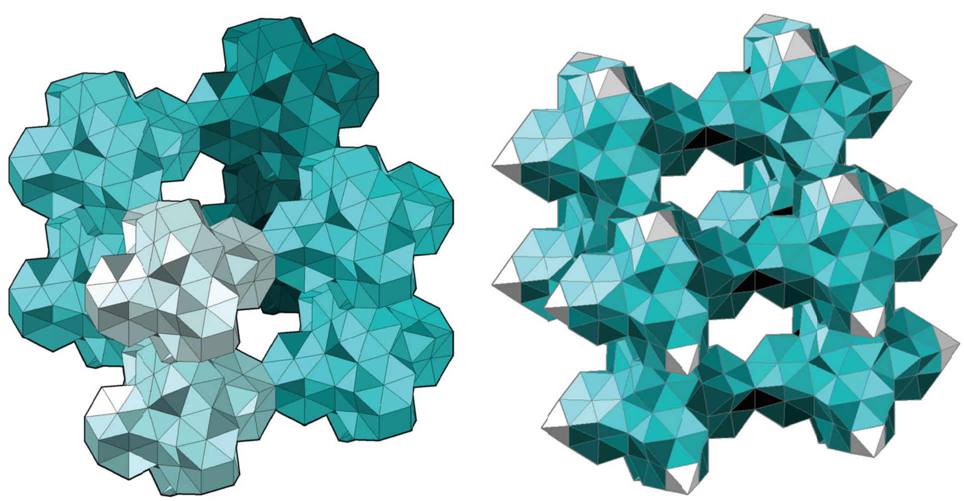


Figure 7: The axonometric view of the cluster polyhedron from the Fig. 6

1.3. Truncated dodecahedron and pentagonal concave cupola of II sort

Consider, analogous to the previously exposed combination of TC and CC II-4, a way of joining pentagonal concave cupolae of II sort (CC II-5) with another Archimedean solid - truncated dodecahedron.

Truncated dodecahedron (TD in further text) (Fig. 8) is a solid that belongs to icosahedral symmetry group. It originates by such a truncation of dodecahedron which gives a 3D figure consisted of regular decagons (12) and triangles (20). Decagonal faces are connected by equilateral triangles. If we use the decagonal faces for augmentations by the pentagonal Concave cupolae of II sort (CC II-5, in further text), we will form new composite, augmented polyhedra.

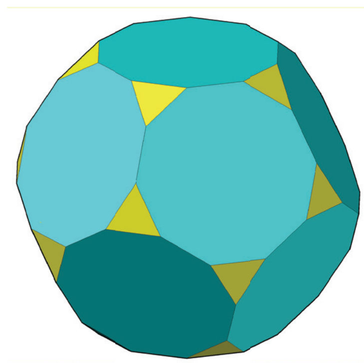
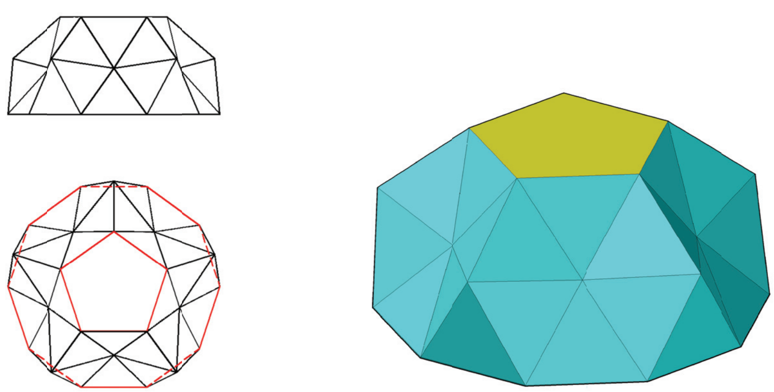


Figure 8: Truncated dodecahedron

Pentagonal concave cupola of II sort (CC II-5) is a polyhedron obtained in the same way as CC II-4, by connecting two polygons in parallel planes: the initial n-sided polygon, pentagon, and a parallel and co-axial 2n-sided polygon – decagon, by the two row sequence of equilateral triangles. An image of this polyhedron is shown in Fig. 9.



a) b)

Figure 9: Pentagonal cupola of II sort: a) top and front view b) axonometric view

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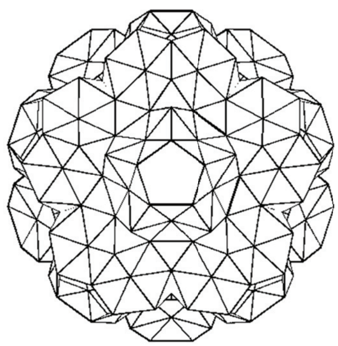
Combining the matching decagonal faces of these two solids, forms a composite polyhedron which we call: *concaugmented truncated dodecahedron*.

If we add now several CC II-5 on the other faces of the TD, we get concaugmentations of TD, similar to those listed in **Table 1**, except for the number of combinations which would be now even higher, according to a number of TD faces.

In Fig. 10, we give only an example of concaugmented truncated dodecahedron, which is further augmented by five-sided, regular pyramids, so that the entire composite polyhedron is composed of equilateral triangles.



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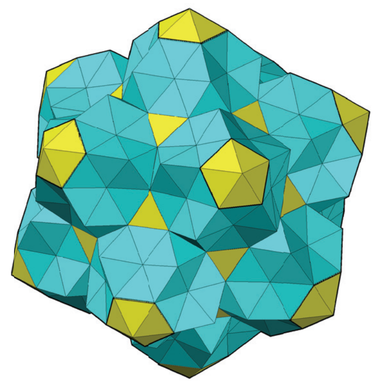


b)

Figure 10: Dodeca-concaugmented truncated dodecahedron: a) top view and b) axonometric view with pyramidal augmentations

1.4. Great Rhombicosidodecahedron and Pentagonal Concave Cupola of II Sort

Great rhombicosidodecahedron (otherwise known as: truncated icosidodecahedron) is an Archimedean solid that belongs to icosahedral symmetry group. As its faces, there appear regular decagons (12), hexagons (20) and squares (30). Decagonal sides are connected by a series of alternating hexagons and squares, as shown in Fig. 11. If we use the decagonal faces again to add the equivalent bases of CC II-5 we will form additional augmented polyhedra.

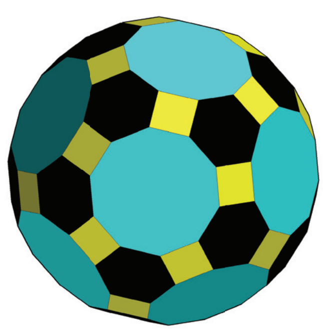
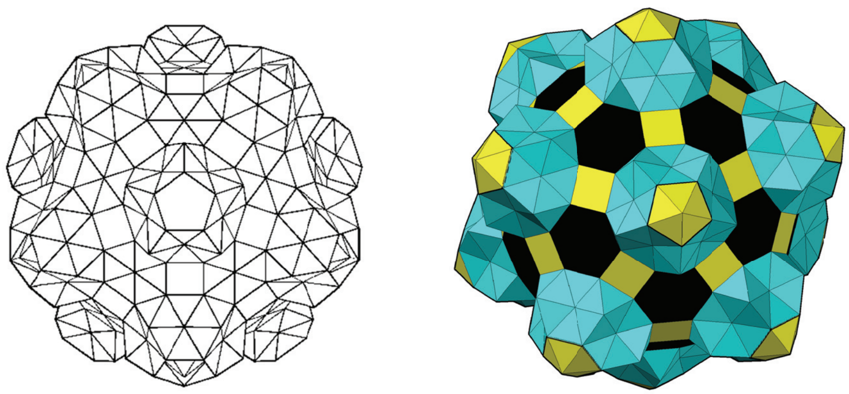


Figure 11: Great rhombicosidodecahedron

In Fig. 12, an example is given, which shows dodeca-concaugmented great rhombicosidodecahedron, which is further augmented by five-sided regular pyramids. Thus, the composite polyhedron is obtained, composed of equilateral triangles, squares and equilateral hexagons. These hexagons can be further triangulated, so that this structure would be consisted only of triangles and squares (Figure 13).



a) *b)*

Figure 12: Dodeca-concaugmented great rhombicosidodecahedron: a) top view and b) axonometric view with pyramidal augmentations

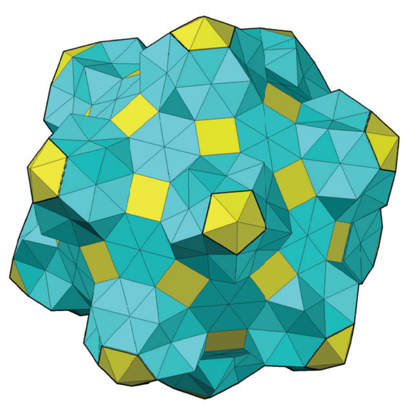


Figure 13: Dodeca-concaugmented great rhombicosidodecahedron with triangulated hexagonal sides

Such a polyhedral structure (or its fragments) could function as a dome with added conchal pockety dents. Its futuristic form could be suitable for different purposes, such as pavilions, exhibition halls, pool roofing structures, arenas, theaters, etc., and the polyhedron itself can be used as an element of urban exterior design, from fountains to sculptures, etc.

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By adding, subtracting, or cutting certain parts of this structure, as well as those described in previous examples, we can get a wide variety of shapes, interesting for discussion of possible applications in architectural practice.

An example of such a possibility is shown in Fig. 14, where a cluster of 8 composite polyhedra, hexa-concaugmented TC (presented in Fig.7) is used as a possible form of an object, for example: market - business center.

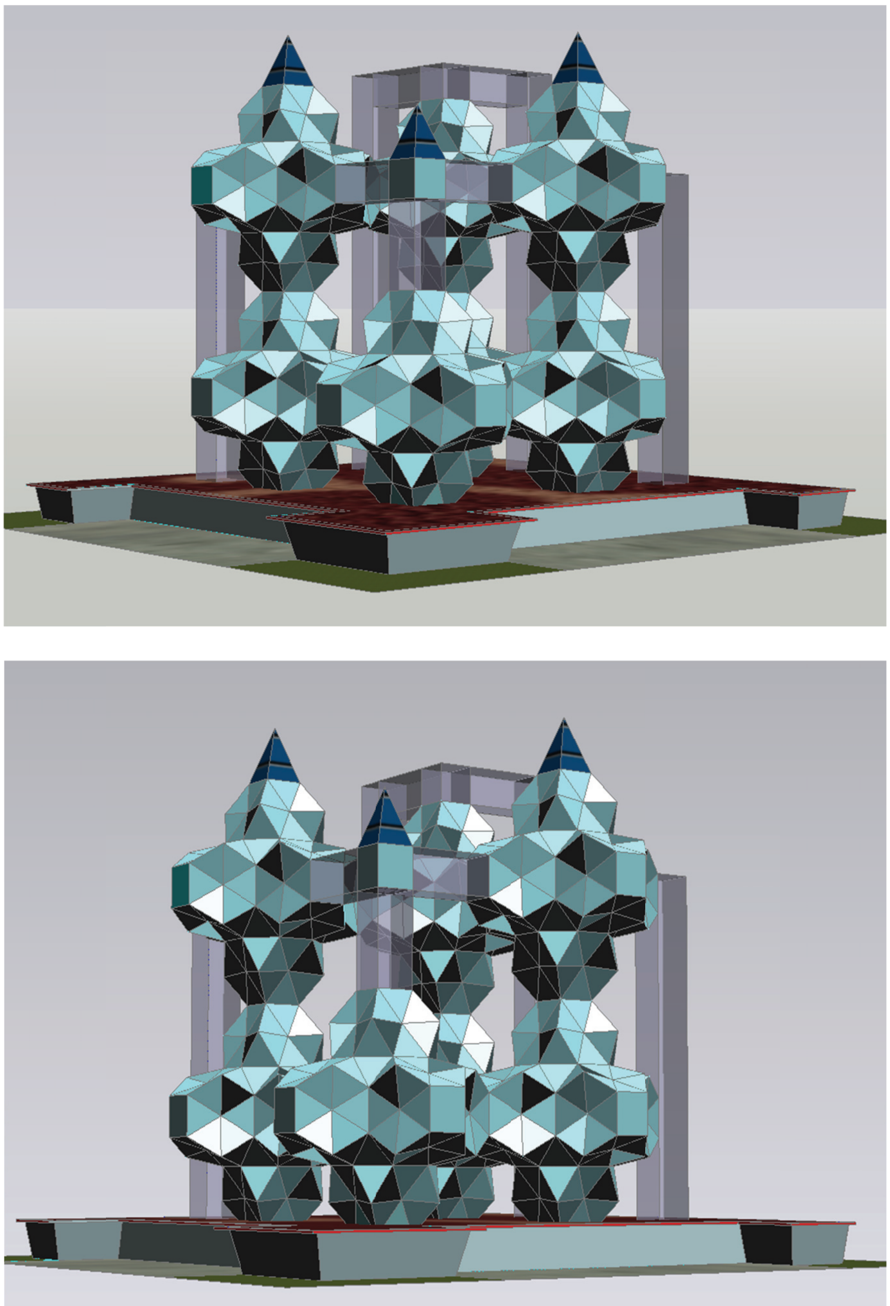

Figure 14: An example of CC II-4 cluster application in architecture

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CONCLUSIONS

Design of polyhedral structures is a source of inspiration for research in the design of various forms of engineering structures. This leads us to look beyond the usual rectangular forms to which we are accustomed, and to turn to the world of more versatile shapes, including triangle, hexagon and other regular polygons. Some of polyhedral shape, such as Archimedean solids, thanks to its uniformity provide still under-utilized potential for use in engineering structures.

Combinations of compatible polyhedral forms are not yet exhausted, and provide many opportunities for research. Also, the concave and complex polyhedra are being increasingly used in architecture, from the interior design, to the overall modeling of space and the structure of the building. Larger surface of those structures (as compared to convex) can be an advantage in recent architectural technologies, if we start from the assumption that it could be used in so-called. smart, self-sustainable buildings, which would provide space for the implementation of solar cells or the like.

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