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A GROUP OF POLYHEDRA ARISED AS VARIATIONS OF CONCAVE BICUPOLAE OF SECOND SORT

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Abstract

The paper presents the variations of polyhedral forms based on the geometry of concave cupolae of II sort. There were considered only variations that include bicupolae of II sort, with bases connected directly or indirectly - using prism or concave antiprism of II sort elongations, and giroelongations by antiprisms.

Considering that there are 14 (i.e. 7 x 2) concave cupolae of II sort, from n = 4 to n = 10, for each base we get 30 variations, making a total of 210 different solids.

Key words: concave bicupola, concave antiprism, elongation, giroelongation

1. INTRODUCTION

The method of originating concave cupolae of second sort has been elaborated in detail in some previous research [3], [4], [5], [6]. In this paper, only the brief overview will be given.

If we take a double row of equilatheral triangles so that they form a strip of hexa-triangular cells, as shown in the **figure 1**[5], and then fold it by the edges in order to assemble the opposite sidess, we can make a concave shell – a lateral area of a concave polyhedron. This polyhedron will be regular faced, if

[5] In order to form a concave cupola of II sort, we need to take out every second triangle in one of the rows of triangular net. If we do the same for the both rows, we will obtain a concave antiprism of II sort [5].

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we assign its bases to be regular polygons. These polygons have to be n-sided and 2n-sided (Fig. 1) and will be set in the parallel planes.

There are two ways of folding the given double row net. Folding it in the manner of retracting central vertex (G) of spatial hexa-triangular cell, we will obtain a polyhedron of major height [6] (Fig. 2- a), while by folding it in the maner of extracting the central vertex, the polyhedron will be of minor height [6] (Fig. 2 -b).

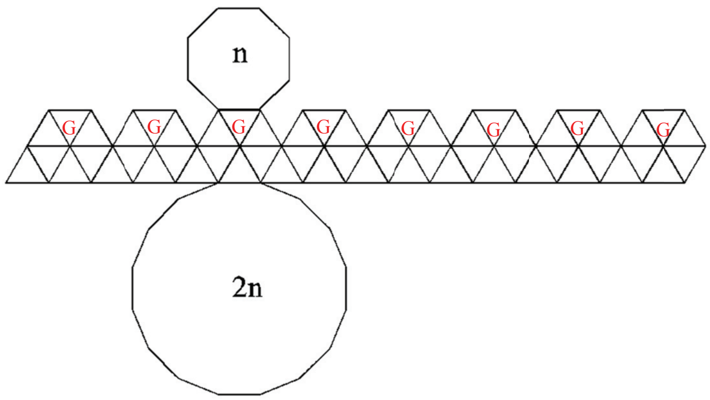

Figure 1 – Double row triangular strip which makes the cupola's lateral surface net

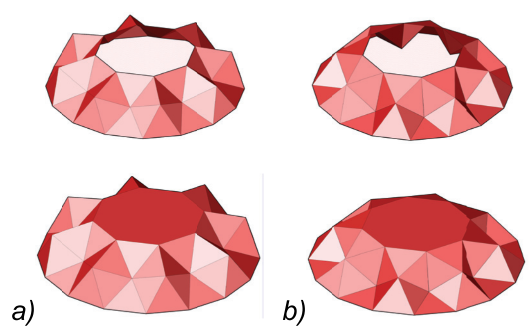

Figure 2 – Two ways of folding the triangulat net to obtain the cupola's lateral surface

The polyhedron obtained in this manner is named: *concave cupola of second sort* [4] (CC II hereinafter). The term cupola is taken as akin to Johnson's solids' cupola, which also have two reular polygons, n-sided and 2n-sided one in paralel planes, connected by triangles (and squares). The sufix „of second sort“ is introduced to denote a particular type of concave polyhedra which lateral area is consisted strictly od two rows od equilateral triangles.

These polyhedra are statically stable if the upper and the lower regular polygons (actualy, minor and major bases) are considered as rigid plates.

As described, there can be formed 14 different concave cupolae of second sort: with the initial (n-sided) polygon from n=4 to n=10. For n>10, two rows of equilateral triangles in the net, wouldn't be sufficient to enclose the

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solid. Thus, there are 7 representatives of CC II of the major height, and another 7 representatives of the minor height.

If we join two cupolae with compatible bases, orderly central or plane symmetrical solids, we obtain a polyhedron called bicupola (Fig. 3).

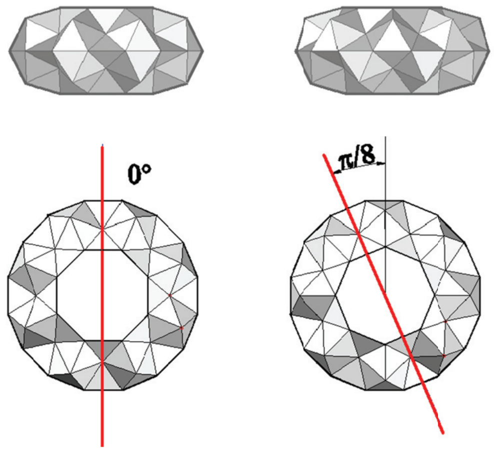

Figure 3 - Ortobicupola and girobicupola

In this paper, some variations of these solids will be presented and illustrated. The main principle of variating these cupolae are the same principle [8] [9] [10] [11] used to extend the list of regular-faced *convex* polyhedra.

2. BICUPOLA

Ortobicupola will be obtained as plane reflexive to the initial cupola, whereat they share the same 2n-sided polygon, as the cross section. Such a polyhedron will have 2n +1 planes of symmetry, one of the common 2n-sided polygon, and one for each side of it (Fig. 4-a)[6].

If we girate one of the obtained cupolae (in this paper, allways the top cupola will be girated), i.e. rotate it arround the common axis for the angle π /n, we will obtain *girobicupola*. Such a polyhedron will be centrally symmetric, according to the central point (centroid) of 2n-sided polygon, and will have no planes of symmetry (Fig. 4-b).

[6] For example, there is presented octagonal CC II -major.

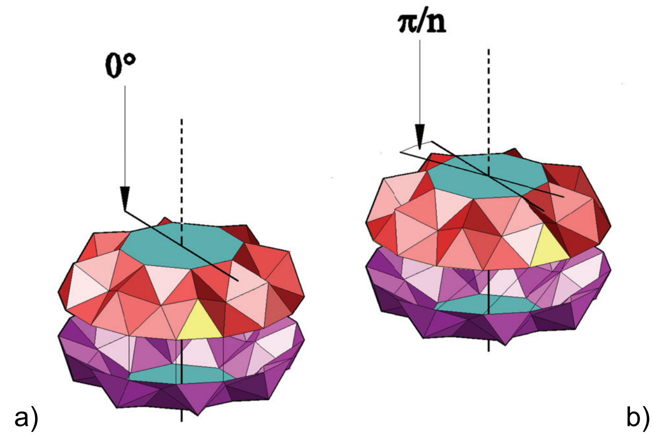

Figure 4 – Formation of bicupolae: a) ortobicupola and b) girobicupola

3. ELONGATIONS

The bicupola can be elongated by adding a ring that interconnects the two halves of the solid. This ring has to be also regular faced, equilateral, and with corresponding base: 2n-sided polygon. For this ring, 2n-sided antiprisms, and 2n-sided regular prisms can be used.

In the first case, the bicupola will be giroelongated, whereat we can distinguish two different cases: a) the upper cupola is girated by the angle $3\pi/2n$, and b) the upper cupola is girated by the angle $\pi/2n$. These two cases are equivalent to the left (L) and right (R) case of girated bicupola for $\pi/2n$, and $-\pi/2n$. Therefore, these two types are chiral.

In the case of inserting 2n-sided prism, bicupola will be just elongated – the upper half is translatory lifted for the height of the edge *a* (side of the polygon n). Thus, there can be obtained c) elongated ortobicupola and d) elongated girobicupola (Fig. 5).

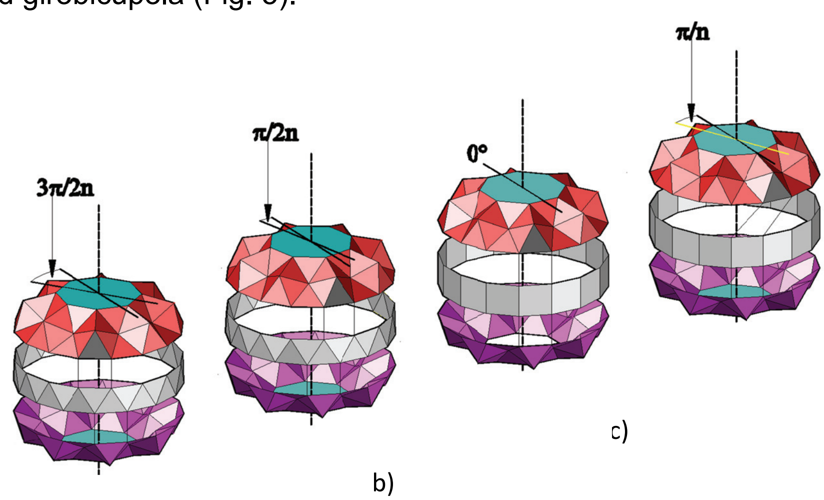

Figure 5 – Addition a) :iprisms and prisms in order to obtain regular faced polyhedral variation of bicupolae: a), b) giroelongations, c), d) elongations

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4. CONCAELONGATIONS

According to the previous description, this paper introduces another simmilar method of elongating bicupolae. We can use, as well as prisms and antiprisms, polyhedral rings of concave deltahedral lateral surface, obtained in a very simmilar manner as the concave cupolae of second sort – the concave antiprisms of second sort [5]. Instead of taking n-sided and 2n-sided polygon as bases of the polyhedron, we will take two identical (2n-sided) polygons. They are connected by the double row of equilateral triangles, folded analogously to the method of forming the CC II. The complete method of forming such polyhedra is described in [5].

There originate two types of antiprisms, similarly to the CC II: concave antiprisms of second sort with major height, and concave antiprisms of second sort with minor height, dependingly on the way of folding the net strip.

Both of them can be used as mesial belt, for further elongation of the bicupolae. Let us name such bicupolae – **concaelongated**.

Fig. 6 illustrates the method of elongating octagonal bicupolae with concave antiprisms of second sort: a) concaelongated ortobicupola major, b) concaelongated girobicupola major, c) concaelongated ortobicupola minor, d) concaelongated girobicupola minor.

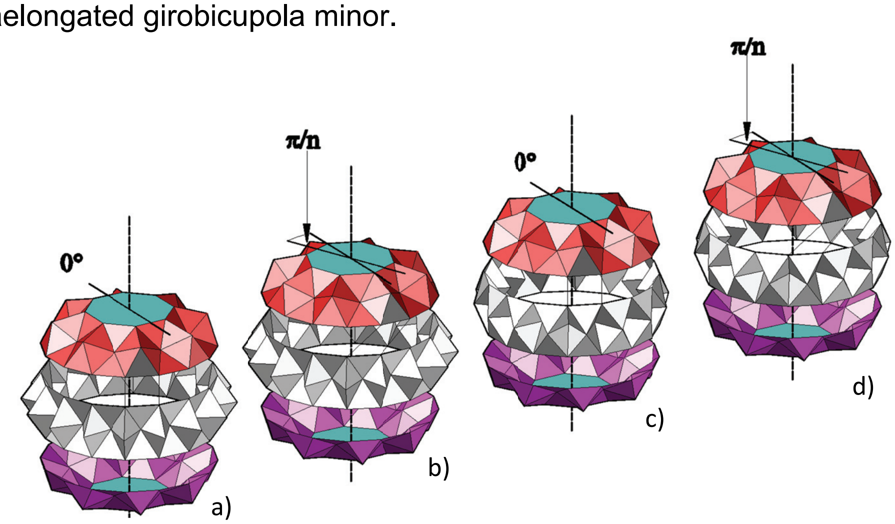

Figure 6 – Addition of concave antiprisms of second sort as a complement to bicupolae

Ten possible variations of octagonal CbC II (8) – M(ajor) (without augmentations) are presented in the Fig. 7.

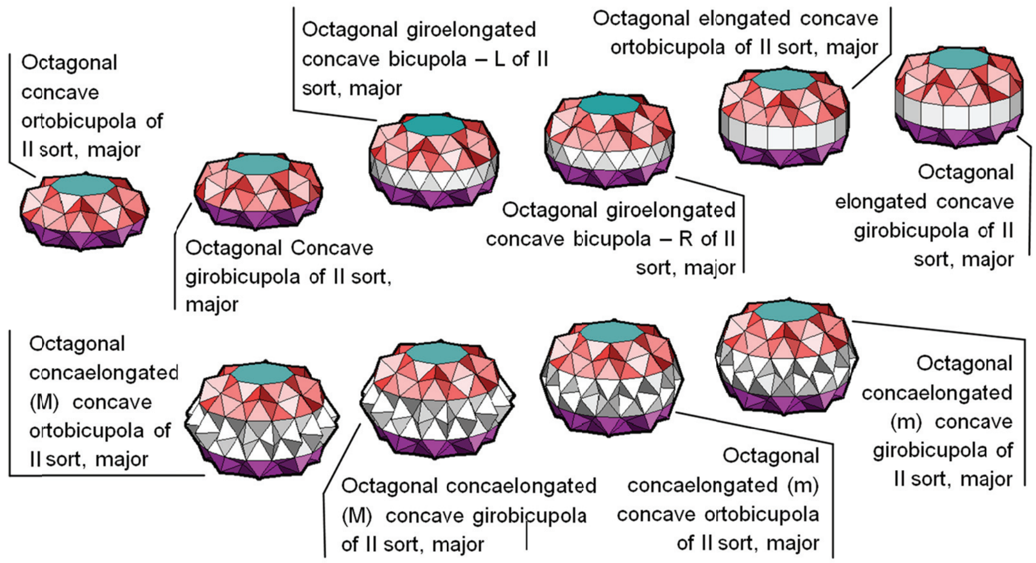

*Figure 7 – 10 variations of the octagonal concave bicupola of II sort with major height (CbC II -8M)*T

5. NUMBER OF VARIATIONS OF CONCAVE BICUPOLAE OF II SORT

Based on the previous, there can be set the final number of concave bicupolae of second sort variations (without augmentations).

As mentioned in the introduction, there are **14** primary representatives of concave cupolae of second sort: 7 of the major type: from CC II-4M[7] to CC II-10M (n=4, n=5, n=6, n=7, n=8, n=9, n=10) and 7 of minor type: from CC II-4m[8] to CC II–10m. Each of them, the minor cases as well as the major, can be constituent of one of 10 variations described in the previous chapters. This makes **14 x 10 = 140** different concave bicupolae of second sort (without augmentations).

Also, these two types of concave cupolae II can be mixed (crossed), so the number of variations rises for another 7 cases: from (CC II-4M + CC II-4m) to (CC II-10M + CC II-10m) (n=4, n=5, n=6, n=7, n=8, n=9, n=10) wich can be variated with the same 10 methods.

Thus, we have another **7x10=70** cases of concave bicupolae variations, which makes, with previously mentioned: **140+70=210** cases of Concave bicupolae of second sort (CbC II) variations.

Augmentations by pyramids or another corresponding CC II were not counted (although some of them are presented in illustrations) as explained below.

[7] Square concave cupola of second sort – Major: CC II-4-M
[8] Square concave cupola of second sort – minor: CC II-4-m.

6. OVERVIEW OF SOME MAIN VARIATIONS OF CONCAVE BICUPOLAE OF SECOND SORT (MAJOR)

In the following figures, some main cases of CbC II – Major will be shown.

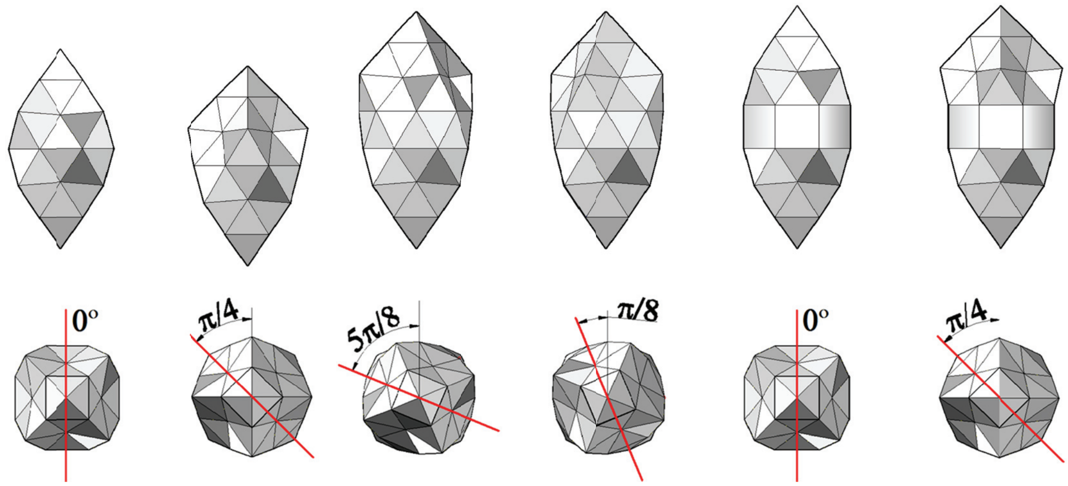

Figure 8 – Square concave bicupola of II sort (biaugmented) in top and front view

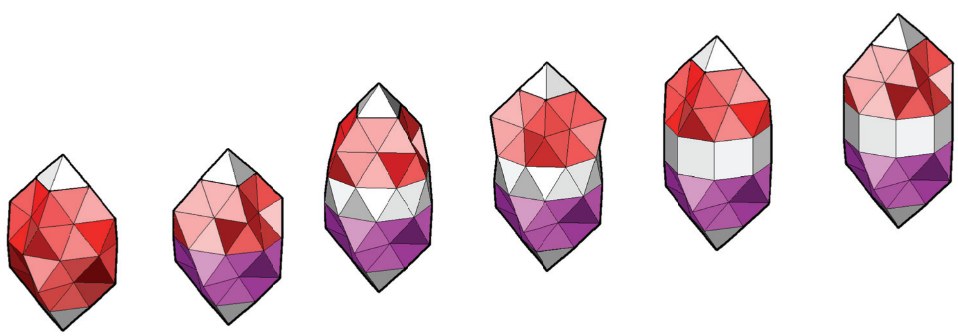

Figure 9 - Square concave bicupola of II sort (biaugmented) in axonometric view

In cases of **square CbC II** (Fig. 8 and Fig. 9) and **pentagonal CbC II** (Fig. 10 and Fig.12) we can transpose the whole bicupola into a deltahedron, by adding regular-faced pyramids on the non-triangular sides[9]. Such obtained augmentations would make another 20 cases (10 biaugmentations and 10 one–side augmentations) for both square and pentagonal base, i.e. **40** new cases.

Although augmentations were not considered in the final number of the variations, they offer some interesting shapes which resemble triangulated geometrical surfaces.

[9] Even the prismatic sides (squares) can be transposed into triangular by augmentations, but this paper does not consider these cases, as trivial. Furthermore, in some aesthetic sence, they would violate the wholeness of the solids' shapes.

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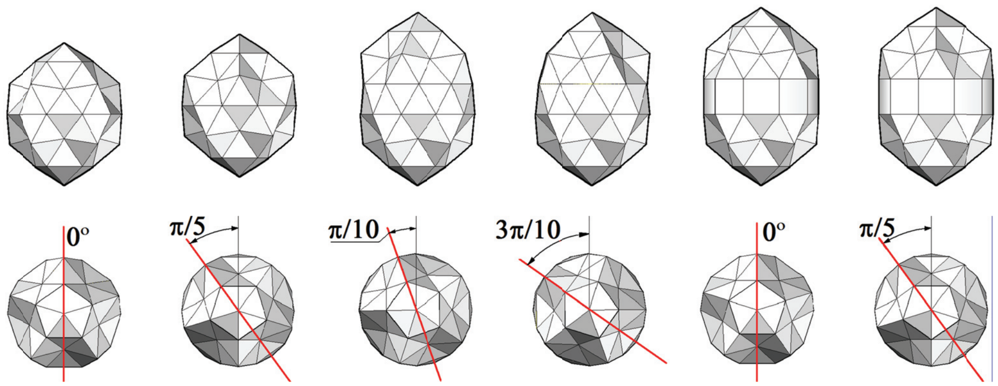

Figure 10 - Pentagonal concave bicupola of II sort (biaugmented) in top and front view

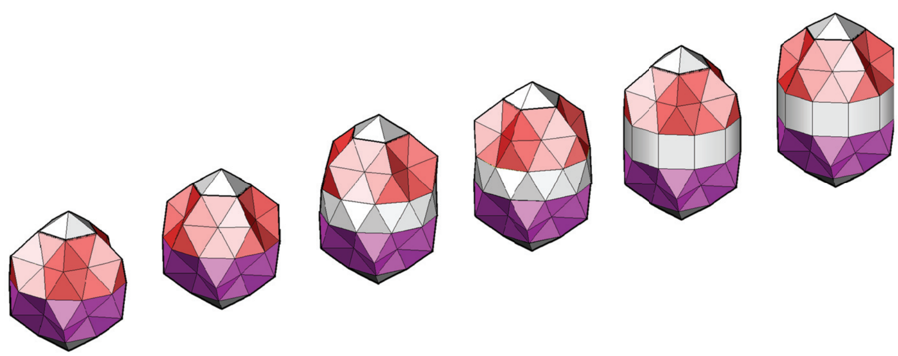

Figure 11 - Pentagonal concave bicupola of II sort (biaugmented) in axonometric view

As we can see in the Fig. 12, although the **hexagonal CbC II** can not be augmented with regular-faced pyramids at the bases, the siluettes of those solids suggest that there would exist some vertices that would fit into the whole envelope of the solid, even without forming the regular faced polyhedron.

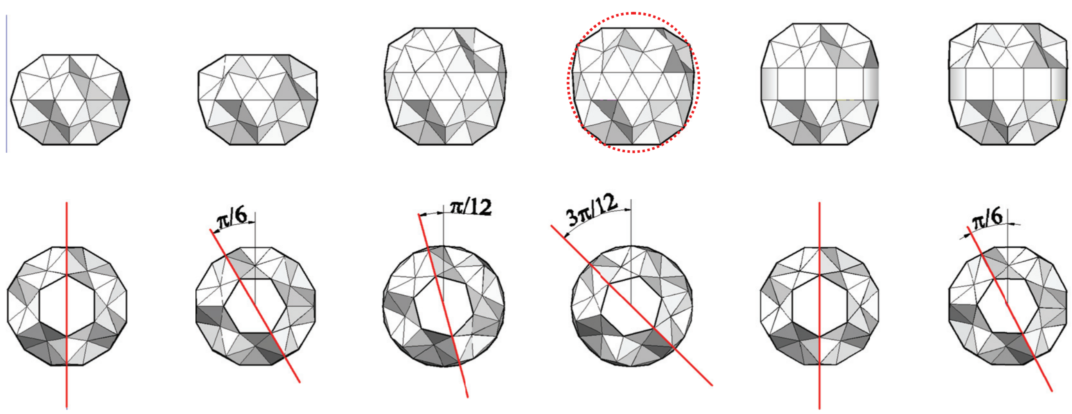

Figure 12 - Hexagonal concave bicupola of II sort in top and front view

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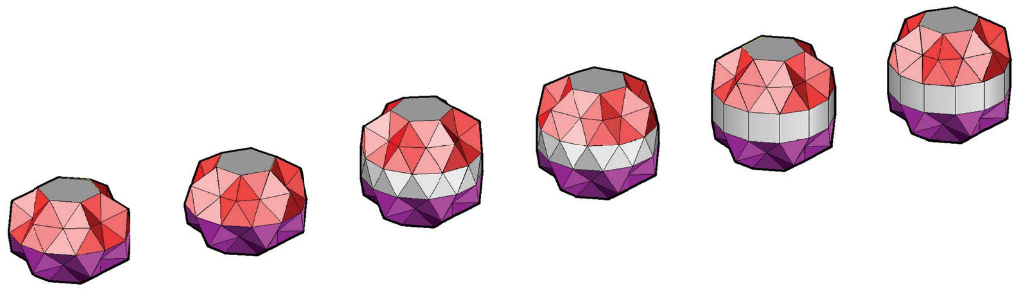

Figure 13 - Hexagonal concave bicupola of II sort in axonometric view

For the matter of diversity, the **heptagonal CbC II** is given as the minor variety. The Fig. 14 and Fig. 15 present the examples of CbC II obtained by opposite folding of triangular net. Since the differences between major and minor heights of the CC II of the same base are not significant, we can observe the gradual decresing of bicupolas' heights, as the number n (the sides of the base polygon) rises. The greatest height occurs at n=4, the lowest at n=10.

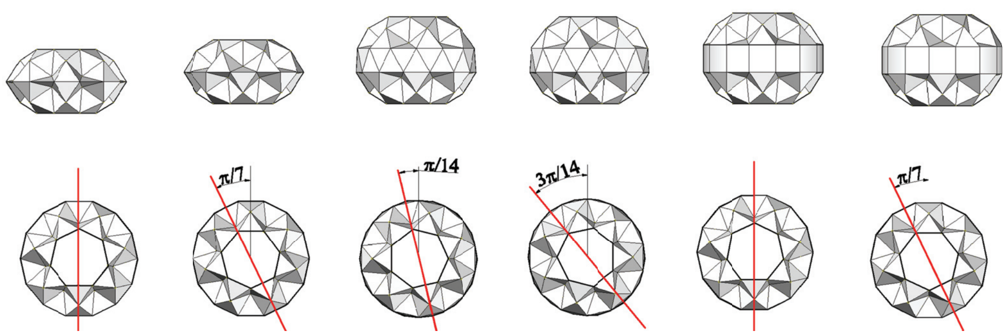

Figure 14 - Heptagonal concave bicupola of II sort in top and front view

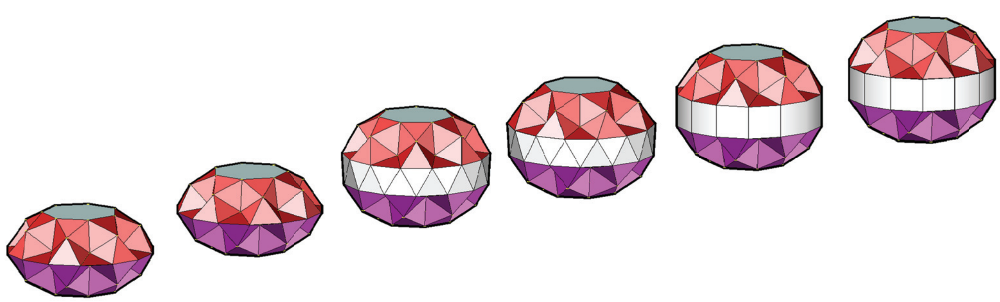

Figure 15 - Heptagonal concave bicupola of II sort in axonometric view

Octagonal CbC II-8 is the first which can be augmented by another concave cupola: CC II-4, which has the octagonal major basis. If we considered these cases, the number of concave polyhedral variations would rise for another: 30 cases (Major x10, minor x10, Major/minor x10) augmentations, only for biaumentations, and for one-sided cases the number would rise for another

20 (just one augmented side, with CC II-4 M or CC II-4m). So, **30+20=50** new variations per base would appear, which makes **50 x 2= 100** new solids, as this kind of variations would be possible also for decagonal CC II. However, as **40** pyramidal augmentations were not counted, these cases are just to be mentioned, and may be used in further research.

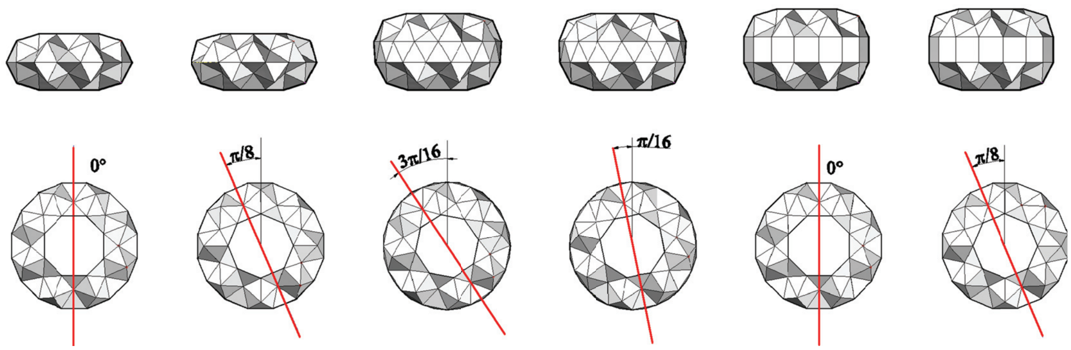

Figure 16 - Octagonal concave bicupola of II sort in top and front view

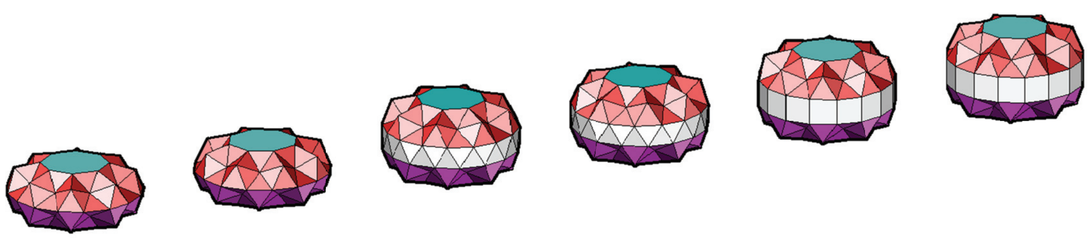

Figure 17 - Square concave bicupola of II sort in axonometric view

Speking of concaelongated bicupolae (ce-CbC II-8M), notice that in the presented Fig. 18 and Fig.19, we can perceive that the concave antiprism of II sort with minor height, somehow more „naturaly“ compement the bicupola with the major height, than the combination of both major heights (and vice cersa).

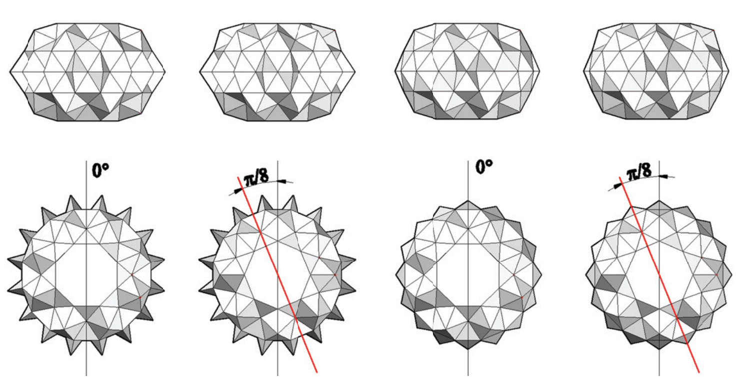

Figure 18 – Octagonal concaelongated concave bicupola of II sort in top and front view

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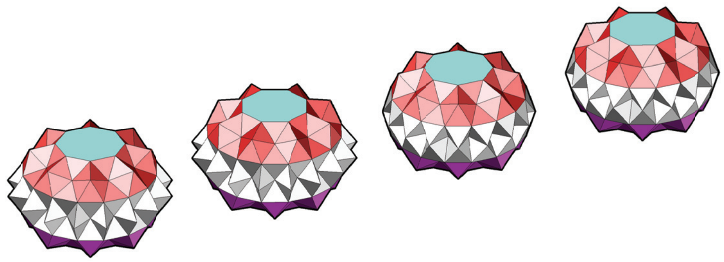

Figure 19 – Octagonal concaelongated concave bicupola of II sort in axonometric view

Nonagonal ortobicupola and girobicupola (Fig. 20 and Fig. 21) are the last of non-elongated CbC II which can be used as outer lateral surface of toroidal deltahedra [6].

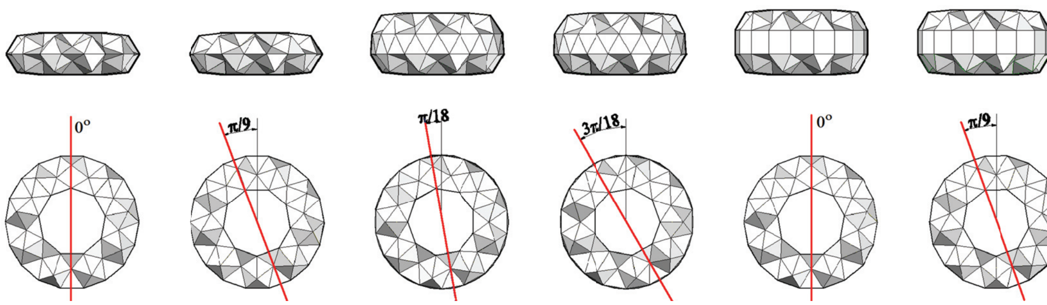

Figure 20 – Nonagonal concave bicupola of II sort in top and front view

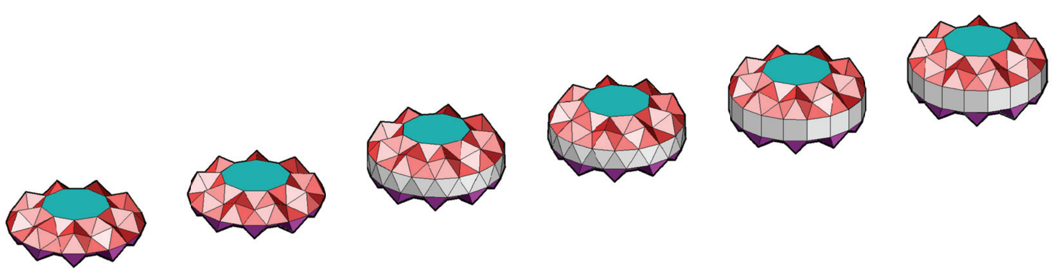

Figure 21 – Nonagonal concave bicupola of II sort in axonometric view

Decagonal CbC II-10 (Fig. 22 and Fig. 23) is the second representative which can be augmented by another concave cupola: CC II-5, as mentioned above.

If we would include even augmentations, 40 by pyramids (for square and pentagonal CbC II), 100 by the CbC II-4 and CbC II-5 (for octagonal and decagonal CbC II), and those 100 enlarged twice: for pyramidal augmentations

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(another 100 biaugmentations and new 100 one-sided augmentations), not counting the augmentations of square sides of elongating prisms, the final number of variations would rise for:

40+100+200=340

which would make, with presented **210** cases in this paper, **550** cases of concave bicupolae od II sort variations, and different polyhedral solids.

Observing the front view of decagonal concave bicupolae (Fig. 22) we can notice that their contures are now becoming hyperbola-like, while the previous representatives had ellipse-like shaped contures. These features of the concave bicupolae of second sort are yet to be explored.

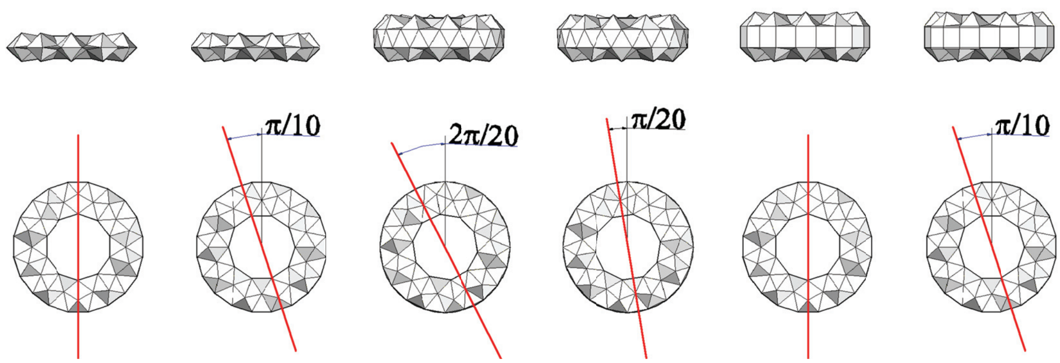

Figure 22 – Decagonal concave bicupola of II sort in top and front view

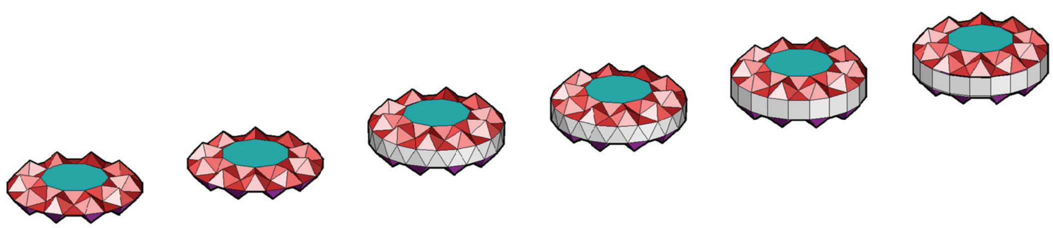

Figure 23 – decagonal concave bicupola of II sort in axonometric view

Note: All the pictures presented in the paper were made as 3D models in AutoCAD 2007, using the data from the tables 7 and 8, [6], pg. 395-402.

These models proved the full-scale accuracy od the algorithm and data used in the research [6].

7. CONCLUSIONS

The paper presents a group of polyhedral solids, based on the geometry of concave cupolae of II sort and gives an effort to systemize them in the simmilar way as convex polyhedra.

Not considering augmentations, it is shown that there are 210 different types of CbC II.

The paper presents, not only a galery of some of the most characteristic types of these solids, but also shows some interesting observations regarding the height and shape of the solids.

Some of these observations may serve as a starting point for further research.

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