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FREE VIBRATION OF PLATE ASSEMBLIES USING SPECTRAL ELEMENT METHOD

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Abstract –*In this paper, the Spectral Element Method (SEM) is applied to analyze free vibration of two plates perpendicularly assemblied, so-called L plate. In order to obtain necessary results, the transformation matrices have been developed for two positions of rectangular plates. Numerical example is conducted for L plate assemblies consisting of rectangular plates with same mechanical and geometrical properties. The accuracy of the results obtained by the SEM is verified by comparing them with the solutions obtained by conventional Finite Element Method (FEM).*

1. INTRODUCTION

Plates form many structural components ranging from walls and floors of high-rise buildings, panels in ship hull and aircraft, to printed circuit boards and silicon chips. It is rare that plates are independent in the structures, but assembled. The most common solution method for analysing these types of structures is the Finite Element Method (FEM) [2]. The size of the finite element depends on the highest frequency in the analysis. Consequently, to gain accurate results for large structures with high eigenfrequencies it is required a lot of finite elements and the increase the number of finite elements takes greater computer time and effort to solve the problem. As an alternative to the FEM in dynamic analysis the Spectral Element Method can be used. The SEM is based on the spectral representation of the displacement field and on the exact solution of the governing equations of motion defined in the frequency domain. Consequently, the dynamic stiffness matrix is frequency dependent, i.e. the analysis is performed in the frequency domain. The SEM is especially useful for one-dimensional elements where the accurate solutions for governing equations of motions are obtained. However, for two-dimensional elements, it is not possible to obtain exact solutions of partial differential equations that satisfy arbitrary boundary conditions. In order to find a solution of a problem, plate displacements are presented as infinite Fourier type series. For practical purposes, the series have to be truncated, which introduces an error. Consequently, the solutions are approximate and satisfy the prescribed degree of accuracy.

The procedure for the development of the dynamic stiffness matrix for rectangular plate undergoing in-plane and transverse vibration can be found in the literature, [4], [7]. Earlier studies of modal characteristics of plate assemblies were conducted for plates with specific boundary conditions. Bercin [2] analyzed plate assemblies that were simply-supported along longitudinal edges.

The main objective of this paper is to present the development of the transformation matrix necessary for obtaining the dynamic stiffness matrix of L plate assembly for general case of boundary conditions. The obtained results were compared with the results obtained by the FEM.

2. DYNAMIC STIFFNESS MATRIX OF PLATES

General form of equation of motion of the plates in the frequency domain without presence of external load can be given as:

$$L(\mathbf{u}) + \rho h \omega^2 \mathbf{u} = 0 \qquad (1)$$

where $\mathbf{u} = \mathbf{u}(x,y)$ is displacement vector, ρ is the mass density, h is the plate thickness, ω is the circular frequency and L is the differential operator.

In order to find a solution of equation of motion, plate displacements are presented as series:

$$u(x,y) \approx \sum_{m=1}^{M} C_m f_m(x,y) \qquad (2)$$

where C_m are integration constants and $f_m(x,y)$ are base functions that satisfy Eq. **(1)**.

Relation between the displacements $\hat{q}(s)$ and forces $\hat{Q}(s)$ along the boundaries is obtained by performing so-called projection method [1]. This method is based on projections of the displacements and forces on the boundaries onto a set of functions $h(s)$:

$$\hat{q}(s) \approx \sum_{n=1}^{M} \left\langle \hat{q}, h_n \right\rangle h_n(s)$$
$$\hat{Q}(s) \approx \sum_{n=1}^{M} \left\langle \hat{Q}, h_n \right\rangle h_n(s) \qquad (3)$$

where $\tilde{q}_n = \left\langle \hat{q}, h_n \right\rangle$ is projection of displacements and $\tilde{Q}_n = \left\langle \hat{Q}, h_n \right\rangle$ is projection of forces along boundaries.

Projections of the displacements and forces along boundaries are collected into vector:

$$\tilde{\mathbf{q}} = \left[\left\langle \hat{q}, h_n \right\rangle\right] \qquad \tilde{\mathbf{Q}} = \left[\left\langle \hat{Q}, h_n \right\rangle\right] \tag{4}$$

Now, it is possible to define a relation between displacements and forces for general case by using a diagonal dynamic stiffness matrix:

$$\tilde{\mathbf{K}}_{\mathbf{D}} = \begin{bmatrix} \tilde{\mathbf{K}}_{\mathbf{D_t}} & 0 \\ 0 & \tilde{\mathbf{K}}_{\mathbf{D_i}} \end{bmatrix} \tag{5}$$

where $\mathbf{K}_{\mathbf{D_t}}$ is dynamic stiffness matrix for out-of-plane vibration and $\mathbf{K}_{\mathbf{D_i}}$ is dynamic stiffness matrix for in-plane vibration. Presented dynamic stiffness matrix **(5)** gives the relation between the displacement vector $\tilde{\mathbf{q}}$ and the force vector $\tilde{\mathbf{Q}}$:

$$\tilde{\mathbf{Q}} = \tilde{\mathbf{K}}_{\mathbf{D}} \tilde{\mathbf{q}} \tag{6}$$

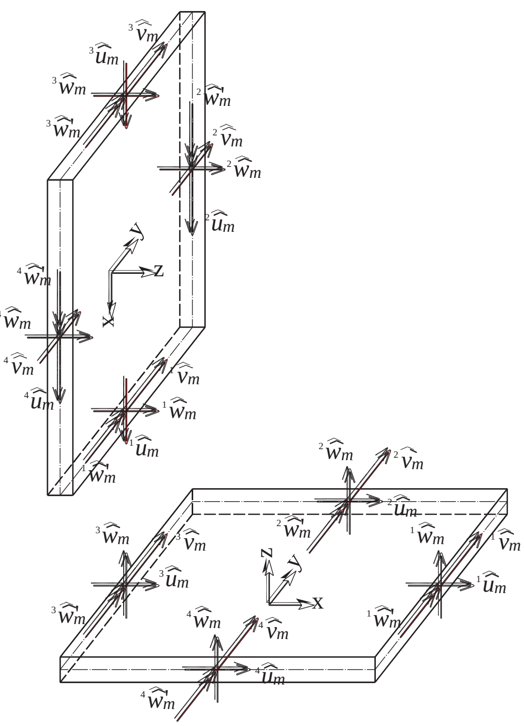

Figure 1. Edge displacements for L plate in local coordinate systems

The force vector $\tilde{\mathbf{Q}}$ is defined as:

$$\begin{gathered}
\tilde{\mathbf{Q}}^{\mathbf{T}} = \begin{bmatrix} \tilde{\mathbf{Q}}_{\mathbf{t}} & \tilde{\mathbf{Q}}_{\mathbf{i}} \end{bmatrix} \\
\tilde{\mathbf{Q}}_{\mathbf{t}}^{\mathbf{T}} = \begin{bmatrix} \tilde{\mathbf{Q}}_{\mathbf{0,t}} & \tilde{\mathbf{Q}}_{\mathbf{1,t}} & \cdots & \tilde{\mathbf{Q}}_{\mathbf{m,t}} & \cdots & \tilde{\mathbf{Q}}_{\mathbf{M,t}} \end{bmatrix} \\
\tilde{\mathbf{Q}}_{\mathbf{m,t}}^{\mathbf{T}} = \begin{bmatrix} {}^{\mathbf{1}}\tilde{\mathbf{Q}}_{\mathbf{m,t}}^{\mathbf{T}} & {}^{\mathbf{2}}\tilde{\mathbf{Q}}_{\mathbf{m,t}}^{\mathbf{T}} & {}^{\mathbf{3}}\tilde{\mathbf{Q}}_{\mathbf{m,t}}^{\mathbf{T}} & {}^{\mathbf{4}}\tilde{\mathbf{Q}}_{\mathbf{m,t}}^{\mathbf{T}} \end{bmatrix} \\
{}^{\mathbf{j}}\tilde{\mathbf{Q}}_{\mathbf{0,t}}^{\mathbf{T}} = \begin{bmatrix} {}^{j}\overline{T}_{x_{S_m}} & {}^{j}M_{x_{S_m}} \end{bmatrix} \; {}^{\mathbf{k}}\tilde{\mathbf{Q}}_{\mathbf{0,t}}^{\mathbf{T}} = \begin{bmatrix} {}^{j}\overline{T}_{y_{S_m}} & {}^{j}M_{y_{S_m}} \end{bmatrix} \\
{}^{\mathbf{j}}\tilde{\mathbf{Q}}_{\mathbf{m,t}}^{\mathbf{T}} = \begin{bmatrix} {}^{j}\overline{T}_{x_{S_m}} & {}^{j}\overline{T}_{x_{A_m}} & {}^{j}M_{x_{S_m}} & {}^{j}M_{x_{A_m}} \end{bmatrix} \\
{}^{\mathbf{k}}\tilde{\mathbf{Q}}_{\mathbf{m,t}}^{\mathbf{T}} = \begin{bmatrix} {}^{k}\overline{T}_{y_{S_m}} & {}^{k}\overline{T}_{y_{A_m}} & {}^{k}M_{y_{S_m}} & {}^{k}M_{y_{A_m}} \end{bmatrix}
\end{gathered} \tag{7}$$

$$\begin{gathered}
\tilde{\mathbf{Q}}_{\mathbf{i}}^{\mathbf{T}} = \begin{bmatrix} \tilde{\mathbf{Q}}_{\mathbf{0,i}} & \tilde{\mathbf{Q}}_{\mathbf{1,i}} & \cdots & \tilde{\mathbf{Q}}_{\mathbf{m,i}} & \cdots & \tilde{\mathbf{Q}}_{\mathbf{M,i}} \end{bmatrix} \\
\tilde{\mathbf{Q}}_{\mathbf{0,i}}^{\mathbf{T}} = \begin{bmatrix} {}^{1}N_{x_{S_0}} & {}^{2}N_{y_{S_0}} & {}^{3}N_{x_{S_0}} & {}^{4}N_{y_{S_0}} \end{bmatrix} \\
\tilde{\mathbf{Q}}_{\mathbf{m,i}}^{\mathbf{T}} = \begin{bmatrix} {}^{\mathbf{1}}\tilde{\mathbf{Q}}_{\mathbf{m,i}}^{\mathbf{T}} & {}^{\mathbf{2}}\tilde{\mathbf{Q}}_{\mathbf{m,i}}^{\mathbf{T}} & {}^{\mathbf{3}}\tilde{\mathbf{Q}}_{\mathbf{m,i}}^{\mathbf{T}} & {}^{\mathbf{4}}\tilde{\mathbf{Q}}_{\mathbf{m,i}}^{\mathbf{T}} \end{bmatrix} \\
{}^{\mathbf{j}}\tilde{\mathbf{Q}}_{\mathbf{m,i}}^{\mathbf{T}} = \begin{bmatrix} {}^{j}N_{x_{S_m}} & {}^{j}N_{x_{A_m}} & {}^{j}N_{xy_{S_m}} & {}^{j}N_{xy_{A_m}} \end{bmatrix} \\
{}^{\mathbf{k}}\tilde{\mathbf{Q}}_{\mathbf{m,i}}^{\mathbf{T}} = \begin{bmatrix} {}^{k}N_{y_{S_m}} & {}^{k}N_{y_{A_m}} & {}^{k}N_{xy_{S_m}} & {}^{k}N_{xy_{A_m}} \end{bmatrix} \\
j = 1,3; \;\; k = 2,4
\end{gathered}$$

The displacement vector $\tilde{\mathbf{q}}$ is defined as:

$$\begin{gathered}
\tilde{\mathbf{q}}^{\mathbf{T}} = \begin{bmatrix} \tilde{\mathbf{q}}_{\mathbf{t}} & \tilde{\mathbf{q}}_{\mathbf{i}} \end{bmatrix} \\
\tilde{\mathbf{q}}_{\mathbf{t}}^{\mathbf{T}} = \begin{bmatrix} \tilde{\mathbf{q}}_{\mathbf{0,t}} & \tilde{\mathbf{q}}_{\mathbf{1,t}} & \cdots & \tilde{\mathbf{q}}_{\mathbf{m,t}} & \cdots & \tilde{\mathbf{q}}_{\mathbf{M,t}} \end{bmatrix} \\
\tilde{\mathbf{q}}_{\mathbf{m,t}}^{\mathbf{T}} = \begin{bmatrix} {}^{\mathbf{1}}\tilde{\mathbf{q}}_{\mathbf{m,t}} & {}^{\mathbf{2}}\tilde{\mathbf{q}}_{\mathbf{m,t}} & {}^{\mathbf{3}}\tilde{\mathbf{q}}_{\mathbf{m,t}} & {}^{\mathbf{4}}\tilde{\mathbf{q}}_{\mathbf{m,t}} \end{bmatrix} \\
{}^{\mathbf{j}}\tilde{\mathbf{q}}_{\mathbf{0,t}}^{\mathbf{T}} = \begin{bmatrix} {}^{j}w_{S_0} & {}^{j}w'_{S_0} \end{bmatrix} \\
{}^{\mathbf{j}}\tilde{\mathbf{q}}_{\mathbf{m,t}}^{\mathbf{T}} = \begin{bmatrix} {}^{j}w_{S_m} & {}^{j}w_{A_m} & {}^{j}w'_{S_m} & {}^{j}w'_{A_m} \end{bmatrix} \\
\tilde{\mathbf{q}}_{\mathbf{i}}^{\mathbf{T}} = \begin{bmatrix} \tilde{\mathbf{q}}_{\mathbf{0,i}} & \tilde{\mathbf{q}}_{\mathbf{1,i}} & \cdots & \tilde{\mathbf{q}}_{\mathbf{m,i}} & \cdots & \tilde{\mathbf{q}}_{\mathbf{M,i}} \end{bmatrix} \\
\tilde{\mathbf{q}}_{\mathbf{0,i}} = \begin{bmatrix} {}^{1}u_{S_0} & {}^{2}v_{S_0} & {}^{3}u_{S_0} & {}^{4}v_{S_0} \end{bmatrix} \\
\tilde{\mathbf{q}}_{\mathbf{m,i}}^{\mathbf{T}} = \begin{bmatrix} {}^{\mathbf{1}}\tilde{\mathbf{q}}_{\mathbf{m,i}} & {}^{\mathbf{2}}\tilde{\mathbf{q}}_{\mathbf{m,i}} & {}^{\mathbf{3}}\tilde{\mathbf{q}}_{\mathbf{m,i}} & {}^{\mathbf{4}}\tilde{\mathbf{q}}_{\mathbf{m,i}} \end{bmatrix} \\
{}^{\mathbf{j}}\tilde{\mathbf{q}}_{\mathbf{m,i}}^{\mathbf{T}} = \begin{bmatrix} {}^{j}u_{S_m} & {}^{j}u_{A_m} & {}^{j}v_{S_m} & {}^{j}v_{A_m} \end{bmatrix} \\
j = 1,2,3,4
\end{gathered} \tag{8}$$

Represented vectors are given in the local coordinate system. The plate's displacements in the local coordinate system are given in Fig. 1.

3. TRANSFORMATION OF PLATE DYNAMIC STIFFNESS MATRIX

As shown in the previous section, in the SEM the force-displacement relation of plate is defined by the dynamic stiffness matrix. Therefore, the same assemblage procedure is used as in the FEM. First, it is necessary to transform displacements and forces along the edges from local coordinate system to global coordinate system. Fig. 1. shows the system consisting of two plates, so-called L plate, with edge displacements in the local coordinate systems. It is necessary to transform the displacements in the global coordinate system, Figure 2.

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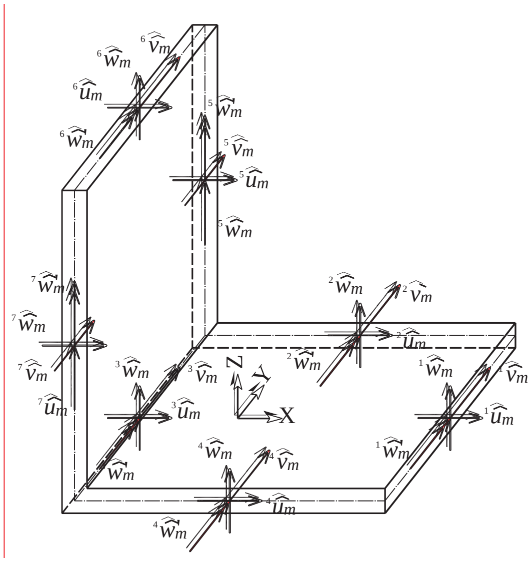

Figure 2. Edge displacement of L plate in global coordinate system

Using the transformation matrix the displacements have been transformed:

$$\tilde{\mathbf{q}} = \mathbf{T_T}\tilde{\mathbf{q}}_\mathbf{T} \tag{9}$$

where vector $\tilde{\mathbf{q}}$ is given in Eq. **(8)**, and displacement vector in global coordinate system is defined as:

$$\begin{aligned}
\tilde{\mathbf{q}}_\mathbf{T}^\mathbf{T} &= \begin{bmatrix} \tilde{\mathbf{q}}_\mathbf{0} & \tilde{\mathbf{q}}_\mathbf{1} & \cdots & \tilde{\mathbf{q}}_\mathbf{m} & \cdots & \tilde{\mathbf{q}}_\mathbf{M} \end{bmatrix} \\
\tilde{\mathbf{q}}_\mathbf{0}^\mathbf{T} &= \begin{bmatrix} {}^\mathbf{1}\tilde{\mathbf{q}}_\mathbf{0} & {}^\mathbf{2}\tilde{\mathbf{q}}_\mathbf{0} & {}^\mathbf{3}\tilde{\mathbf{q}}_\mathbf{0} & {}^\mathbf{4}\tilde{\mathbf{q}}_\mathbf{0} \end{bmatrix} \\
{}^\mathbf{j}\tilde{\mathbf{q}}_\mathbf{0}^\mathbf{T} &= \begin{bmatrix} {}^j u_{S_0} & {}^j v_{S_0} & {}^j w'_{S_0} \end{bmatrix} \\
{}^\mathbf{k}\tilde{\mathbf{q}}_\mathbf{0}^\mathbf{T} &= \begin{bmatrix} {}^k v_{S_0} & {}^k w_{S_0} & {}^k w'_{S_0} \end{bmatrix} \\
\tilde{\mathbf{q}}_\mathbf{m}^\mathbf{T} &= \begin{bmatrix} {}^\mathbf{1}\tilde{\mathbf{q}}_\mathbf{m} & {}^\mathbf{2}\tilde{\mathbf{q}}_\mathbf{m} & {}^\mathbf{3}\tilde{\mathbf{q}}_\mathbf{m} & {}^\mathbf{4}\tilde{\mathbf{q}}_\mathbf{m} \end{bmatrix} \\
{}^\mathbf{i}\tilde{\mathbf{q}}_\mathbf{m}^\mathbf{T} &= \begin{bmatrix} {}^\mathbf{i}\mathbf{u}_\mathbf{m} & {}^\mathbf{i}\mathbf{v}_\mathbf{m} & {}^\mathbf{i}\mathbf{w}_\mathbf{m} & {}^\mathbf{i}\mathbf{w}'_\mathbf{m} \end{bmatrix} \\
{}^\mathbf{i}\mathbf{r}_\mathbf{m} &= \begin{bmatrix} {}^i r_{S_m} & {}^i r_{A_m} \end{bmatrix} \quad i = 1,2,3,4; \quad r = u, v, w, w'
\end{aligned} \tag{10}$$

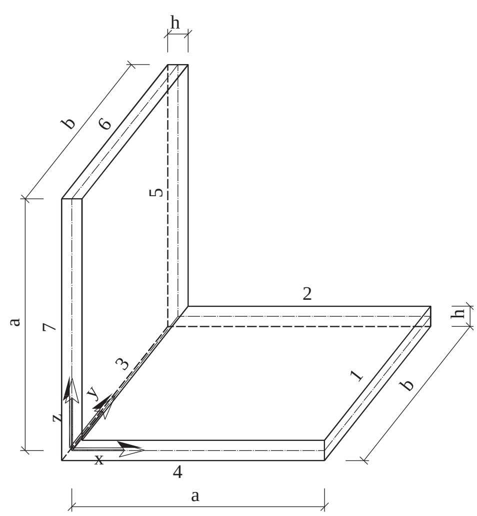

Figure 3. L plate with dimensions and edge numeration

Table 1. Eigenfrequencies of L plate assembly

SEM			FEM			
			Mesh size			
M=1	M=3	M=10	5x10	10x20	20x40	40x80
15.0	15.1	15.1	14.8	15.1	15.1	15.1
19.3	19.3	19.3	18.7	19.1	19.2	19.3
52.7	53.0	53.1	49.7	52.2	52.9	53.0
65.7	65.9	65.9	60.8	64.5	65.5	65.8
66.9	67.2	67.3	62.7	66.1	67.0	67.2
70.4	70.6	70.3	66.0	69.4	70.3	70.6
117.3	116.0	115.8	106.5	115.3	115.7	115.8
118.4	119.5	119.6	112.7	116.2	118.8	119.4
130.8	131.6	131.7	116.6	127.5	130.6	131.4
133.8	135.1	135.2	126.0	133.1	134.7	135.1

It is necessary to transform edge forces too, using the same transformation matrix:

$$\tilde{\mathbf{Q}} = \mathbf{T_T}\tilde{\mathbf{Q}}_\mathbf{T} \tag{11}$$

where vector $\tilde{\mathbf{Q}}$ is given in Eq. **(7)**, and force vector in global coordinate system is defined as:

$$\begin{aligned}
\tilde{\mathbf{Q}}_\mathbf{T}^\mathbf{T} &= \begin{bmatrix} \tilde{\mathbf{Q}}_\mathbf{0} & \tilde{\mathbf{Q}}_\mathbf{1} & \cdots & \tilde{\mathbf{Q}}_\mathbf{m} & \cdots & \tilde{\mathbf{Q}}_\mathbf{M} \end{bmatrix} \\
\tilde{\mathbf{Q}}_\mathbf{0}^\mathbf{T} &= \begin{bmatrix} {}^\mathbf{1}\tilde{\mathbf{Q}}_\mathbf{0} & {}^\mathbf{2}\tilde{\mathbf{Q}}_\mathbf{0} & {}^\mathbf{3}\tilde{\mathbf{Q}}_\mathbf{0} & {}^\mathbf{4}\tilde{\mathbf{Q}}_\mathbf{0} \end{bmatrix} \\
{}^\mathbf{j}\tilde{\mathbf{Q}}_\mathbf{0}^\mathbf{T} &= \begin{bmatrix} {}^j N_{x_{S_0}} & {}^j \overline{T}_{x_{S_0}} & {}^j M_{x_{S_0}} \end{bmatrix} \\
{}^\mathbf{k}\tilde{\mathbf{Q}}_\mathbf{0}^\mathbf{T} &= \begin{bmatrix} {}^k N_{y_{S_0}} & {}^k \overline{T}_{y_{S_0}} & {}^k M_{y_{S_0}} \end{bmatrix} \\
\tilde{\mathbf{Q}}_\mathbf{m}^\mathbf{T} &= \begin{bmatrix} {}^\mathbf{1}\tilde{\mathbf{Q}}_\mathbf{m} & {}^\mathbf{2}\tilde{\mathbf{Q}}_\mathbf{m} & {}^\mathbf{3}\tilde{\mathbf{Q}}_\mathbf{m} & {}^\mathbf{4}\tilde{\mathbf{Q}}_\mathbf{m} \end{bmatrix} \\
{}^\mathbf{j}\tilde{\mathbf{Q}}_\mathbf{m}^\mathbf{T} &= \begin{bmatrix} {}^\mathbf{j}\mathbf{N}_{\mathbf{x_m}} & {}^\mathbf{j}\mathbf{N}_{\mathbf{xy_m}} & {}^\mathbf{j}\overline{\mathbf{T}}_{\mathbf{x_m}} & {}^\mathbf{j}\mathbf{M}_{\mathbf{x_m}} \end{bmatrix} \\
{}^\mathbf{k}\tilde{\mathbf{Q}}_\mathbf{m}^\mathbf{T} &= \begin{bmatrix} {}^\mathbf{k}\mathbf{N}_{\mathbf{y_m}} & {}^\mathbf{k}\mathbf{N}_{\mathbf{xy_m}} & {}^\mathbf{k}\overline{\mathbf{T}}_{\mathbf{y_m}} & {}^\mathbf{k}\mathbf{M}_{\mathbf{y_m}} \end{bmatrix} \\
{}^\mathbf{j}\mathbf{r}_\mathbf{m} &= \begin{bmatrix} {}^j r_{S_m} & {}^j r_{A_m} \end{bmatrix} \; {}^\mathbf{k}\mathbf{r}_\mathbf{m} = \begin{bmatrix} {}^j r_{S_m} & {}^j r_{A_m} \end{bmatrix} \\
& j = 1,3; \; k = 2,4 \\
{}^j r_m &= {}^j N_{x_m}, {}^j N_{xy_m}, {}^j \overline{T}_{x_m}, {}^j M_{x_m} \\
{}^k r_m &= {}^k N_{y_m}, {}^k N_{xy_m}, {}^k \overline{T}_{y_m}, {}^k M_{y_m}
\end{aligned} \tag{12}$$

Substituting Eq. **(9)** and Eq. **(11)** into Eq. **(6)** the relation between the force vector and displacement vector in global coordinate system is obtained:

$$\tilde{\mathbf{Q}}_\mathbf{T} = \mathbf{T}_\mathbf{T}^\mathbf{T}\tilde{\mathbf{K}}_\mathbf{D}\mathbf{T}_\mathbf{T}\tilde{\mathbf{q}}_\mathbf{T} = \tilde{\mathbf{K}}_{\mathbf{D_T}}\tilde{\mathbf{q}}_\mathbf{T} \tag{13}$$

where the dynamic stiffness matrix in the global coordinate system is given as:

$$\tilde{\mathbf{K}}_{\mathbf{D_T}} = \mathbf{T}_\mathbf{T}^\mathbf{T}\tilde{\mathbf{K}}_\mathbf{D}\mathbf{T}_\mathbf{T} \tag{14}$$

In the following, it will be shown the transformation matrix for horizontal plate:

$$\mathbf{T}^\mathbf{T} = \begin{bmatrix} \mathbf{T_1} & \mathbf{T_2} \end{bmatrix} \tag{15}$$

The first sub matrix is defined as:

$$\mathbf{T}_1 = \begin{bmatrix} \mathbf{t}_1 & & & & & \\ & \mathbf{t}_{2,1} & & & & \\ & & \ddots & & & \\ & & & \mathbf{t}_{2,m} & & \\ & & & & \ddots & \\ & & & & & \mathbf{t}_{2,M} \end{bmatrix}$$

$$\mathbf{t}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(16)}$$

$$\mathbf{t}_{2,m} = \begin{bmatrix} \mathbf{t}_0 & \mathbf{t}_{21} & & & & & & \\ & & \mathbf{t}_0 & \mathbf{t}_{21} & & & & \\ & & & & \mathbf{t}_0 & \mathbf{t}_{21} & & \\ & & & & & & \mathbf{t}_0 & \mathbf{t}_{21} \end{bmatrix}$$

$$\mathbf{t}_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{t}_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The second sub matrix is defined as:

$$\mathbf{T}_2 = \begin{bmatrix} \mathbf{t}_3 & & & & & \\ & \mathbf{t}_{4,1} & & & & \\ & & \ddots & & & \\ & & & \mathbf{t}_{4,m} & & \\ & & & & \ddots & \\ & & & & & \mathbf{t}_{4,M} \end{bmatrix}$$

$$\mathbf{t}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \textbf{(17)}$$

$$\mathbf{t}_{4,m} = \begin{bmatrix} \mathbf{t}_{41} & \mathbf{t}_0 & & & & & & \\ & & \mathbf{t}_{41} & \mathbf{t}_0 & & & & \\ & & & & \mathbf{t}_{41} & \mathbf{t}_0 & & \\ & & & & & & \mathbf{t}_{41} & \mathbf{t}_0 \end{bmatrix}$$

$$\mathbf{t}_{41} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{t}_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The same procedure is repeated for the vertical plate, with the corresponding transformation matrix.

4. NUMERICAL EXAMPLE

In order to illustrate the convergence and accuracy of the modal characteristics of L plates using the principles explained above, it will be shown example of two connected rectangular plates with same geometrical and mechanical characteristics, Young's modules E=30GPa, mass density ρ=2.5t/m3, Poisson's ratio ν=0.15, edge length a = b = 3m and plate thickness h=0.15m. Edges 2. and 5 are clamped and the other edges of L plate are completely free, Fig. 3. The results of the first 10 eigenfrequencies are compared with the results obtained using SAP2000 [7], Table 1.

5. CONCLUSION

Calculation of eigenfrequencies of plate assemblies with arbitrary boundary conditions has been presented in this paper. For that purpose the computer code in MATLAB has been developed. The present solution has been validated by comparing it with the results obtained using SAP2000 finite element code. The results indicated high precision of the proposed solution in comparison with the FEM. The number of unknowns was significantly decreased in the SEM, without loosing accuracy. The FEM requires mesh refinement with increasing frequencies. However, the accuracy of the results is not dependent on the mesh refinement in the SEM. In addition, the computation time and memory cost were decreased, too. Consequently, the SEM has demonstrated a great potential in modeling rectangular plate assemblies.

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