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PRIMER PRORAČUNA SPREGNUTOG NOSAČA OD ČELIKA I BETONA PRIMENOM RAZLIČITIH METODA

Rezime:

U radu je prikazan proračun spregnutog nosača od čelika i betona primenom četiri metode proračuna: tzv. tačne metode proračuna, uprošćene metode, metode efektivnog modula i metode predložene standardom Evrokod 4. Tačna metoda primenjuje linearne integralne operatore. Uprošćena metoda je izvedena iz tačne metode i pretpostavke da se generalisana pomeranja menjaju linearno sa funkcijom tečenja betona. Metoda efektivnog modula i metoda predložena Evrokodom 4 su algebarske metode koje se široko koriste u praksi. Na konkretnom brojnom primeru upoređeni su rezultati dobijeni primenom ove četiri metode proračuna.

Ključne reči: viskoelastična analiza, spregnuti nosači, funkcija tečenja betona

COMPOSITE STEEL-CONCRETE BEAM ANALYSIS USING DIFFERENT METHODS

Summary:

The paper compares the following methods for analysis of composite steel-concrete beams: the "exact" analysis method, simplified method, effective modulus method and the method proposed by Eurocode 4 design code. The exact analysis method is based on the application of linear integral operators. Simplified analysis is derived from the exact method and adopts the assumption that generalized displacements change linearly with the concrete creep function. Effective modulus method and the method proposed by the Eurocode 4 design code are algebraic methods, widely used in practice. The results obtained using the mentioned four methods are compared on one example.

Key words: viscoelastic analysis, composite structures, concrete creep function

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1. INTRODUCTION

Composite steel-concrete beams are the most commonly used type of composite structures adopting the most favorable properties of both materials. Steel and concrete as constituent materials united in a same cross section demonstrate fundamentally different behavior. Development and application of currently available calculation methods mostly originates from different approaches to rheological properties of concrete in time.

This paper compares four proposed analysis methods for viscoelastic analysis of composite beams on an example. We initially refer to Deretić-Stojanović [1] where the same numerical example was already calculated using the "exact" method based on linear integral operators. Mandel [6] established linear integro-differential operators in the aging linear viscoelasticity for presentation of the integral relations. Further, Bazant and Huet [7] used matrix and tensor integro-differential operators. Prof Lazic [6] first applied these linear integral operators in force based analysis of composite and prestressed beams. His work was followed by Deretić-Stojanović and Kostić who's "exact" method presented in [4, 5] uses the same operators but involves displacement based method for analysis of composite steel-concrete and prestressed beams. Displacements become unknowns and ultimate equations are nonhomogeneous integral equations. These can be solved with Laplace transformations but only under the assumption of constant concrete modulus of elasticity and creep functions of the hereditary theory or the aging theory.

Next proposed method is a simplified matrix stiffness method which arises from the above described "exact" analysis method [2,3]. The idea is to simplify the calculation by introducing the assumption that unknown displacements, in time, are linear functions of the concrete creep function. The result is transformation of the nonhomogeneous integral equations into simple algebraic equations, but the method keeps good approximate estimates of the creep effects.

Finally, the same example is analyzed by widely used effective modulus method (EM method) for steel-concrete composite structures, and by procedure in current design code - Eurocode 4 (EC4). This paper provides a brief review on application of each of four above mentioned methods and compares and comments obtained results.

2. "EXACT" ANALYSIS METHOD

In the "exact" analysis method, the basic unknowns are displacements. The relations between the generalized element deformations and the generalized element forces are integral. It is shown [3, 4, 8] that these basic relations can be presented in the same form as for the elastic homogeneous frame element using the mathematical theory of linear integral operators.

The system of equations that obtains using this method, has the same form as for homogeneous structure's analysis:

$$[\hat{K}'][q] = [S], \tag{1}$$

where:

$[\hat{K}']$ is the operator stiffness matrix,

$[q]$ is the vector of displacements,

$[S]$ is the vector that includes external nodal forces and nodal forces due to element loads. The main disadvantage of this method is that the closed-form solutions can be found only for some analytical forms of the concrete creep functions, i.e. Rate of Creep Method, Hereditary theory [6]. In other cases, the system needs to be solved numerically.

3. SIMPLIFIED ANALYSIS METHOD

A simplified method for analysis of composite beams [3] introduces further assumption that generalized joint displacements (displacements or rotations) q_λ, change linearly with the concrete creep function F^* over time:

$$q_\lambda = q_{\lambda 0} 1^* + \Delta q_\lambda (F^* - 1^*), \qquad \lambda=1,2,\dots n \tag{2}$$

where:
t_0 is the age of concrete when first stress and deformation appear,
$q_{\lambda 0} = q_{\lambda 0}(t_0, t_0)$ is the unknown displacement at time t_0,
Δq_λ is unknown that should be determined,
1^* is the Heaviside step function.

This way, the ultimate system of nonhomogeneous integral equations changes into the system of nonhomogeneous algebraic equations, without particular mathematical adaptation. Integrals in the element stiffness matrices are replaced with linear combination of function F^* and three other functions. Finally, the unknown Δq_λ that needs to be determined for time interval ($t_0, t \rightarrow \infty$) becomes constant.

4. EM AND EC4 METHOD

The effective modulus method (EM) is considered to be approximate method of calculation. It introduces assumptions regarding rheological properties of concrete, as well as certain mathematical simplifications in the calculation. The creep effect is included trough simple reduction of the concrete modulus of elasticity E_{c0} into effective elastic modulus of concrete $E_{c,eff}$;

$$E_{c,eff} = \frac{E_{c0}}{1+\varphi_r} \tag{3}$$

where:
φ_r is the reduced creep coefficient.

Secondly, the stress-strain relation σ_c-ε_c for concrete is converted from integral into the algebraic form:

$$\sigma_c(t) = E_{c,eff}(\varepsilon_c - \varepsilon_{cs}), \tag{4}$$

where:
ε_{cs} is the concrete shrinkage strain.

This means that the variation of the stress σ_c in the interval t_o- t is neglected. Consequently, the analysis in time $t \rightarrow \infty$ remains exactly the same as the analysis at time t_0, having in mind that effective elastic modulus of concrete $E_{c,eff}$ should be used instead of the initial elastic modulus of concrete E_{c0}. This method of calculation was firstly proposed by Faber [12].

Viscoelastic analysis in accordance with Eurocode 4 (EC4) is based on the EM method. The only difference in calculation is contained in definition of the effective elasticity modulus of concrete $E_{c,eff}$:

$$E_{c,eff} = \frac{E_{cm}}{1+\psi_L \varphi_r} \tag{5}$$

where:
E_{cm} is the secant modulus of elasticity of the concrete for short-term loading,
ψ_L is the creep multiplier that depends on the load type.

5. NUMERICAL EXAMPLE

Methodology and comparison of results of four presented methods is provided on a simple continuous composite beam element, often found in real structures. The beam has different cross sections as showed in Figure 1 and is loaded with uniformly distributed load q. Both cross sections 1-1 and 2-2 consist of concrete and steel profile and geometrical properties are given in Table 1. The cross sections are doubly symmetric about two orthogonal axes, so the centroid lies at the intersection of those axes.

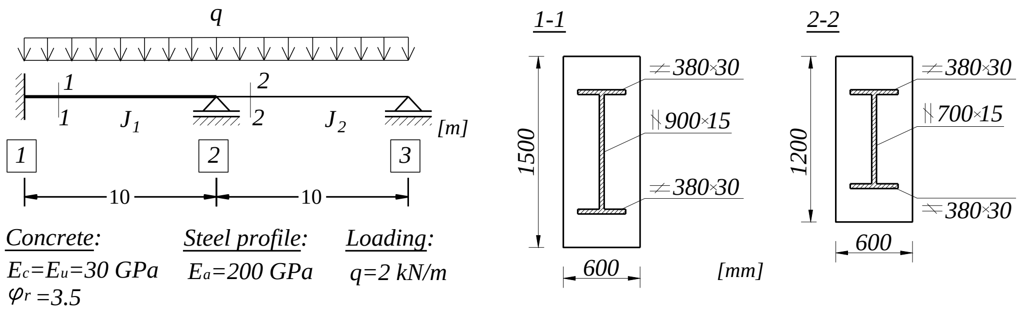

Figure 1 – Continuous composite beam and cross-sections 1-1 and 2-2

The system is statically indeterminate to the second degree while there is only one unknown generalized displacement rotation φ at point 2 (Figure 2). The calculation of the unknown displacement φ and the bending moments at points 1 and 2 will be based on displacement method.

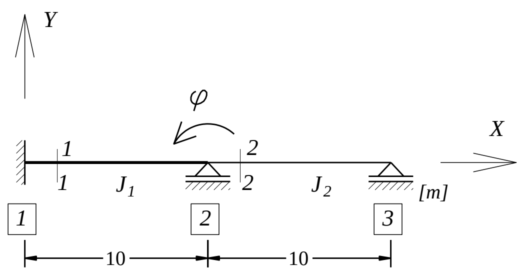

Figure 2 – Beam setup and unknown generalized displacement

Table 1 – Geometrical properties of cross-sections 1-1 and 2-2

Section	1-1	2-2
A_i (m^2)	0,165855	0,136305
J_i (m^4)	$3{,}02789565 \cdot 10^{-2}$	$1{,}59077915 \cdot 10^{-2}$

5.1. Calculation in time t_0 for all proposed methods

Due to the viscoelastic properties of concrete, the calculation of the composite beam is carried out for the time t_0 first (the moment when the first load is applied), and then for the time $t \rightarrow \infty$. In accordance with the linear theory of elasticity, the composite cross section will be replaced with an idealized cross section of homogeneous elastic material of modulus of elasticity E_u, cross sections $A_{i,0}$ and moments of inertia $I_{i,0}$. The beam from Figure 2 can be modeled with one element type "k" fixed on both ends 1 and 2 (element 1: L_1= 10 m), and one element type "g" fixed at end 2 with moment release at end 3 (element 2: L_2= 10m). Further, matrices of stiffness $[K_i]_0$ for each element (i=1,2) and whole system $[K]_0$ are created. The unknown rotation φ_0 at time t_0 is denoted from the expression:

$$\varphi_0 = \left(q\frac{L_1^2}{12} - q\frac{L_2^2}{8}\right) / \left(\frac{4E_cI_{1,0}}{L_1} + \frac{3E_cI_{2,0}}{L_2}\right) \tag{6}$$

were:

E_c is modulus of elasticity for concrete;

$I_{1,0}$ and $I_{2,0}$ are idealized moments of inertia of Sections 1-1 and 2-2, respectfully.

Further, unknown reactions and internal forces can be determined in a vector form $[R_i]_0$ using the well-known expression:

$$[R_i]_0 = [K_i\,]_0 \times [q_i]_0 - [Q_i]_0 \tag{7}$$

where:

$[q_i]_0$ is nodal displacements vector,

$[Q_i]_0$ is nodal load vector dependent on uniformly distributed load q for elements 1 and 2.

Results of the unknown rotation φ_0 at point 2 at time t_0, and bending moments M_1 and M_2 at points 1 and 2 for all four methods are given in Table 2. Values of bending moments M_1 and M_2 represent bending moments at left ends of element 1 and 2, respected.

5.2. Simplified method calculation in time t→∞

As explained above, we start from the "exact" method but incorporate the assumption that the unknown rotation φ changes linearly with the concrete creep function F^* during time, i.e. eq.(2):

$$\varphi = \varphi_0 1^* + \Delta\varphi(F^* - 1^*), \tag{8}$$

Opposite to integral equations as in "the exact" method, the unknown rotation φ is obtained from the following algebraic equation:

$$\left(\hat{A}'_{ki} + \hat{D}'_{ig}\right)\varphi = q\frac{l_1^2}{12}1^* - q\frac{l_2^2}{8}1^*, \tag{9}$$

where:
$\hat{A}'_{ki}$ is the element (*6,6*) of the operator stiffness matrix of the element *1;*
$\hat{D}'_{ig}$ is the element (*3,3*) of the operator stiffness matrix of the element *2.*
The creep function of the aging theory F^* with the constant concrete modulus of elasticity, and corresponding concrete relaxation function R^* are adopted as in [1]:

$$F^* = 1^* + \varphi_r, \qquad R^* = e^{-\varphi_r} \tag{10}$$

After $\Delta\varphi$ is determined, total rotation φ at $t \to \infty$ and M_1 and M_2 are given in Table 2.

5.3. EM method and EC4 calculation in time t→∞

As previously explained, the analysis at time $t \to \infty$ remains exactly the same as analysis at time t_0, perceiving that effective elastic modulus of concrete $E_{c,eff}$ should be used instead of the initial elastic modulus of concrete E_{c0}. Therefore, the composite cross section in above presented algorithm will be replaced with an idealized cross section of modulus of elasticity E_u, only following different rules for generating effective elasticity modulus of concrete $E_{c,eff}$ for EM or EC4. Further, areas of idealized cross sections $A_{i,t}$ and moments of inertia $I_{i,t}$ are calculated. The results of unknown rotation φ and bending moments M_1 and M_2 are shown in Table 2. The methods are listed in descending order of rotation φ accuracy when compared to the "exact" analysis method results.

Table 2 – The results of the rotation φ and moments M_1 and M_2 at time t_0 and $t \to \infty$

Results	t_0	$t \to \infty$			
	all methods	*"exact"*	*simplified*	*EC4*	*EM*
φ *[10^{-6} rad]*	-2,4678	-7,436	-7,0079	-6,7615	-6,5319
M_1 *[kNm]*	13,6777	13,731	13,7264	13,7237	13,7213
M_2 *[kNm]*	22,6445	22,538	22,5472	22,5525	22,5574

Bending moments M_1 and M_2 are following the same order as shown in Figure 3. As expected, the nearest to the "exact" method is simplified method. Nevertheless, proposed approximative methods, EC4 and EM methods, demonstrated high accuracy in calculation.

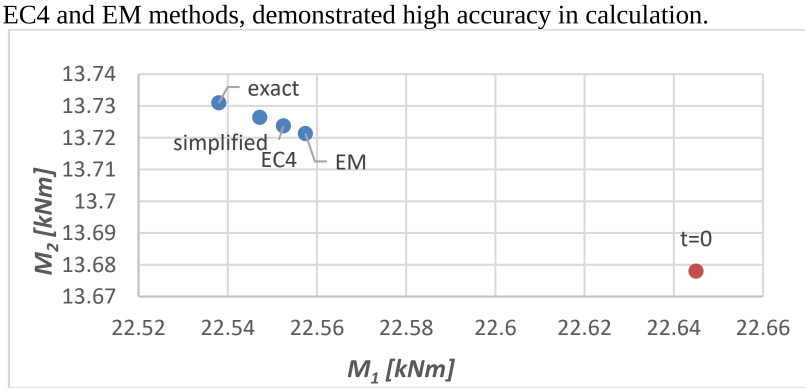

Figure 3 – M_1 and M_2 values in t_0 and $t \to \infty$ for four analysis methods

6. CONCLUSIONS

The aim of this paper was to compare results of different methods for viscoelastic analysis of composite steel-concrete beams. We included the following analysis methods: the exact method, the simplified method, EM method, and EC4 method. The calculation results are compared on the steel-concrete composite beam example. It is shown that despite the introduced simplification compared to the "exact" method, the simplified method preserves a high level of accuracy. It is very convenient as solving the system of nonhomogeneous algebraic equations instead of a system of nonhomogeneous integral equations saves a lot of effort. Moreover, approximate methods EC4 and EM are slightly less accurate than the simplified method. Among these, EC4 method is more accurate than the EM method.

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