



## LOW-CYCLE FATIGUE DAMAGE MODELING WITH HYSTERETIC ENERGY LOSS

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### Abstract

In this paper, hysteretic energy loss in cyclic loading is used as the main parameter in fatigue damage modeling. The analytical expression for hysteretic energy loss and its numerical implementation are features of the powerful hysteretic operator that has the ability to model progressive damage growth based on the maximum strain amplitude. Its scalar damage parameter is further modified in order to produce a reliable estimation of fatigue life under arbitrary loading.

**Key words:** fatigue, damage variable, hysteretic energy loss, mean stress effect

### 1. Introduction

While in general fatigue analysis, the material under cyclic loading can exhibit a various type of failure modes, the following research is concentrated on ductile materials such as typical constructional steel in a uniaxial stress state. Models and the analysis of fatigue in the material are versatile, and they are investigated on various modeling levels, from crack evolution up to the global estimation of fatigue life. Evaluating damage parameters in fatigue analysis based on various continuum damage approaches is developed by [1][2][3], and correlation of specific energy-based approaches is presented in [4].

In the presented analysis, the determination of damage in the material is based on the parameters that define the proposed model of hysteretic (Preisach) operator, particularly applicable for ductile materials. The first set of parameters produces an elastoplastic damage model that complies with Masing-type behavior. The second set, based on dissipation energy, is formed by experimentally obtained fatigue life measures so that material nonlinearity that includes phenomena of plastic deformation and damage is enhanced by the fatigue analysis.

### 2. Fatigue damage model and comparison to experimental results

Energy loss in one cycle for one element (operator) is calculated as a volume with a base between limit values of active the area in the Preisach plane [5], although it may also be approximated with the area of the hysteresis loop [6]. For the presented mechanical model

(infinitely many operators-elements), hysteretic energy loss  $Q_{hys}$  dissipated into heat is calculated analytically as follows:

$$Q_{hys} = \iint_{\Omega} P(\alpha, \beta) \cdot (\alpha - \beta) d\alpha d\beta \quad (1)$$

Preisach function  $P(\alpha, \beta)$  and its domain  $\Omega$  must be determined according to a specific type of experimental curve [7]. The scalar damage parameter presented in [8] is therefore extended in the function of hysteretic energy loss in process of damage analogous to the proposed relation in [9]. A similar approach is used in [10] [11], however, based on the experimental results and numerical models for the evolution of damage parameter presented in [2][12][13][14], this parameter needs to be adjusted with material constants for a specific type of damage growth in low-cycle fatigue loading. Total energy dissipation through cycles to failure is approximately equal to the parameter  $Wf$  for arbitrary cases of uniaxial loading in observed material. Type of the material for experimental data matching used in this paper is constructional steel and the conducted strain-controlled tests [15] with amplitudes 1%, 3%, 5%, and 7%. Different strain histories can be a significant factor in the fatigue analysis of steel specimens. Therefore, variable strain tests are also used to verify adopted parameters obtained in the constant strain amplitude tests. Furthermore, the mean stress effect for the proposed model is investigated through constructing failure curves for different fatigue life ( $N_f=2, 10, 100, 1000$ ) and presented in Fig.1. Results of the proposed model can also be verified through isochronous curves for failure [3].

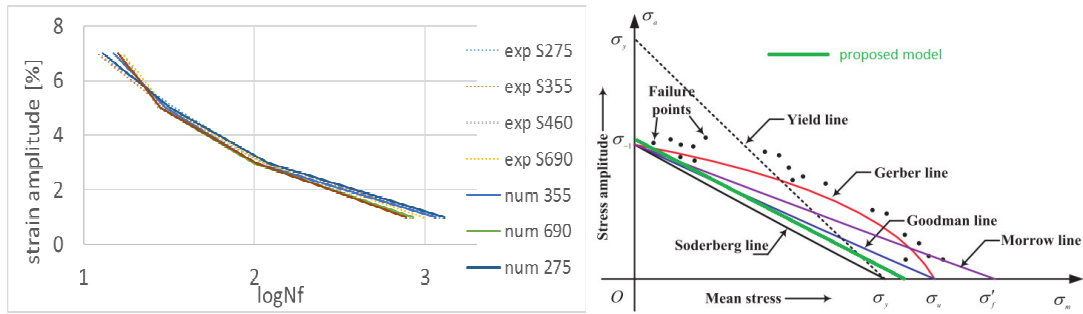


Fig.1 (a) Predicted number of cycles to failure vs experimental results [15] - logarithmic scale; (b) Proposed model mean stress effect vs schematic diagrams [16]

### 3. Conclusions

The presented fatigue damage model can determine fatigue life in the low-cycle regime of loading in a uniaxial stress state. Moreover, the shape of the resulting hysteretic curve is in good agreement with the experimental curve as this model is based on Preisach hysteretic operator, which also enables an analytical expression for calculating hysteretic energy loss. Thus, its numerical implementation provides an efficient solution that can capture various effects in an arbitrary low-cycle fatigue regime of loading.

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