PHYSICS INFORMED NEURAL NETWORKS FOR 1D FLOOD ROUTING

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Abstract:

Machine learning methods have been widely and successfully applied in hydrological problems. Most of the methods, such as artificial neural networks, have been focused on estimating hydrological data based on observation over time. Even though these models provide good results, it can be observed that results become unreliable when the training dataset is small or when input data is significantly out of range compared to the training data. Therefore, a new approach is presented, in which artificial neural networks are trained to satisfy physical laws. This is conducted by a novel method called physics-informed neural networks (PINNs), in which physical principles are embedded in a custom loss function. This paper presents the application of physics informed neural networks for solving 1D flood wave propagation in open channels. The research has shown promising results.

Keywords: physics informed neural networks, flood wave propagation, loss function

1. Introduction

Water resource management requires tools for long-term and, more often, short-term forecasting of various hydrological data (e.g. stage and flow hydrographs). This is significant in the field of flood risk management, hydropower plant control, inland navigation, water supply, etc. Numerous tasks related to hydrological data forecasting have been successfully treated by using a model-driven forecasting approach. Even though this approach provides excellent results and represents an inevitable step, there are still many issues to be addressed. In many cases, computational time is the parameter that can limit a real-world application of physically based hydrological and hydraulic models. Additionally, physically based models are insufficiently flexible in cases of inverse problems, parameter identification, introduction of the experimental data, etc. Therefore, machine learning (ML) methods are rapidly being applied in solving different hydrological-hydraulic problems, where artificial neural networks (ANN) are widely used.

ANN-based forecasting models have been applied in rainfall-runoff modelling [1], early flood warning systems [2] and the modelling of urban water networks [3]. These approaches require a large amount of data for training which can create problems when there is no data. In addition, although these models perform well in the exploitation phase, it can be seen that they are unable to give good results when input data are out of the range of the data used for training. In these cases, ANN models can produce physically impossible results. Hence, a new approach called physics-informed machine learning has been proposed recently [4]. ANNs are trained to emulate physical phenomenon using custom made loss (criteria) function, which specifies a physical law. This paper presents the potential of using physics-informed neural networks (PINNs) for 1D flood wave propagation in open channels by joining the physical law with the initial and boundary conditions described by kinematic wave propagation in the loss function.

2. Materials and methods

2.1 Flood routing – physical law

Flood wave propagation (flood routing) in open channels is described using two equations: the mass conservation principle, a continuity equation (1), and the momentum conservation principle, a dynamic equation (2). This equation contains impacts of friction, gravity, pressure force as well as local and convective acceleration. In this research, flood routing in a rectangular channel is represented by a kinematic wave which simplifies the dynamic equation using only friction and gravity impacts:

$$\frac{\partial h(x,t)}{\partial t} + c \cdot \frac{\partial h(x,t)}{\partial x} = 0 \tag{1}$$

$$Q(x,t) = \frac{1}{n} \cdot B \cdot h(x,t)^{\frac{5}{3}} \cdot \sqrt{I_d}, \tag{2}$$

where h $[m]$ represents water depth, t $[s]$ represents time, x $[m]$ represents spatial coordinate, c $[m/s]$ represents disturbance propagation velocity, Q $[m^3/s]$ represents flow (discharge), n $[m^{-\frac{1}{3}}s]$ represents Manning's roughness, B [m] represents cross-section width and Id [/] represents longitudinal slope. The goal of flood routing is to estimate water depth change along the channel $h(x,t)$ caused by the flood wave represented by flow hydrograph $Qin(t) = Q(0,t)$ at the upstream boundary.

2.2 Implementation of physics into neural networks – custom loss function

Physics-informed neural networks (PINNs) are trained to solve supervised learning tasks while respecting any given law of physics described by general nonlinear partial differential equations [4]. PINNs combine two networks together: an approximator network and a residual network. The approximator network undergoes training after which it provides a solution $\tilde{h}(x,t)$ at a given input point (x,y) called the *collocation point*, in the simulation domain [5]. The major innovation with PINN is the introduction of a residual network that encodes the governing differential equations, takes the output of an approximator network, and calculates a residual value r which acts as a loss function in deep-learning terminology. The residual network is not trained and its only function is to provide the approximator network with the residual:

$$r = \nabla h(x,t) - f(x,t), \tag{3}$$

where $f(x,t)$ is defined by the right-hand-side of equation (1). To solve PDE (1), the initial condition $h(x, t = 0) = 1.751\ m$ is required. Additionally, we impose the boundary condition at $(x = 0)$ derived from eq. (2):

$$h(0,t) = \left(\frac{Q_{in}(t) \cdot n}{B \cdot \sqrt{I_d}} \right)^{3/5}. \tag{4}$$

In addition to the governing equation (1), the residual network includes boundary and initial conditions, and their residues are also calculated and sent to the approximator network [5]. In the approximator model, the Mean Squared Error (MSE)

$$MSE = MSE_r + MSE_0 + MSE_b \tag{5}$$

is minimized, where:

$$MSE_r = \frac{1}{N_{x_f,y_f}} \sum \left| r(x_f, y_f) \right|^2,$$

$$MSE_0 = \frac{1}{N_{x_0,y_0}} \sum \left| \tilde{h}(x_0, 0) - h(x_0, 0) \right|^2,$$

and

$$MSE_b = \frac{1}{N_{x_b,y_b}} \sum \left|\tilde{h}(0,t_b) - h(0,t_b)\right|^2.$$

Here, N_{x_f,y_f}, N_{x_0,y_0} and N_{x_b,y_b} are the total number of collocation points on $h(x,t)$, initial and boundary conditions, respectively. To implement PINN with the given equations, we used SciANN [6]. SciANN is a Python package for scientific computing and physics-informed deep learning using artificial neural networks. SciANN employs widely used deep-learning packages Tensorflow and Keras to build deep neural networks and optimization models. It is designed to abstract neural network construction for scientific computations and solutions, but also for the discovery of partial differential equations (PDE) [6].

2.3 Test case

Physics informed neural network (PINN) application for flood routing was tested on a hypothetical test case where flood wave is propagated through a 1600m long prismatic channel (Fig. 1) with a rectangular cross-section (15 m wide). Manning's roughness was set to $n = 0.015\ m^{-\frac{1}{3}}s$, the longitudinal slope is set to $Id = 0.005$ and disturbance propagation velocity is set to c = 15 m/s. The goal was to calculate the water depth change along the channel, induced by flood wave generated as upstream boundary conditions (x = 0):

$$Qin(t) = Q(0,t) = 180 \cdot \left(1 + \left(-\frac{sgn(t-600)}{2} + 0.5\right) \cdot \sin\left(\frac{t \cdot \pi}{600}\right)\right). \tag{6}$$

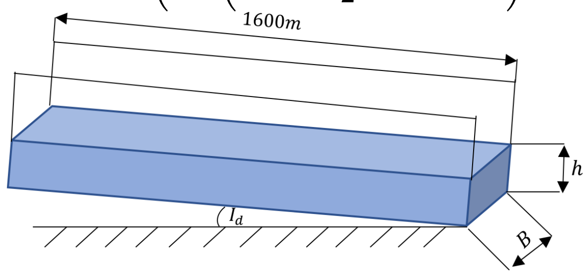

Fig 1. Test case: Flow through a prismatic rectangular channel

1.1 Physics informed neural network – model configuration

Physics-informed neural networks require independent variables x and t as inputs to predict the value of h. Those inputs are presented in the network as a grid of points x_i and y_i, whereby arranged pairs (x_i, y_i), $x_i = 0, \Delta x, 2 \cdot \Delta x, \ldots, 1600m$ and $y_j = 0, \Delta t, 2 \cdot \Delta t, \ldots, 999s$, represent the dataset for PINN model training. The values $\Delta x = 50$ m and $\Delta t = 3$ s represent the spatial step and time step, respectively. The artificial neural network, built using SciANN package, consists of 10 hidden layers, each containing 20 neurons. The activation of each of these neurons was defined by the Rectified Linear Units activation function. We used Adam optimizer with a learning rate of 10^{-3}, a batch size of 128 and taught the network for 1000 epochs. If the number of sampling points where boundary conditions are imposed is a small portion, the batch size should be set to a large value to always include some of these points and guarantee consistent optimization. The network was trained simultaneously on both training data and equation (1) by minimizing the loss (5) and by satisfying initial and boundary conditions [6].

3. Results and discussion

Once the network was trained, we compared the values $h(x,t)$ predicted by the PINN model to a numerical solution obtained by applying the finite difference method. For each value $x \in \{0m,\ 800m, 1600m\}$, the obtained results are given in Fig. 2.

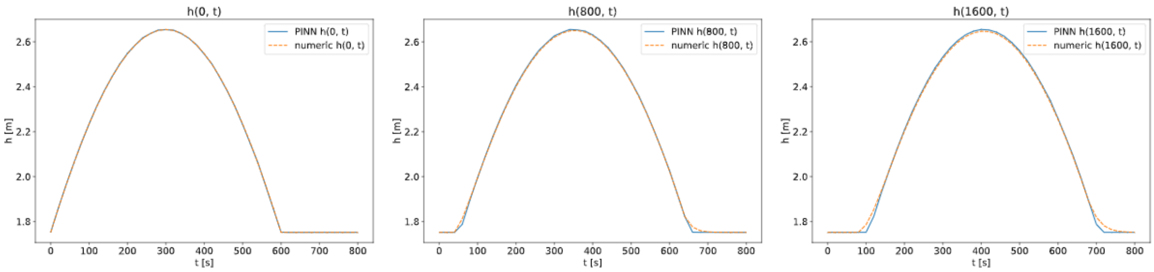

Fig 2. Stage hydrographs calculated using the finite difference method and PINN

The stage hydrographs provided by the PINN and by the finite difference method are very similar (**Fig 2**). These results are obtained for different values of Δx and Δt than those used during the training, which means that PINN generalizes sufficiently well. The values of Root Mean Squared Error (RMSE) of function h for each value $x \in \{0m,\ 800m, 1600m\}$ are given in Table 1.

Function	RMSE [m]
$\boldsymbol{h(0,t)}$	0.0009
$\boldsymbol{h(800,t)}$	0.0031
$\boldsymbol{h(1600,t)}$	0.0063

Table 1. Error of the PINN model

4. Conclusions

Based on the results, the PINN model for solving 1D flood wave propagation in open channels gives exceptional results compared to the results obtained by applying the finite difference method. The presented PINN-based approach for solving flood wave propagation requires less time and less input data, thus showing a high potential for application in hydrology modelling. However, for any real-world must of PINN models, a more complex physical law and real river geometry must be implemented, which will be must subject must forthcoming research.

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