Applied Mechanics and Materials Vols. 725-726 (2015) pp 869-874
Online available since 2015/Jan/29 at www.scientific.net
© (2015) Trans Tech Publications, Switzerland
doi:10.4028/www.scientific.net/AMM.725-726.869

Submitted: 13.10.2014
Revised: 06.11.2014
Accepted: 08.11.2014

Elasto-Plastic Stability Analysis of the Frame Structures Using the Tangent Modulus Approach

Stanko Ćorić[1,a*], Stanko Brčić[2,b], Nikolay Vatin[3,c]

[1,2]Faculty of Civil Engineering, University of Belgrade,11000 Beograd, Serbia

[3]St.Petersburg State Polytechnical University, 195251, Saint-Petersburg, Russia

[a]cstanko@grf.bg.ac.rs, [b]stanko@grf.bg.ac.rs, [c]vatin@mail.ru

Keywords: stability of structures, elastic-plastic analysis, tangent modulus.

Abstract. This paper presents the procedure for stability analysis of frames in elastic-plastic domain using the concept of the tangent modulus. When the buckling of structure occurs in plastic domain, it is necessary to replace the constant modulus of elasticity E with the tangent modulus E_t. Tangent modulus is stress dependent function and takes into account the changes of the member stiffness in the inelastic range. Formulation of the corresponding stiffness matrices is based upon the solution of the equation of bending of the beam according to the second order theory. Numerical analysis was performed using the code ALIN, developed in the C++ programming language.

Introduction

Calculation based on the theory of elastic stability is widely applied in the engineering practice. Namely, it can be assumed that engineering structures have generally elastic behavior when they are subjected to the usual working loads. Therefore, it is clear that such theory is the basis of the standards related to stability analysis of the framed structures [1, 2]. This calculation is defined by the determination of the effective buckling length of the compressed columns. However, stability calculation becomes more complicated if, before the critical load is achieved, some compressed members enter into the phase of nonlinear material behavior. It means that stresses in such columns become higher than the proportionality limit. Therefore, such calculation obtains another type of nonlinearity and it becomes also materially (or physically) nonlinear problem. Investigation of the buckling in the elastic-plastic domain has always been of interest to the researchers in the field of steel structures. Many authors have dealt with such kind of problems, for example [3, 4, 5], and many corresponding solutions have been suggested. In this paper the problem is analysed using the tangent modulus concept [6]. It means that elastic modulus will be replaced by the tangent modulus (E_t) to represent the distributed plasticity along the length of the member due to yielding caused by the axial force. The value of tangent modulus is a function of the member's axial loading state, and is often evaluated from the capacity specification equations of the column.

Stability analysis of the plane frame structures in inelastic domain.

In the elastic range there is a linear relationship between stresses and strains, so the modulus of elasticity E is constant. Taking into consideration well known expression for the Euler's critical force, critical stress in a member may be expressed as a function of the modulus of elasticity (E) and the slenderness ratio (λ_i):

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{l_i^2 A} = \pi^2 \frac{E}{\lambda_i^2} \tag{1}$$

This equation of hyperbola is valid until the critical stress is less than a proportionality limit, as it is shown in Figure 1.

When this stress is exceeded, the member is buckling in a plastic range. Many scientists were dealing with this problem. Bauschinger first made, at the end of the nineteenth century, an experimental study of this problem. On the basis of this results and his own research, Tetmajer later suggested expression for the linear relation between stress and slenderness in the plastic domain. Engesser had noticed in all the tests performed [7] the deviation of the column buckling curve from the Euler's theoretical one. Many other scientists also investigated these problems, as Karman and Shanley who modified Engeser's curve. Some of the most significant buckling curves in the plastic domain are given in Fig. 1.

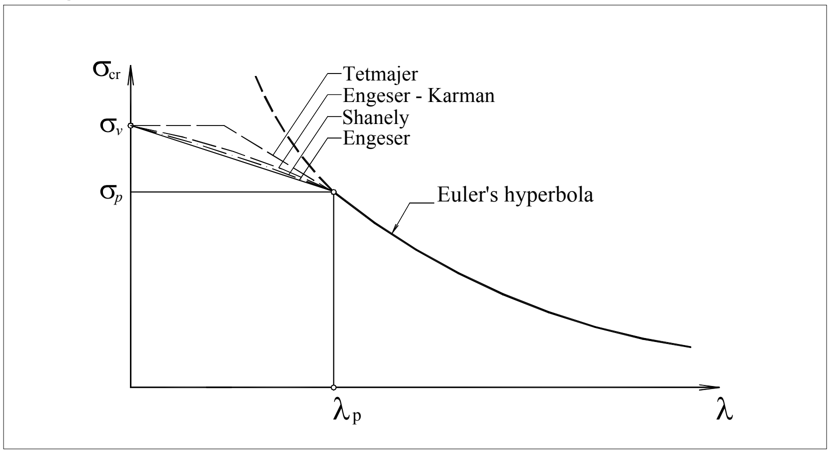

Figure 1. Buckling curves in the plastic domain

As it is already mentioned, in the case of elastic stability problem, the modulus of elasticity E has a constant value. But, elastic-plastic analysis is more complicated. For the structural member where the proportionality limit is exceeded, for each new load increment the member stiffness has to be changed and the corresponding tangent modulus E_t should be used for that member. The generally accepted approach applied to this problem is the tangent modulus concept [6]. It is based on the observation that the load-shortening relationship, for a section made of a material such as structural steel, is affected by the residual stresses that result inevitably form the manufacturing process. Thus, the effective stress-strain diagram of the material is as in Fig. 2.

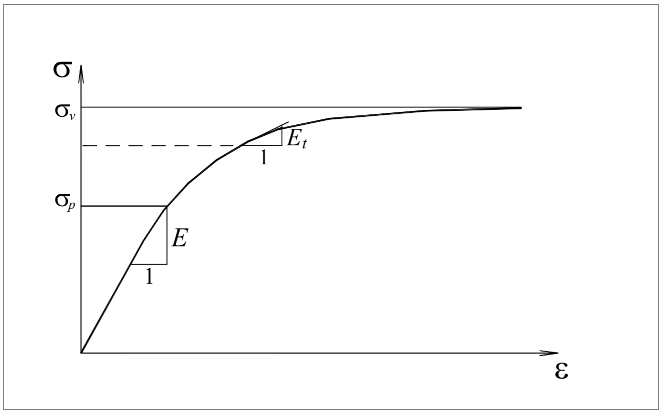

Figure 2. Stress-strain diagram of structural steel

Below a proportionality limit σ_p, it is elastic. Above that point it is inelastic with gradually decreasing resistance, measured by the tangent modulus E_t. The theory postulates that, for an ideally straight column with an elastic critical stress greater than σ_p, bifurcation of equilibrium can occur and the column will start to buckle at the load [8]

$$P_{cr,i} = \frac{\pi^2 E_t I}{L^2} \tag{2}$$

where E_t is the tangent modulus corresponding to the stress $\sigma_{cr,i} = P_{cr,i} / A$.

Eq.(2) is mathematically identical to the expression for the Euler load ($P_{cr} = \pi^2 EI/L^2$). But the difference is in the fact that E is only a function of the type of material and E_t is also a stress dependent function. A frequently used [6] relationship between the two moduli is

$$E_t = 4E \cdot \left[\frac{\sigma}{\sigma_y} \left(1 - \frac{\sigma}{\sigma_y} \right) \right] \tag{3}$$

This is an empirical expression designed to represent the behavior of structural steel columns in the inelastic range. Implicitly this expression takes the assumption that the proportionality limit is half of the yield point. This expression was used in development of the program ALIN related to the nonlinear elastic-plastic analysis of frame structures.

The finite element method, as the most efficient numerical method for the solution of various numerical problems, is applied in this paper. As it is well known, when using the finite element method, the critical load may be obtained from the homogeneous matrix equation as the non-trivial solution:

$$\mathbf{K} \cdot \mathbf{q} = 0 \tag{4}$$

In Eq. (4) **K** is the global stiffness matrix for the whole frame, including the corresponding boundary conditions, while q represents the vector of generalized coordinates. This problem can be solved by an incremental process, by increasing the load at the specified increments until the critical value is reached, i.e. until det **K** = 0 is obtained.

In order to formulate the exact matrix stability analysis, it is necessary to obtain the corresponding stiffness matrix. Interpolation functions should be derived from the solution of the differential equation of bending according to the second order theory. Such interpolation polynomials are obtained as trigonometric or hyperbolic functions of the axially loaded element. The advantage of such approach is in the fact that only one finite element is needed for each beam or column, so the total number of finite elements is 5-10 times less than in the usual approach based on the geometric stiffness matrix.

Stiffness matrix for the member of the so-called type “k“ (i.e. clamped at both ends), subjected to compressive force is given by [9]:

$$K = \frac{E_t I}{l^3 \Delta_t} \begin{bmatrix} \omega_t^3 \sin\omega_t & \omega_t^2 l(1-\cos\omega_t) & -\omega_t^3 \sin\omega_t & \omega_t^2 l(1-\cos\omega_t) \\ & \omega_t l^2(\sin\omega_t - \omega_t\cos\omega_t) & -\omega_t^2 l(1-\cos\omega_t) & \omega_t l^2(\omega_t - \sin\omega_t) \\ & & \omega_t^3 \sin\omega_t & -\omega_t^2 l(1-\cos\omega_t) \\ & \text{symm.} & & \omega_t l^2(\sin\omega_t - \omega_t\cos\omega_t) \end{bmatrix} \tag{5}$$

where:

$$E_t = 4E \cdot \left[\frac{P_{cr,i}}{A \cdot \sigma_y} \left(1 - \frac{P_{cr,i}}{A \cdot \sigma_y} \right) \right] \tag{6}$$

$$\Delta_t = 2 \cdot (1 - \cos\omega_t) - \omega_t \cdot \sin\omega_t \tag{7}$$

$$\omega_t = \sqrt{\frac{P_{cr,i}}{E_t \cdot I}} \cdot l = \frac{1}{2} A\sigma_v l \cdot \sqrt{\frac{1}{EI(A\sigma_v - P_{cr,i})}} \tag{8}$$

It is obvious that stiffness matrix for nonlinear material behavior has the same form as for the linear behavior of the material, but they are essentially very different. Namely, the difference is primarily in the fact that constant modulus E is replaced by stress dependent tangent modulus E_t.

The critical load for the whole frame is then obtained as the solution of the corresponding stability equation:

$$\det \mathbf{K}(\omega_t) = 0 \qquad (9)$$

On the basis of this theoretical approach, a numerical example will illustrate this elasto-plastic stability analysis of steel frames. For the numerical analysis, the corresponding computer code ALIN is used. The code, developed using C++ language, enables the complex linear analysis of the plane and space frames. The basic possibilities of this program are analysis according to the first and the second order theory, dynamic analysis and stability analysis, i.e. calculation of the critical load in the elastic and inelastic domains.

Numerical analysis

The six-story three-bay non-sway steel frame is analyzed. The frame is clamped at the base, with the load on each column at each story. The span lengths are 10m, and the height of stories is 5m, as it is given in Fig. 3.

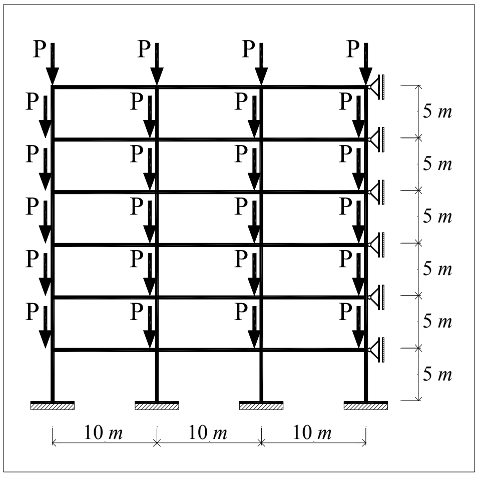

Figure 3. Numerical examples – six-story three-bay non-sway frame

The numerical analysis is performed for five different cross-sections. So, it is assumed that all columns and girders in the frame have cross-sections 2 [8, 2 [12, 2 [16, 2 [20 and 2 [26. First, applying the elastic analysis, the critical load is obtained. In that case the modulus of elasticity has a constant value E = 210,000,000 kN/m^2. But, when the stresses in the columns are higher than the proportionality limit, the inelastic stability analysis is performed. Then the modulus of elasticity (that is now tangent modulus) becomes stress dependent. The yield stress of the steel is σ_v = 240,000 kN/m^2. Table 1 presents obtained results of the critical load for all six considered cross sections.

Table 1. Values of critical load for the frame given in Figure 3

	elastic analysis	inelastic analysis
2[8	$P_{cr,el}$ = 64.91 kN	$P_{cr,inel}$ = 61.10 kN
2[12	$P_{cr,el}$ = 161.03 kN	$P_{cr,inel}$ = 116.65 kN
2[16	$P_{cr,el}$ = 323.37 kN	$P_{cr,inel}$ = 175.87 kN
2[20	$P_{cr,el}$ = 596.29 kN	$P_{cr,inel}$ = 242.99 kN
2[26	$P_{cr,el}$ = 1304.03 kN	$P_{cr,inel}$ = 371.98 kN

From the Table it can be seen what differences in results are obtained when the calculation is performed in elastic and elastic-plastic domain respectively. This differences increase for the frames with larger stiffness.

The results of the elastic modulus and tangent modulus at the moment of buckling are presented in the Tab. 2. Since the axial force in columns is not constant, the elastic-plastic stability analysis leads to different behavior of the columns in the different floors. Therefore, the results for the columns in all stories of the analyzed frame are given separately.

Table 2. Values of tangent modulus for the frame given in Figure 3

	1st floor	2nd floor	3rd floor
2[8	E_t = 178,264,572 kN/m^2	E_t = 204,804,847 kN/m^2	E = 210,000,000 kN/m^2
2[12	E_t = 102,512,698 kN/m^2	E_t = 171,256,295 kN/m^2	E_t = 205,668,274 kN/m^2
2[16	E_t = 64,644,826 kN/m^2	E_t = 151,756,926 kN/m^2	E_t = 199,714,531 kN/m^2
2[20	E_t = 44,940,849 kN/m^2	E_t = 141,258,497 kN/m^2	E_t = 196,053,029 kN/m^2
2[26	E_t = 30,175,286 kN/m^2	E_t = 133,268,263 kN/m^2	E_t = 193,112,364 kN/m^2
	4th floor	5th floor	6th floor
2[8	E = 210,000,000 kN/m^2	E = 210,000,000 kN/m^2	E = 210,000,000 kN/m^2
2[12	E_t = 205,748,633 kN/m^2	E = 210,000,000 kN/m^2	E = 210,000,000 kN/m^2
2[16	E_t = 208,517,641 kN/m^2	E_t = 178,166,256 kN/m^2	E = 210,000,000 kN/m^2
2[20	E_t = 209,324,446 kN/m^2	E_t = 181,072,747 kN/m^2	E = 210,000,000 kN/m^2
2[26	E_t = 209,707,588 kN/m^2	E_t = 183,053,935 kN/m^2	E_t = 143,151,406 kN/m^2

From these results it is possible to notice the usefulness of applying the inelastic stability analysis for considered frame structures. It is clear that frames with larger stiffness can be exposed to larger critical load, so the stress in their columns can exceed the proportionality limit of the material. Therefore, it is necessary to perform stability analysis in inelastic domain. So, the physical properties of materials are changed and the corresponding tangent modules should be calculated. Again, it should be emphasized that the values of this module depend on the axial force in the columns. Tab. 2 also shows the difference in the behavior of columns in different floors. Obviously, the overall buckling of the multi-story frame is governed by the columns in the first, most loaded floor.

Summary

The paper is presenting the procedure for stability analysis of the frame structures in elastic-plastic domain. Performed numerical analyses show the advantages of this procedure when compared to the standard elastic stability analysis. Stiffness matrices are derived using the tangent modulus which is stress dependant and follows changes of the member stiffness in the inelastic field. These matrices have been implemented in the computer code ALIN. The presented algorithm introduces more accurate calculation of buckling in plastic domain. It allows monitoring the phenomena of stability loss of the frame structure in the plastic domain and direct determination of the critical force at the moment of buckling.

Acknowledgement.
Authors are grateful for the financial support of the Ministry of education, science and technological development of the Republic of Serbia within the project TP 36043.

References

[1] Eurocode 3. Design of Steel Structures - ENV 1993-1-1: 2005, European Committee for Standardization, (2005).

[2] Zarić, B., Buđevac, D., Stipanić, B. Steel Structures in Civil Engineering (2000) Građevinska knjiga, Belgrade, 373 p.

[3] Allen, H. G., Bulson, P. S. Background to buckling (1980) McGraw Hill, U.K., 284 p.

[4] Gaylord, E. H. Design of steel structures, (1991) McGraw-Hill Co, New York, 427 p.

[5] Galambos, T. V. Guide to Stability Design Criteria for Metal Structures, 5th edition (1998) John Wiley and Sons, New York, USA, 273 p.

[6] Engesser, F. Ueber die Knickfestigkeit Gerader Stabe, Zeistschrift fur Architektur und Ingenieurwesen (1892) 382 p.

[7] McGuire, W., Gallagher, R., Ziemain, R. Matrix Structural Analysis, 2nd edition (2000) John Wiley and Sons, New York, USA, 398 p.

[8] Ćorić, S. Nonlinear stability analysis of the frame structures, doctoral thesis (2013) Faculty of Civil Engineering University of Belgrade, 185 p.

[9] Horne, M. R., Merchant, W. The stability of frames (1965) Pergamon, Oxford, 352 p.

Innovative Technologies in Development of Construction Industry
10.4028/www.scientific.net/AMM.725-726

Elasto-Plastic Stability Analysis of the Frame Structures Using the Tangent Modulus Approach
10.4028/www.scientific.net/AMM.725-726.869