Critical state constitutive model for overconsolidated clays

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Abstract. The paper presents a simple critical state bounding surface constitutive model for describing the mechanical behaviour of overconsolidated clays. Keeping the simplicity and the same set of the parameters as Modified Cam Clay model, new model provides a more realistic description of numerous elements of clay behaviour. The novel form of the hardening rule was proposed with the state parameter and the degree of overconsolidation as the state variables. Expressing the hardening parameter through the state parameter of the stress point on a loading surface and the state parameter of a conjugate stress point on the bounding surface, strain hardening and strain softening in drained conditions, as well as negative pore pressure in undrained conditions are well described. Inner loading surface always passes through the current stress point, thus enabling elasto-plastic soil behaviour even in early stages of loading. The model overcomes many deficiencies of the Modified Cam Clay model as demonstrated on a broad experimental evidence.

Keywords: Constitutive model, overconsolidated clays, state parameter

1 INTRODUCTION

The development of advanced, but at the same time, for the use in engineering practise simple constitutive model for soil is essential for the rational design of the geotechnical structures. Bearing in mind that simple expressions and clear physical meaning of the model parameters are an imperative for practical application of constitutive models, the HArdening State Parameter model (HASP) for describing mechanical behaviour of overconsolidated clays is developed on the basis of the critical state theory and within the concept of bounding surface plasticity (Dafalias and Herrmann 1980). The HASP model overcomes many deficiencies of the Modified Cam Clay model (MCC) (Roscoe and Burland 1968): inadequate predictions of the behaviour on dry side, large elastic region, as well as sudden transition from elastic region into plastic region. At the same time, HASP model retains the same simplicity and the same set of parameters as MCC model.

2 HASP MODEL

Relations of the HASP model are based on the following principles: soil is isotropic, plastic strains develop from the very beginning of loading, hardening parameter depends on the increments of plastic volumetric and shear strains. Bounding surface is the MCC surface. Point A (p',q) representing the current stress state is always on the inner yield surface, Figure 1a:

$$\frac{p'}{p'_0} = \frac{M^2}{M^2 + \eta^2} \tag{1}$$

Associated flow rule applies, i.e. plastic strain increment vector is always normal to the yield surface.

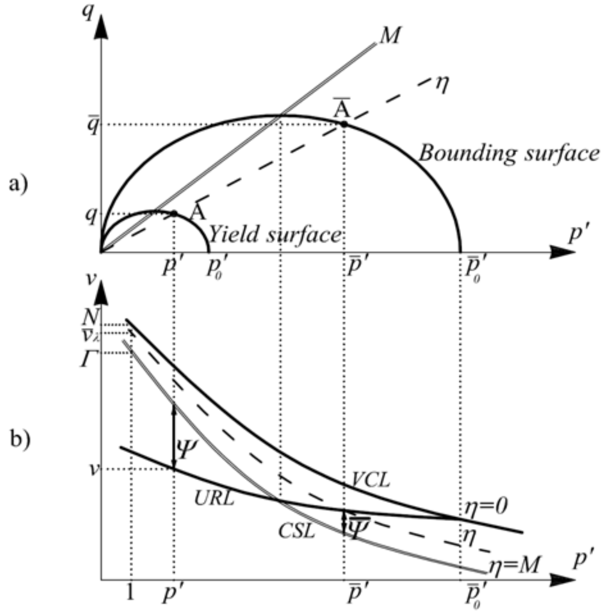


Figure 1. a) Bounding surface concept b) State parameters

Bounding surface possesses all the characteristics of the MCC surface. For stress ratio below the critical state line (CSL), the volume decreases and the surface expands, while for stress ratio above the critical state line, the volume increases and the surface shrinks. Yield surface expands until peak strength is reached at stress ratio $\eta=M_f$, after which it shrinks (softening) until critical state is reached.

2.1 Hardening rule for the HASP model

Volumetric hardening rule does not allow negative dilatancy to develop for overconsolidated soils before the peak strength is reached. In order for the yield surface to continue expanding also for stress ratio values $M<\eta<M_f$, it is necessary to use combined hardening and express the hardening rule as a function of plastic shear strain also (Nova and Wood 1979, Yao et al. 2009) as follows:

$$dp_0' = \frac{v}{\lambda-\kappa} p_0' \left(d\varepsilon_v^p + \xi d\varepsilon_q^p \right) \tag{2}$$

where ξ is a parameter discussed later in the paper, v is specific volume, λ is a slope of the virgin compression line (VCL), κ is a slope of an swelling line (URL) in v-lnp' plane, Figure 1b. The combined hardening significantly affects the stress path. This formulation allows the effective stress path to cross the CSL and reach the peak in drained conditions. In an undrained test, the combined hardening is key to achieve *"S"* shaped effective stress path. If the plastic shear strain increment is expressed through the dilatancy $d = d\varepsilon_v^p / d\varepsilon_q^p$ and if current overconsolidation ratio is defined as:

$$R = \frac{\bar{p}'}{p'} = \frac{\bar{q}}{q} = \frac{\bar{p}_0'}{p_0'} \tag{3}$$

the hardening rule for the yield surface can be written as:

$$dp_0' = \frac{v}{\lambda-\kappa} p_0' d\varepsilon_v^p \left(1 + \frac{\xi}{d} \right) R = \frac{v}{\lambda-\kappa} p_0' d\varepsilon_v^p \omega \tag{4}$$

where ω is the *hardening coefficient*:

$$\omega=\left(1+\frac{\xi}{d}\right)R \tag{5}$$

Expressions for plastic strains thus becomes:

$$d\varepsilon_v^p=\frac{\lambda-\kappa}{v}\frac{1}{p'}\frac{1}{\omega}\left(\frac{M^2-\eta^2}{M^2+\eta^2}dp'+\frac{2\eta}{M^2+\eta^2}dq\right) \tag{6}$$

$$d\varepsilon_q^p=\frac{\lambda-\kappa}{v}\frac{1}{p'}\frac{1}{\omega}\left(\frac{2\eta}{M^2+\eta^2}dp'+\frac{4\eta^2}{\left(M^2+\eta^2\right)\left(M^2-\eta^2\right)}dq\right) \tag{7}$$

The hardening coefficient is at the same time the reduction coefficient for plastic strains. It is then possible to assume that soil deforms plastically from the very beginning of loading. When peak strength is reached (transition from hardening to softening), $dp_0'=0$ applies and maximum gradient of volume change (negative dilatancy) is noticeable, marked as d_{min} by the compression-positive convention. Based on Eq. (4) it can be concluded that if ω=0 then $\xi=-d_{min}$ i.e. parameter ξ is the absolute value of dilatancy at peak strength in drained conditions, which is in line with the considerations stated in Nova (2006).

2.2 State parameter

According to works of Parry (1958), Li and Dafalias (2000), Jefferies and Been (2006), dilatancy is not a function of the stress ratio η only, it also depends on the state parameter Ψ. State parameter represents the difference between the current specific volume and the specific volume on the reference state line (CSL) at the same mean effective stress, Figure 1b, (Been and Jefferies 2006). State parameter for the current stress state, i.e. point on the yield surface, can be expressed as:

$$\Psi=v+\lambda \ln p'-\Gamma \tag{8}$$

where Γ is specific volume on the CSL for reference pressure (p'=1kPa). State parameter is negative for highly overconsolidated clays $\Psi<0$, while for lightly overconsolidated and normally consolidated clays state parameter is positive $\Psi>0$. When stress point reaches the CSL then $\Psi=0$. State parameter for conjugate point on the bounding surface can be expressed as:

$$\overline{\Psi}=(\lambda-\kappa)\ln\left(\frac{2M^2}{M^2+\eta^2}\right) \tag{9}$$

Also, the current overconsolidation ratio via state parameters can be expressed as:

$$R=\frac{\overline{p'}}{p'}=\frac{\overline{q}}{q}=\exp\left(\frac{\overline{\Psi}-\Psi}{\lambda-\kappa}\right) \tag{10}$$

In the expression for the hardening coefficient (5) it is necessary to define the ratio ξ/d. Detailed explanation is given in Jocković and Vukićević (2017) and the following expression for the hardening coefficient is proposed:

$$\omega=\left(1+\frac{\overline{\Psi}-\Psi}{\overline{\Psi}}\right)R \tag{11}$$

For normally consolidated clays, the HASP model automatically transforms into the MCC model since $\Psi = \overline{\Psi}$ and hardening coefficient is then $\omega=1$.

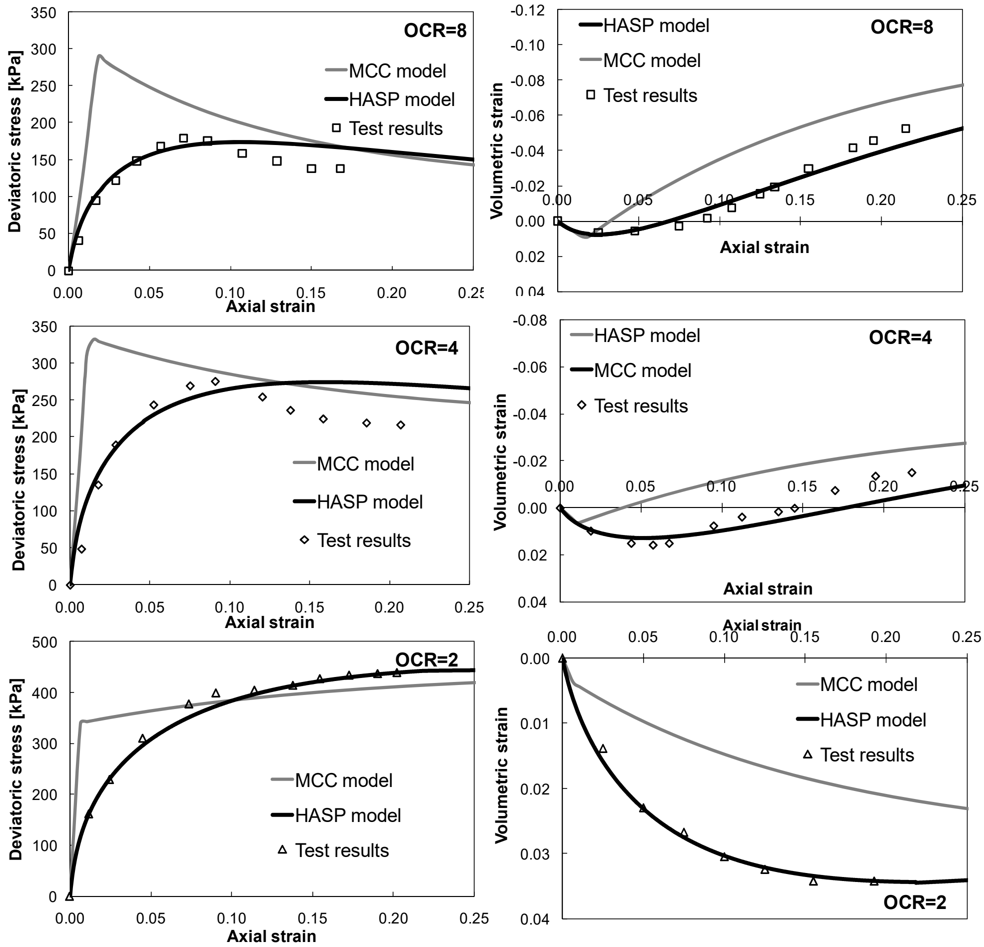
3 VALIDATION OF THE HASP MODEL

Validation was done against published results of drained and undrained tests in triaxial compression and extension, on the samples with various overconsolidation ratios. The HASP model requires five material parameters for describing stress – strain relations. All the parameters can be easily determined by the conventional laboratory tests and they are summarized in Table 1. Considering that the HASP model is formulated to overcome the basic drawbacks of the MCC model, comparison has been made between experimental results, HASP model and MCC model.

Table 1. Parameters of the HASP model

	λ	κ	M_c	M_e	Γ	μ
Kaolin clay (Biarez & Hicher 1994), CD tests	0.230	0.030	0.81	/	3.44	0.2
Cardiff clay (Banerjee & Stipho 1979), CU tests	0.140	0.050	1.05	0.85	2.63	0.2

The hardening behaviour of Kaolin clay in drained conditions is well predicted by the HASP model for all overconsolidation ratios (OCR=8, 4, 2). For heavily overconsolidated tests, the HASP model predicts a drop in strength – softening, Figure 2a. The dilatant behaviour has been observed and excellent prediction of the change of volumetric strains is achieved, Figure 2b.



a) b)

Figure 2. CD tests, Kaolin clay a) Stress-strain relations b) Volumetric strains

Large deviations from experimental results were recorded for the MCC model, especially for high overconsolidation ratio. Peak strength is overestimated up to two times.

Results of two undrained triaxial compression tests for remoulded samples of Cardiff Kaolin clay with overconsolidation ratios 5 and 12 are presented, as well as results of two undrained triaxial extension tests for remoulded samples with overconsolidation ratios 6 and 10. Very good agreement with experimental results is noticeable for HASP model (stress-strain relations – Figure 3 and porewater pressure changes – Figure 4), for all overconsolidation ratios in triaxial compression and extension tests. General form of the effective stress paths (normalized with the equivalent mean effective stress on the VCL) depending on the overconsolidation ratios is well predicted, Figure 5. Disadvantages of the MCC model can be seen in undrained conditions, also.

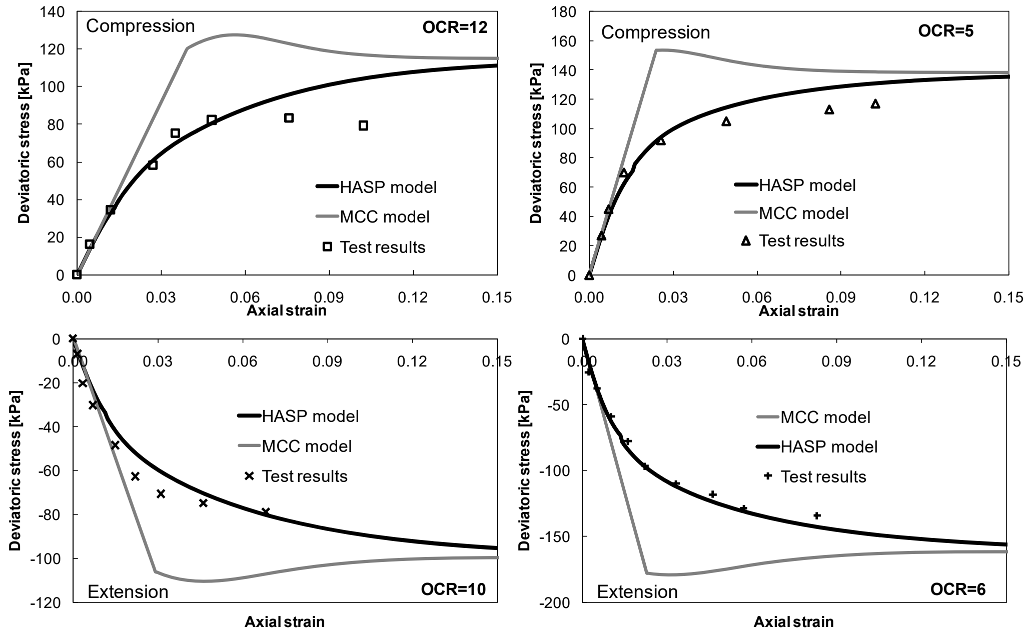


Figure 3. CU tests, Cardiff clay – stress-strain relations

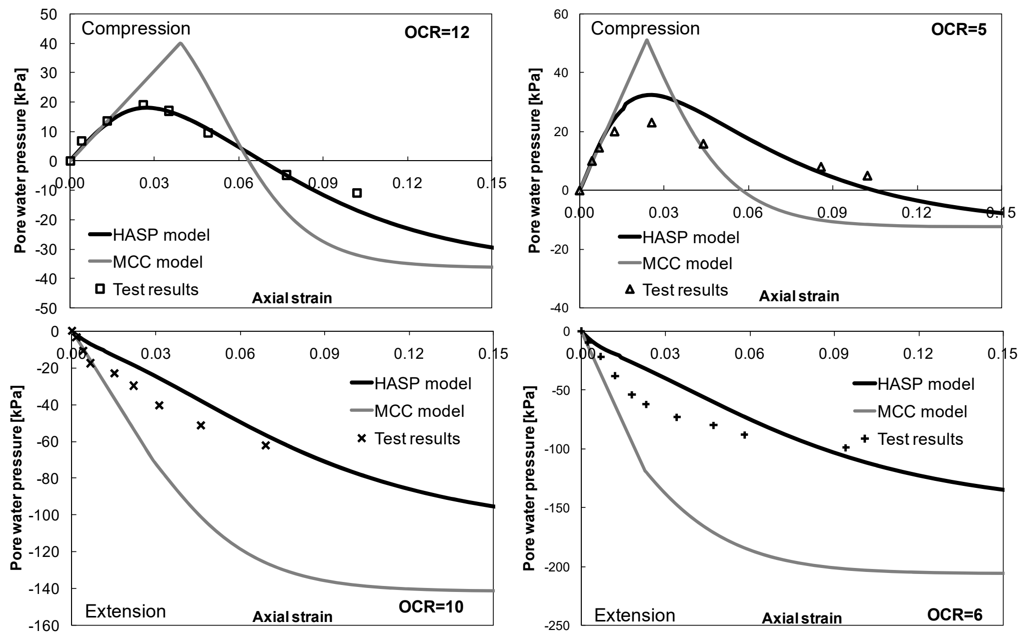


Figure 4. CU tests, Cardiff clay – pore water pressure

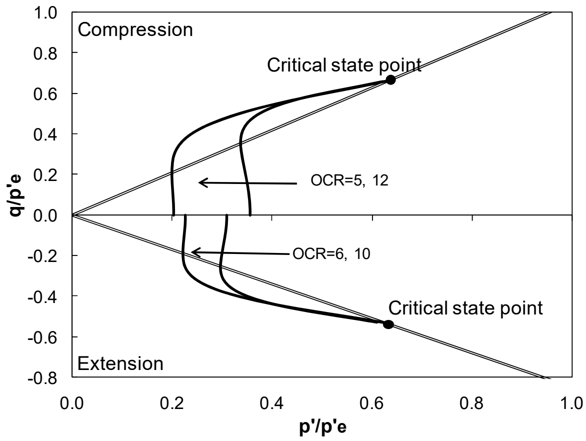


Figure 5. CU tests, Cardiff clay – normalized effective stress paths

4 CONCLUSIONS

With novel form of the hardening rule it is possible to describe a number of elements of the mechanical behavior of overconsolidated clays. In drained conditions, unlike the MCC model, HASP model predicts the smooth transition from contractive to dilatant behaviour before the peak strength is reached and a smooth transition from hardening to softening, without mathematical description. In undrained conditions, the general form of the effective stress paths depending on the overconsolidation ratio is well predicted, as well as pore water pressure. There is no pure elastic domain, but the hardening coefficient reduces plastic strains depending on the current degree of overconsolidation. For normally consolidated clays the HASP model transforms into the Modified Cam Clay model.

REFERENCES

Banerjee, P.K. and Stipho, A.S. (1979). Elastoplastic model for undrained behavior of heavily overconsolidated clays. *Int J Numer Anal Mech Geomech*; 3(1): 97-103.

Been, K. and Jefferies, M.G. (1985). A state parameter for sands. *Géotechnique*, 35(2): 99-112.

Biarez, J. and Hicher, P.Y. (1994). *Elementary Mechanics of Soil Behaviour. Saturated Remoulded Soils*. A. A. Balkema, Rotterdam.

Dafalias, Y.F. and Herrmann, L.R. (1980). A bounding surface soil plasticity model. *Proc. Int. Symp. on Soils under Cyclic and Transient Loading*, Vol. 1, Swansea, U.K., 335–345.

Jefferies, M.G. and Been, K. (2006). *Soil Liquefaction: A critical state approach*. Taylor and Francis, Abingdon.

Jocković, S. and Vukićević, M. (2017). Bounding surface model for overconsolidated clays with new state parameter formulation of hardening rule. *Comput Geotech*, 83:16–29.

Li, X.S. and Dafalias, Y.F. (2000). Dilatancy for cohesionless soils. *Géotechnique*, 50:449–60.

Nova, R. and Wood, D.M. (1979). A constitutive model for sand in triaxial compression, *Int J Numer Anal Methods Geomech*, 3: 255–78.

Nova, R. (2006). Modelling of bonded soils with unstable structure. International Workshop on Modern Trends in Geomechanics – Vienna, Springer.

Roscoe, K.H. and Burland, J.B. (1968). On the generalized stress-strain behaviour of wet clay. *Engineering Plasticity*, Cambridge University Press, 535 – 609.

Yao, Y.P., Hou, W., Zhou, A.N. (2009). UH model: three-dimensional unified hardening model for overconsolidated clays, *Géotechnique*, 59:451–69.