UDC: 69.059:510.644
Izvorni naučni članak

APPLICATION OF FUZZY AHP METHOD FOR CHOICE OF OBJECTS FOR MAINTENANCE AND RECONSTRUCTION

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SUMMARY

The choice of objects for maintenance and reconstruction using Fuzzy AHP (Analytic Hierarchy Process) method will be presented in this paper. This method, as one of the efficient methods for multicriteria decision making (MCDM), will be shortly described. Since in these and other similar problems, relevant input data and parameter are usually imprecise and could not be exactly determined, they are expressed in this work by the triangular fuzzy numbers. The authors have developed corresponding computer program according to given procedure, that has been used several times in the practice, and one example of choice of bridge structures based on a risk assessment during exploitation, will be presented in the paper.

KEY WORDS: AHP method, Maintenance and rehabilitation, Risk assesment

PRIMENA RASPLINUTE (FUZZY) AHP METODE ZA IZBOR OBJEKATA ZA ODRŽAVANJE I REKONSTRUKCIJU

REZIME

Izbor objekata za održavanje i rekonstrukciju, koristeći Rasplinutu (Fuzzy) metodu Analitičkog hijerarhijskog procesa (AHP) biti će prikazan u ovom radu. Ova metoda, kao jedana od efikasnih metoda za višekriterijumsko donošenje odluka (MKDO), će biti ukratko izložena. Pošto su u ovim i drugim sličnim problemima relevnatni ulazni podaci i parametri najčešće veličine koje se ne mogu precizno odrediti, to se one u ovom radu izražavaju pomoću trouglastih rasplinutih (fuzzy) brojeva. Autori su prema ovoj proceduri razvili odgovarajući računarski program, koji je nekoliko puta bio korišćen u praksi, i jedan primer izbora mostovskih konstrukcija za rekonstrukciju na osnovu procenjenog rizika tokom eksploatacije će biti prikazan u ovome radu.

KLJUČNE REČI: AHP metod, Održavanje i rehabilitacija, Procena rizika

INTRODUCTION

The Analytic Hierarchy Process (AHP), as one of important methods for multicriteria decision making (MCDM), was proposed by Thomas Saaty (Saaty, 1980). In this process factors are selected on different levels in a hierarchy structure descending from one overall goal to criteria and alternatives, as it shown in Fig. 1.

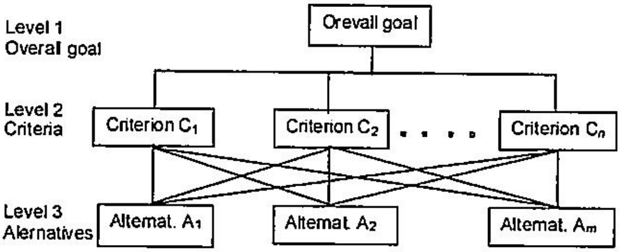

Figure 1. Hierarchical levels
Slika 1. Hijerarhijski nivoi

Each level may represent different factors (economical, technical, social, etc.) that should be evaluated by experts. It provides an overall view of the complex relationships inherent in a considered situation. It helps the decision maker to assess whether the issues in each level are the same order of magnitude, so he can compare such homogeneous elements accurately.

Elements that have a global character can be represented at the higher levels of the hierarchy. The fundamental approach of AHP is to decompose a "big" problem into several smaller problems that are solved separately to determine their priority vectors. At each level the elements that concerns to the criteria and alternatives are arranged in so called *comparison priority matrices*. Unlike other methods of MCDM, here is not necessary to know the exact numerical values of the factors being considered. It is enough to assess a good value of its comparisons or quotients. For these matrices are calculated separate priority vectors for criteria and alternatives, and then the final priority vector for ranking alternatives (Saaty, 1980, 1991). Ishizaka (2012) has developed method with clusters and pivots for larger number of alternatives and high number of judgments in the comparison matrix. The AHP method with crisp (non fuzzy), and fuzzy numbers is explained in the our previous works (Praščevic and Praščevic, 2012,2015), and here will be given necessary final formulas and procedure for solving this problem.

FUZZY AHP

Some of decision criteria are subjective and qualitative by nature, so the decision maker cannot easily express strengths of his preferences or provide exact pairwise comparison. Hence, the crisp numbers are not so suitable to express these pairwise comparison values due to their vagueness. Since decision maker's or his team judgments are uncertain and imprecise, it is much better to take pairwise comparisons as fuzzy values than as crisp ones. To overcome these shortcomings with crisp numbers, the Fuzzy AHP was developed for solving these problems of multicriteria decision making. Many authors have used triangular fuzzy numbers to express these imprecise values.

Triangular fuzzy number and fuzzy priority matrix

A triangular fuzzy number, as special type of a fuzzy set over the set of real numbers (real line) R, is shown in Figure 2.

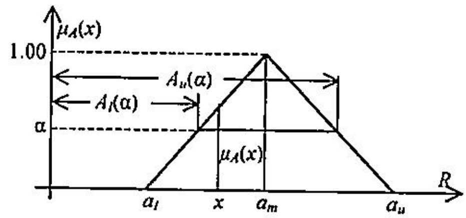

Figure 2. Triangular fuzzy number $\widetilde{A}$
Slika2. Trouglasti rasplinuti broj

A membership function of the triangular fuzzy number is

$$\begin{aligned} &\mu(x) = 0, \text{ for } x \leq a_l \text{ and } x \geq a_u : \\ &\mu(x) = (x - a_l)/(a_m - a_l), \text{ for } a_l \leq x \leq a_m; \\ &\mu(x) = (a_u - x)/(a_u - a_m), \text{ for } a_m \leq x \leq a_u. \end{aligned} \quad (1)$$

A triangular fuzzy number is usually described by three characteristic values a_l, a_m and a_u that are real numbers.

$$\widetilde{A} = (a_l, a_m, a_u), \quad a_l \leq a_m \leq a_u. \quad (2)$$

Parametric presentation of a triangular fuzzy number $\widetilde{A}$ at level α is

$$A_\alpha = [A_l(\alpha), A_u(\alpha)], \quad (3)$$

Where

$$A_l(\alpha) = a_l + (a_m - a_l)\alpha, \quad A_u(\alpha) = a_u - (a_u - a_m)\alpha, \quad 0 < \alpha \leq 1. \quad (4)$$

Reciprocal fuzzy number $\widetilde{A}^{-1}$ to $\widetilde{A}$ is for $a_l > 0$ is

$$\begin{aligned} &\widetilde{A}^{-1} = 1/\widetilde{A} = [1/A_u(\alpha), 1/A_l(\alpha)], \\ &\widetilde{A}^{-1} = [1/a_u, 1/a_m], \text{ for } \alpha = 0 \text{ and } \widetilde{A}^{-1} = [1/a_m, 1/a_m] \text{ for } \alpha = 1. \end{aligned} \quad (5)$$

Reciprocal triangular fuzzy number is usually presented in the form

$$\tilde{A}^{-1} = (1/a_u, 1/a_m, 1/a_l)\,. \tag{6}$$

The pairwise comparison judgments, that express relative importance between factors F_i and F_j in the hierarchy, in this work are triangular fuzzy numbers $\tilde{f}_{ij}$

$$\tilde{f}_{ij} = (f_{ij,l}, f_{ij,m}, f_{ij,u}) \tag{7}$$

which constitute a fuzzy comparison matrix $\tilde{\mathbf{F}}$ with elements

$$\tilde{f}_{ij} = 1/\tilde{f}_{ji},\quad i = 1,2,\ldots,k;\ j = 1,2,\ldots,k. \tag{8}$$

Fuzzy matrix can be expressed, according to (7) by three characteristic non fuzzy matrices

$$\tilde{\mathbf{F}} = (\mathbf{F}_l, \mathbf{F}_m, \mathbf{F}_u)\,, \tag{9}$$

where, taking into account (7) and (8)

$$\mathbf{F}_l = \begin{bmatrix} 1 & f_{12,l} & \cdots & f_{1k,l} \\ 1/f_{12,u} & 1 & \cdots & f_{2k,l} \\ . & . & \cdots & . \\ 1/f_{1k,u} & 1/f_{2k,u} & \cdots & 1 \end{bmatrix}, \mathbf{F}_m = \begin{bmatrix} 1 & f_{12,m} & \cdots & f_{1k,m} \\ 1/f_{12,m} & 1 & \cdots & f_{2k,m} \\ . & . & \cdots & . \\ 1/f_{1k,m} & 1/f_{2k,m} & \cdots & 1 \end{bmatrix}, \mathbf{F}_u = \begin{bmatrix} 1 & f_{12,u} & \cdots & f_{1k,u} \\ 1/f_{12,l} & 1 & \cdots & f_{2k,u} \\ . & . & \cdots & . \\ 1/f_{1k,l} & 1/f_{2k,l} & \cdots & 1 \end{bmatrix} \tag{10}$$

Some authors have proposed triangular fuzzy numbers for expression of the intensity of importance on Saaty's absolute scale (Saaty,1980). In this paper are proposed and used fuzzy numbers

$$\tilde{1} = (1,1,\alpha_g),\quad \tilde{x} = (x-\alpha_d,\, x,\, x+\alpha_g),\ x = 2,\ldots,8,\quad \tilde{9} = (9-\alpha_d, 9, 9);\ \alpha_d \ge 0,\ \alpha_g \ge 0. \tag{11}$$

Fuzzy eigenvalues and eigenvectors

Since Saaty's AHP method is based on finding the eigenvalue and eigenvectors of the fuzzy matrix $\tilde{\mathbf{F}}$ at the corresponding hierarchical level, authors of this paper have proposed one method for solving the fuzzy eigenvalue and eigenvector problem and finding solutions of the system of homogenous fuzzy linear equations (Praščević and Praščević, 2015)

$$\tilde{\mathbf{F}} \otimes \tilde{\mathbf{w}} = \tilde{\lambda} \otimes \tilde{\mathbf{w}}\,, \tag{12}$$

where a sign $\otimes$ denotes the fuzzy product.

Elements of the fuzzy matrix $\tilde{\mathbf{F}}$, fuzzy vector $\tilde{\mathbf{w}}$ and the eigenvalue $\tilde{\lambda}$ are assumed as triangular fuzzy numbers, that may be denoted according to (2) and (7) as

$$\tilde{\mathbf{w}} = (\mathbf{w}_l, \mathbf{w}_m, \mathbf{w}_u),\quad \tilde{\lambda} = (\lambda_l, \lambda_m, \lambda_u). \tag{13}$$

The proposed method is based on the calculation of expected values of fuzzy numbers and their products. Expected value $EV(\tilde{A})$ of a fuzzy number $\tilde{A} = (a_l, a_m, a_u)$, written in the parametric forms (4) is (Chanas, 2001)

$$EV(\tilde{A}) = 1/2\int_0^1 [A_l(\alpha) + A_u(\alpha)]d\alpha$$

Substituting $A_l(\alpha)$ and $A_u(\alpha)$ by expressions (9) after integration obtains

$$EV(\tilde{A}) = (a_l + 2a_m + a_u)/4. \tag{14}$$

As it shown in the previous author's work (Prascevic and Prascevic, 2015) the fuzzy eigenvalues problem (12) is transformed to the solution of next non fuzzy (crisp) eigenvalue problem

$$\overline{\mathbf{F}}_l \mathbf{w}_l + \overline{\mathbf{F}}_m \mathbf{w}_m + \overline{\mathbf{F}}_u \mathbf{w}_u = \overline{\lambda}_l \mathbf{w}_l + \overline{\lambda}_m \mathbf{w}_m + \overline{\lambda}_u \mathbf{w}_m, \tag{15}$$

$$\overline{\mathbf{F}}_l = 2\mathbf{F}_l + \mathbf{F}_m, \quad \overline{\mathbf{F}}_m = \mathbf{F}_l + 4\mathbf{F}_m + \mathbf{F}_u, \quad \overline{\mathbf{F}}_u = \mathbf{F}_m + 2\mathbf{F}_u. \tag{16}$$

Since all the values in these equations are nonnegative ones, this system of equations may be decomposed into three systems, which represent three crisp eigenvalue problems

$$\overline{\mathbf{F}}_l \mathbf{w}_l = \overline{\lambda}_l \mathbf{w}_l, \quad \overline{\mathbf{F}}_m \mathbf{w}_m = \overline{\lambda}_m \mathbf{w}_m, \quad \overline{\mathbf{F}}_u \mathbf{w}_u = \overline{\lambda}_u \mathbf{w}_u. \tag{17}$$

By solving these three eigenvalue problems, eigenvectors $\mathbf{w}_l$, $\mathbf{w}_m$ and $\mathbf{w}_u$ and auxiliary eigenvalues $\overline{\lambda}_l, \overline{\lambda}_m$ and $\overline{\lambda}_u$ are obtained and then the requested eigenvalues λ_l, λ_m and λ_u by solving linear equations

$$2\lambda_l + \lambda_m = \overline{\lambda}_l, \quad \lambda_l + 4\lambda_m + \lambda_u = \overline{\lambda}_m, \quad \lambda_m + 2\lambda_u = \overline{\lambda}_u. \tag{18}$$

To meet the requirements $\overline{\mathbf{w}}_l \leq \overline{\mathbf{w}}_m \leq \overline{\mathbf{w}}_u$ for $\lambda_l \leq \lambda_m \leq \lambda_u$, the calculated eigenvectors $\mathbf{w}_l$, $\mathbf{w}_m$ and $\mathbf{w}_u$ should be normalized according to the following formulas

$$\overline{\mathbf{w}}_l = \mathbf{w}_l \lambda_l /(s_l \lambda_m), \quad \overline{\mathbf{w}}_l = \mathbf{w}_m / s_m, \quad \overline{\mathbf{w}}_u = \mathbf{w}_u \lambda_u /(s_u \lambda_m), \tag{19}$$

where

$$s_l = \sum_{i=1}^{k} w_{j,l}, \quad s_m = \sum_{j=1}^{k} w_{j,m}, \quad s_u = \sum_{j=1}^{k} w_{j,u}. \tag{20}$$

<u>Steps in the execution of Fuzzy AHP</u>

Fuzzy AHP is carried out in several steps in a similar way as the procedure with non fuzzy (crisp) numbers that will be briefly explained here.

The first step. Define the problem, the overall goal that have to be attained, the criteria and alternatives.

The second step. Define the hierarchy structure from the top level through intermediate levels that contains the criteria and sub criteria to the lowest level, which are usually related to the alternatives, as it shown in Figure 1.

The third step. Formulate the pair-wise comparison reciprocal fuzzy matrix $\tilde{\mathbf{C}}$ for the criteria $C_1, C_2, \ldots, C_n$ by assessing the priority values as fuzzy numbers $\tilde{c}_{ij} = (c_{ij,l}, c_{ij,m}, c_{ij,u})$ (i=1,2,...,n; j=1,2,...,n) using Saaty's fundamental comparison scale adjusted to fuzzy values according to (11). Express the fuzzy matrix $\tilde{\mathbf{C}}$ by three matrices $\mathbf{C}_l, \mathbf{C}_m$ and $\mathbf{C}_u$ according to (9). Solve the fuzzy eigenvalue problem $\tilde{\mathbf{C}} \otimes \tilde{\mathbf{w}} = \tilde{\lambda}\tilde{\mathbf{w}}$, as it described in the previous section, and determine the principal fuzzy eigenvalue $\tilde{\lambda} = (\lambda_l, \lambda_m, \lambda_u)$ and the corresponding fuzzy eigenvectors $\tilde{\mathbf{w}} = (\mathbf{w}_l, \mathbf{w}_m, \mathbf{w}_u)$ and then normalize these vectors by formulas (19) and (20) to obtain the *fuzzy priority vectors of criteria* $\tilde{\bar{\mathbf{w}}} = (\bar{\mathbf{w}}_l, \bar{\mathbf{w}}_m, \bar{\mathbf{w}}_u)$.

For the matrix $\mathbf{C}_m$, calculate the consistency index *CI* and consistency ratio *CR*. If $CR \le 0.10$, accept the assessed fuzzy elements of the pairwaise matrix $\tilde{\mathbf{C}}$ and obtained eigenvalues and eigenvectors. If *CR*>0.10, improve consistency of the fuzzy matrix $\tilde{\mathbf{C}}$ by changing some of its elements and repeat the procedure until this condition is satisfied. This index and ratio are explained in previous works (Saaty,1980, Prascevic and Prascevic, 2013 and others).

The fourth step. Formulate the pairwise comparison matrices for the alternatives $\tilde{\mathbf{A}}^{(j)}$ related to the criterion C_j (j = 1,2,..., n)

$$\tilde{\mathbf{A}}^{(j)} = \begin{bmatrix} 1 & \tilde{a}_{12}^{(j)} & \cdots & \tilde{a}_{1m}^{(j)} \\ 1/\tilde{a}_{12}^{(j)} & 1 & \cdots & \tilde{a}_{2m}^{(j)} \\ & & \cdots & \\ 1/\tilde{a}_{1m}^{(j)} & 1/\tilde{a}_{2m}^{(j)} & \cdots & 1 \end{bmatrix} \text{ or } \tilde{\mathbf{A}}^{(j)} = (\mathbf{A}_l^{(j)}, \mathbf{A}_m^{(j)}, \mathbf{A}_u^{(j)}) \tag{21}$$

Solve the fuzzy eigenvalue problem $\tilde{\mathbf{A}}^{(j)} \otimes \tilde{\mathbf{p}}^{(j)} = \tilde{\lambda}^{(j)} \otimes \tilde{\mathbf{p}}^{(j)}$, $j = 1,2,\ldots,m$; to find the fuzzy principal eigenvalues $\tilde{\lambda}_{\max}^{(j)} = (\lambda_l^{(j)}, \lambda_m^{(j)}, \lambda_u^{(j)})$ and the fuzzy eigenvectors $\tilde{\mathbf{p}}^{(j)} = (\mathbf{p}_l^{(j)}, \mathbf{p}_m^{(j)}, \mathbf{p}_u^{(j)})$, consistency indices $CI^{(j)}$ and consistency ratios $CR^{(j)}$ for matrices $\mathbf{A}_m^{(j)}$, (j=1,2,..., m). If the consistency ratio is $CR^{(j)} > 0.10$, change some of the assessed values $\tilde{a}_{u,m}$ to obtain the satisfactory consistency of this matrix. Normalize vectors $\tilde{\mathbf{p}}^{(j)} = (\mathbf{p}_l^{(j)}, \mathbf{p}_m^{(j)}, \mathbf{p}_u^{(j)})$ by the formulas (19) and (20) to obtain normalized local priority vectors $\tilde{\bar{\mathbf{p}}}^{(j)} = (\bar{\mathbf{p}}_l^{(j)}, \bar{\mathbf{p}}_m^{(j)}, \bar{\mathbf{p}}_u^{(j)})$. This procedure is the same as in the step 3.

The fifth step. Formulate local priority fuzzy matrix $\tilde{\mathbf{P}} = (\mathbf{P}_l, \mathbf{P}_m, \mathbf{P}_u)$, that contains normalized local priority vectors, where

$$\mathbf{P}_l = [\overline{\mathbf{p}}_l^{(1)}\ \overline{\mathbf{p}}_l^{(2)}\ \ldots\ \overline{\mathbf{p}}_l^{(n)}]\ ,\ \mathbf{P}_m = [\overline{\mathbf{p}}_m^{(1)}\ \overline{\mathbf{p}}_m^{(2)}\ \ldots\ \overline{\mathbf{p}}_m^{(n)}]\ ,\ \mathbf{P}_u = [\overline{\mathbf{p}}_u^{(1)}\ \overline{\mathbf{p}}_u^{(2)}\ \ldots\ \overline{\mathbf{p}}_u^{(n)}]\ . \tag{22}$$

Multiply these matrices from the right by the priority vectors of the criteria respectively, which are determined in the third step

$$\overline{\mathbf{w}}_l = \left[\overline{w}_{1,l}\quad \overline{w}_{2,l}\quad \ldots\quad \overline{w}_{n,l}\right]^T, \overline{\mathbf{w}}_m = \left[\overline{w}_{1,m}\quad \overline{w}_{2,m}\quad \ldots\quad \overline{w}_{n,m}\right]^T, \overline{\mathbf{w}}_u = \left[\overline{w}_{1,u}\quad \overline{w}_{2,u}\quad \ldots\quad \overline{w}_{n,u}\right]^T.$$

and obtain *vectors of global priorities* $\mathbf{g}_l, \mathbf{g}_m$ and $\mathbf{g}_u$

$$\begin{aligned}
\mathbf{g}_l &= \mathbf{P}_l\overline{\mathbf{w}}_l = \left[g_{1,l}\quad g_{2,l}\quad \cdots\quad g_{m,l}\right]^T,\\
\mathbf{g}_m &= \mathbf{P}_m\overline{\mathbf{w}}_m = \left[g_{1,m}\quad g_{2,m}\quad \cdots\quad g_{m,m}\right]^T.\\
\mathbf{g}_u &= \mathbf{P}_u\overline{\mathbf{w}}_u = \left[g_{1,u}\quad g_{2,u}\quad \cdots\quad g_{m,u}\right]^T.
\end{aligned} \tag{23}$$

These vectors constitute fuzzy matrix of global priorities $\tilde{\mathbf{G}} = [\mathbf{g}_l\, \mathbf{g}_m\, \mathbf{g}_u]$ of alternatives $A_1, A_2, \ldots, A_m$. For every alternative A_i ($i = 1,2,\ldots,m$), elements of these vectors are expressed by the corresponding approximate triangular fuzzy numbers

$$\tilde{g}_i = (g_{i,l}, g_{i,m}, g_{iu});\quad i = 1,2,\ldots,m. \tag{24}$$

The sixth step. Alternatives A_i (i=1,2,...,m) are ranked in this step according to their global priorities that are expressed by triangular fuzzy numbers $\tilde{g}_i$. There are in the literature more proposals for ranking of fuzzy numbers. Here, is used Lee and Le's (1988) method improved by Cheng (1992).

In this paper, comparison of the fuzzy numbers is based on the probability measure of fuzzy events, which was introduced by Zadeh (1968). The fuzzy numbers are ranked according to the generalized fuzzy mean (expected value) and generalized fuzzy spread (standard deviation). For the triangular probability distribution of the triangular fuzzy number as a fuzzy event, these values for $\tilde{g}_i$ are calculated by the following formulas (Cheng, 1992):

- generalized fuzzy mean (expected value)
  $$g_{i,c} = (g_{i,l} + 2g_{i,m} + g_{i,u})/4,\ i = 1,2,\ldots,m; \tag{25}$$

- generalized spread (standard deviation)
  $$\sigma_i = [\frac{1}{80}(3g_{i,l}^2 + 4g_{i,m}^2 + 3g_{i,u}^2 - 4g_{i,l}g_{i,u} - 2g_{i,l}g_{i,m} - 4g_{i,m}g_{i,u})]^{1/2},\ i = 1,2,\ldots,m. \tag{26}$$

According to Lee and Li (1998), a fuzzy number with a higher mean value and at the same time a lower spread is ranked better. Due to some shortcoming of this ranking, Cheng (1992) proposed to rank fuzzy numbers according to the coefficient of variation CV_i

$$CV_i = \sigma_i / g_{e,i}, \quad i = 1,2,...,m. \tag{27}$$

A fuzzy number or an alternative with a smaller CV_i is ranked better, and the best ranked alternative A^* is alternative A_i with minimal CV_i.

In this paper is proposed and used formula for ranking of alternatives on the basis of the *modified expected value*

$$g_{i,me} = g_{i,e} - k_m\sigma_i, \; i = 1,2,...,m. \; k_m = 0.50 - 1.50. \tag{28}$$

In the literature exist many proposals for ranking fuzzy numbers. One of very efficient methods is proposed by Chang (1996), based on his *extend analysis*.

According to this procedure, the authors have developed a corresponding computer program *Fuzzy AHP* in MATLAB, which has been used to solve several problems of ranking alternatives in the construction industry.

EXAMPLE

This example, which is related to bridge risk assessment, is taken from papers written by Wang and Ehlang (2003,2007), where this problem is solved by Fuzzy TOPSIS method. In the example are considered five bridge structures $BS_1,BS_2,...,BS_5$ which represent alternatives $A_1,A_2,...,A_5$. All consequences and probabilities of the risk events are assessed on the base of evidence and engineering judgment by three experts against four criteria: *safety* (C_1), *functionality* (C_2), *sustainability* (C_3) and *envinronment* (C_4). The coefficients of significance of alternatives are also assessed by the experts. These values are assessed as linguistic and numeric variables that are finally transformed into triangular fuzzy numbers. The goal is to determine the levels of risk of the destruction of structures and rank them for reconstruction according to this level.

According to given values in the mentioned papers, authors of this work have formulated comparison fuzzy matrices for these criteria $\tilde{C} = (C_l, C_m, C_n)$ and for the alternatives related to the criteria C_j, j=1,2,3,4.

$$\tilde{C} = \begin{bmatrix} (1, 1, 1) & (0.93, 1.33, 1.73) & (1.60, 2.00, 2.40) & (2.60, 3.00, 3.40) \\ (0.58, 0.76, 1.08) & (1, 1, 1) & (1.00, 1.40, 1.80) & (1.93, 2.00, 2.73) \\ (0.42, 0.50, 0.63) & (0.56, 0.71, 1.00) & (1, 1, 1) & (1.27, 1.67, 2.07) \\ (0.29, 0.33, 0.38) & (0.37, 0.43, 0.52) & (0.47, 0.60, 0.79) & (1, 1, 1) \end{bmatrix}$$

$$\tilde{A}^{(j)} = (A_l^{(j)}, A_m^{(j)}, A_u^{(j)})$$

$$
\tilde{A}^{(1)} = \begin{bmatrix}
(1,1,1) & (1.00, 1.00, 1.30) & (1.07, 1.37, 1.67) & (8.70, 9.00, 9.30) & (8.70, 9.00, 9.30) \\
(1.00, 1.00, 1.30) & (1, 1, 1,) & (1.07, 1.37, 1.67) & (8.70, 9.00, 9.30) & (8.70, 9.00, 9.30) \\
(0.60, 0.73, 0.93) & (0.60, 0.73, 0.93) & (1,1,1) & (8.70, 9.00, 9.30) & (8.70, 9.00, 9.30) \\
(0.11, 0.11, 0.11), & (0.11, 0.11, 0.11) & (0.11, 0.11, 0.11), & (1,1,1) & (1.00,.1.00, 1.30) \\
(0.11, 0.11, 0.11), & (0.11, 0.11, 0.11), & (0.11, 0.11, 0.11) & (1.00, 1.00, 1.30). & (1,1,1)
\end{bmatrix}
$$

$$
\tilde{A}^{(2)} = \begin{bmatrix}
(1,1,1) & (0.68, 0.86, 1.16) & (0.68, 0.86, 1.16) & (0.68, 0.86, 1.16) & (8.70, 9.00, 9.30) \\
(0.86, 1.16\ 1.46) & (1, 1, 1,) & (1.00, 1.10, 1.30) & (1.00, 1.00, 1.00, 1.30) & (8.70, 9.00, 9.30) \\
(0.86, 1.16, 1.46) & (100, 1.00, 1.30) & (1, 1, 1) & (1.00, 1.00, 1.30) & (8.70, 9.00, 9.30) \\
(0.86, 1.16, 1.46), & (1,00, 1.00,.1.30) & (1.00, 1.00, 1.30), & (1, 1, 1) & (8.70,.9.00, 9.30) \\
(0.11, 0.11, 0.11), & (0.11, 0.11, 0.11), & (0.11, 0.11, 0.11) & (0.11, 0.11, 0.10). & (1, 1, 1)
\end{bmatrix},
$$

$$
\tilde{A}^{(4)} = \begin{bmatrix}
(1, 1, 1) & (1.00, 0.86, 1.16) & (1.00, 0.86, 1.16) & (0.65, 0.86, 1.16) & (0.50, 9.00, 9.30) \\
(1.00, 1.16, 1.46) & (1, 1, 1,) & (1.00, 1.00, 1.30) & (0.65, 1.00, 1.00, 1.30) & (0.50, 9.00, 9.30) \\
(1.00, 1.16, 1.46) & (100, 1.00, 1.30) & (1, 1, 1) & (0.65, 1.00, 1.00, 1.30) & (0.50, 9.00, 9.30) \\
(0.94, 1.16. 1.46), & (0.94, 1.00,.1.30) & (0.94, 1.00, 1.30), & (1, 1, 1) & (0.60, 9.00, 0,9.30) \\
(1.40,.1.70, 2.00), & (1.40, 1.70, 2.00), & (1.40, 1.70, 2.00) & (1.07, 1.37, 1.67). & (1, 1, 1)
\end{bmatrix}
$$

Applying computer program *Fuzzy AHP*, developed by the authors, for every alternative A_i (i=1,2,...,5) are obtained triangular fuzzy numbers $\tilde{g}_i = (g_{i,l}, g_{i,m}, g_{i,u})$ as its global priorities, and then calculated corresponding generalized expected values $g_{i,e}$, generalized standard deviations σ_i and coefficients of variations CV_i according to expressions (27), (28) and (29) respectively. These values are given in the next table.

Table 1. Characteristic global priority values
Tabela 1. Karakteristicne vrednosi globalnog prioriteta

Alternative (A_i)	$g_{i,l}$	$g_{i,m}$	$g_{i,u}$	$g_{i,e}$	σ_i	CV_i
Alternative(A1)	0.2050	0.2614	0.3581	0.2715	0.0246	0.0907
Alternative(A2)	0.2075	0.2628	0.3592	0.2731	0.0244	0.0894
Alternative(A3)	0.1755	0.2205	0.2988	0.2288	0.0198	0.0867
Alternative(A4)	0.1150	0.1492	0.2105	0.1560	0.0154	0.0988
Alternative(A5)	0.0815	0.1060	0.1474	0.1102	0.0106	0.0961

Table 2. Ranking of alternatives
Tabela 2. Rangiranje alternativa

Rank	Alternative	Expected value $g_{i,e}$	Standard deviat. σ_i	Modified exp. value $g_{i,me}$
1	Altern.(A2)	0.2715	0.0244	0.2365
2	Altern.(A1)	0.2731	0.0246	0.2345
3	Altern.(A3)	0.2288	0.0198	0.1991
4	Altern.(A4)	0.1560	0.0154	0.1329
5	Altern.(A5)	0.1102	0.0106	0.0944

The risk levels of considered alternatives A_i are triangular fuzzy numbers with maximal posibilitywith expected values $g_{i,e}$ (i=1,2,...,5). The risk at every alternative may take value from $g_{i,l}$ to $g_{i,u}$, with different possibility μ and minimal possibility μ=1.00 for $g_{i,m}$. In this case alternatives A_2=BS_2 and A_1=BS_1 have similar and bigger risk in comparison with other alternatives. Available amount of money for the maintenance of bridges may be distributed according to expeted values of risk, as it shown in te previous authors work (Prascevic and Prascevic, 2011).Very similar results of ranking these alternative were obtained in that paper.

CONCLUSION

Fuzzy AHP method, enables more complete and flexible modeling of the multiple criteria decision making problems then by the Crisp AHP method. In Fuzzy AHP can be introduced comparisons of imprecise input factors for the chosen criteria and alternatives. Unlike other methods of MCDM, here is not necessary to know the exact numerical values of the factors being considered. It is enough to assess a good value of its comparisons or quotients. This method, like Fuzzy TOPSIS method, may be successfully used for ranking alternatives and optimally deliver investments on projects, optimal risk assessment of different type of objects, optimal choice of objects for reconstruction, choice of appropriate contractor on tendering procedure and in many other cases of multiple criteria decision making. It is recommendable in the practice for MCDM to use and compare several methods, especially when difference between two or more alternatives is not significant.

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