ONE PROCEDURE OF FUZZY AHP BASED ON EIGENVALUES

NATAŠA PRAŠČEVIĆ[1], ŽIVOJIN PRAŠČEVIĆ[2],

[1] Građevinski fakultet, Beograd, natasa@grf.rs
[2] Akademija inženjerskih nauka Srbije, zbprascevic@gmail.com

Abstract: *One procedure of fuzzy AHP that is based on eigenvalues is presented in this paper. According to expected values of fuzzy numbers, one method for determination of fuzzy eigenvalues and eigenvectors is proposed and used in the method. According to this procedure, corresponding computer program is written out, which is used in solving several problems in practice.*

Keywords: *Fuzzy AHP, Eigenvalues, Multicriteria decision making, Construction industry.*

1. INTRODUCTION

The Analytic Hierarchy Process (AHP) for choosing factors that are important for decision making (DM) was proposed by Thomas Saaty (1980). This is one of the useful methods in multi criteria decision making (MCDM), which has found wide application in many areas of activities. There is a huge number of references about AHP. In this process factors are selected and formulated in a hierarchy structure descending from one overall goal to criteria and alternatives, as it shown in Fig. 1.

Each level may represent different factors (economical, technical, social, etc.) that are evaluated by experts. It provides an overall view of the complex relationships inherent in a considered situation. It helps the decision maker to assess whether the issues in each level are the same order of magnitude, so he can compare such homogeneous elements accurately. As Saaty (1980) emphasises, "the most effective way to concentrate judgments is to take a pair of elements and compare them on a single property without concern for other properties or other elements.

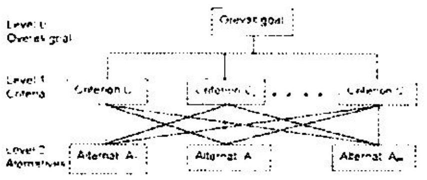

Figure 1: Hierarchical levels

Elements that have a global character are represented at the higher levels of the hierarchy. The fundamental approach of AHP is to decompose a "big" problem into several smaller problems that are solved separately to determine their priority vectors. According to these values of the separate priority vectors, the final priority vector of the alternatives is calculated taking into account relationships between hierarchy levels (Saaty, 1980, 1990).

2. NONFUZZY AHP

In the first Saaty's works is proposed and developed AHP with nonfuzzy (crisp) data on several levels and many other authors have used this procedure to solve different problems of decision making. In this paper is considered the problem of multicriteria decision making in which given alternatives $A_1, A_2, \ldots, A_m$ are ranked for prescribed criteria $C_1, C_2, \ldots, C_n$. One model with three levels for solving these problems is shown in Figure 1. Level 0 is related to the overall goal, which includes ranking of alternatives and determination of the best or most appropriate alternative. Level 1 encompasses prescribed criteria and level 2 contains alternatives that are related to these criteria. Unlike of other methods of multicriteria decision making, relative weights w_i of factors F_i ($i = 1,2,\ldots,k$), which in this case are criteria or alternatives, are compared in dependence on corresponding level. These weights are assessed usually by the decision making team. According to these values is determined the *priority matrix* $\mathbf{F} = [f_{ij}]_{k \times k}$, with elements

$$f_{ij} = w_i/w_j, \quad i=1,2,\ldots,k; \quad j=1,2,\ldots,k. \qquad (1)$$

where w_i and w_j are *weights* of corresponding criteria C_i and C_j. This matrix is known as a *reciprocal matrix*, since it has positive entries everywhere *and* satisfies the reciprocal property

$$f_{ji} = 1/f_{ij},\ i = 1,2,\dots,k;\ j = 1,2,\dots,k. \quad (2)$$

This matrix is *consistent*, because the following conditions are satisfied

$$f_{ip} = f_{ij}/f_{pj},\quad i = 1,2,\dots,k;\ j = 1,2,\dots,k,\ p = 1,2,\dots,k.$$

According to Saaty (1980), necessary and sufficient condition for consistency is that the principal eigenvalue λ_{max} of matrix **F**, for the eigenvalue problem

$$\mathbf{F\,w} = k\mathbf{w} \quad (3)$$

Has value $\lambda_{max} = k$.

To make vector **w** unique, it is necessary to normalize its elements by dividing each element by their sum

$$\overline{w}_i = w_i / (w_1 + w_2 + \dots + w_k),\quad i = 1,2,\dots,k. \quad (4)$$

The values f_{ij}, according to Saaty (1980,1990), represents the *pairwise comparison* or importance of the factor F_i compared to the factor F_j at a certain level of the hierarchy. Hence, matrix **F** is called *pairwise comparison matrix*. As Saaty (1990) emphasizes in a general decision making it is impossible to give precise values of elements f_{ij} according to formula (1), but only estimate them. For elicitation of pairwise comparison judgments of criteria, he proposed fundamental scale of measurements. The differences $\Delta_{ij} = f_{ij} - w_i/w_j$ cause inconsistency of the matrix **F**, and its principal eigenvalue is

$$\lambda_{max} \geq k. \quad (5)$$

To every eigenvalue λ_i corresponds eigenvector **w**, that represents one solution of the system of k homogeneous linear equations (3). Maximal positive real eigenvalue λ_{max} and corresponding eigenvector **w** are accepted for further calculation. Since estimated matrix **F** is not consistent one, Saaty (1990) introduced *consistency index CI* and *consistency ratio CR* for this matrix, that are calculated by the formulas

$$CI = (\lambda_{max} - k)/(k-1), \quad (6)$$

$$CR = CI/RI. \quad (7)$$

RI is called *random consistency*, which depends on the size of matrix k, and its values are given in special table proposed by Saaty (1990). If $CR \leq 0.10$, the estimates of the elements of the vector **w** are acceptable. Otherwise, the consistency of the matrix **F** should be improved by changing values of some its elements, taking into account that this matrix must be reciprocal. Saaty's method is based on calculation of the maximal eigenvalue and corresponding eigenvector, and hence is known as the *eigenvector method*.

3. FUZZY AHP

Some of decision criteria are subjective and qualitative by nature, so the decision maker can not easily express strengths of his preferences or provide exact pairwise comparison. Hence, the crisp numbers are not so suitable to express these pairwise comparison values due to their vagueness. Since decision maker's or his team judgments are uncertain and imprecise, it is much better to give pairwise comparisons as fuzzy values than as crisp ones. To overcome these shortcomings with crisp numbers, the Fuzzy AHP was developed for solving these problems of multicriteria decision making. The first solution of Fuzzy AHP was proposed by Van Laarhoven and Pedrycz (1983). Buckley (1985) used trapezoidal fuzzy numbers to express pairwise comparison values. In many papers are used extent analysis, proposed by Chang (1992, 1998) with triangular fuzzy numbers for handling fuzzy AHP and ranking alternatives.

The triangular fuzzy number, as special type of a fuzzy set over the set of real numbers (real line) R, is shown in Figure 2. Parametric presentation of a triangular fuzzy number $\tilde{A}$ at level α is

$$A_\alpha = [A_l(\alpha),\ A_u(\alpha)] \quad (8)$$

where $A_l(\alpha) = a_l + (a_m - a_l)\alpha,\quad A_u(\alpha) = a_u - (a_u - a_m)\alpha,\ 0 < \alpha \leq 1,\quad a_l \leq a_m \leq a_u.$

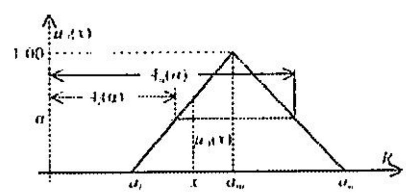

Figure 2: Triangular fuzzy number $\tilde{A}$

Triangular fuzzy number is usually described by three characteristic values

$$\tilde{A} = (a_l, a_m, a_u). \quad (9)$$

while reciprocal fuzzy number $\tilde{A}^{-1}$ to $\tilde{A}$ is for $a_l > 0$

$$\tilde{A}^{-1} = 1/\tilde{A} = (1/a_u, 1/a_m, 1/a_l). \quad (10)$$

The pairwise comparison judgments, that express relative importance between factors F_i and F_j in the hierarchy, in this work are triangular fuzzy numbers $\tilde{f}_{ij}$, $\tilde{f}_{ij} = (f_{ij,l}, f_{ij,m}, f_{ij,u})$, which constitute a fuzzy comparison matrix $\tilde{\mathbf{F}}$ with elements

$$\tilde{f}_{ij} = 1/\tilde{f}_{ji}, \quad i = 1,2,...,k; \quad j = 1,2,...,k. \quad (11)$$

Fuzzy matrix can be expressed, according to (9) by three characteristic nonfuzzy matrices

$$\tilde{\mathbf{F}} = (\mathbf{F}_l, \mathbf{F}_m, \mathbf{F}_u), \quad (12)$$

where, taking into account (9) and (10),

$$\mathbf{F}_l = \begin{bmatrix} 1 & f_{12,l} & \cdots & f_{1k,l} \\ 1/f_{12,u} & 1 & \cdots & f_{2k,l} \\ \cdot & \cdot & \cdots & \cdot \\ 1/f_{1k,u} & 1/f_{2k,u} & \cdots & 1 \end{bmatrix}, \mathbf{F}_m = \begin{bmatrix} 1 & f_{12,m} & \cdots & f_{1k,m} \\ 1/f_{12,m} & 1 & \cdots & f_{2k,m} \\ \cdot & \cdot & \cdots & \cdot \\ 1/f_{1k,m} & 1/f_{2k,m} & \cdots & 1 \end{bmatrix}, \mathbf{F}_u = \begin{bmatrix} 1 & f_{12,u} & \cdots & f_{1k,u} \\ 1/f_{12,l} & 1 & \cdots & f_{2k,u} \\ \cdot & \cdot & \cdots & \cdot \\ 1/f_{1k,l} & 1/f_{2k,l} & \cdots & 1 \end{bmatrix}. \quad (13)$$

Some authors have proposed triangular fuzzy numbers for expression of the intensity of importance on Saaty's absolute scale. In this paper are used fuzzy numbers

$$\tilde{1} = (1, 1, \alpha_g), \quad \tilde{x} = (x - \alpha_d, x, x + \alpha_g), \; x = 2,...,8, \quad \tilde{9} = (9 - \alpha_d, 9, 9); \quad \alpha_d \ge 0, \; \alpha_g \ge 0. \quad (14)$$

3.1 Fuzzy eigenvalues and eigenvectors

Since Saaty's AHP method is based on finding eigenvalue and eigenvectors of the fuzzy matrix $\tilde{\mathbf{F}}$ at the corresponding hierarchical level, here is proposed one method to solve the fuzzy eigenvalue and eigenvector problem and find solutions of the system of homogenous fuzzy linear equations

$$\tilde{\mathbf{F}} \otimes \tilde{\mathbf{w}} = \tilde{\lambda} \otimes \tilde{\mathbf{w}}, \quad (15)$$

where the sign $\otimes$ denotes the fuzzy product.

Elements of the fuzzy matrix $\tilde{\mathbf{F}}$, fuzzy vector $\tilde{\mathbf{w}}$ and eigenvalue $\tilde{\lambda}$ are assumed as triangular fuzzy numbers, that may be described according to (9) as

$$\tilde{\mathbf{w}} = (\mathbf{w}_l, \mathbf{w}_m, \mathbf{w}_u), \quad \tilde{\lambda} = (\lambda_l, \lambda_m, \lambda_u) \quad (16)$$

Proposed method is based on the calculation of expected values of fuzzy numbers and their products. Expected value $EV(\tilde{A})$ of a fuzzy number $\tilde{A} = (a_l, a_m, a_u)$, written in the parametric form (8) and (9), is

$$EV(\tilde{A}) = (a_l + 2a_m + a_u)/4. \quad (17)$$

Expected value of a product of two fuzzy numbers $\tilde{A} = (a_l, a_m, a_u)$ and $\tilde{B} = (b_l, b_m, b_u)$ is

$$EV(\tilde{A} \otimes \tilde{B}) = \frac{1}{12}[(2a_l + a_m)b_l + (a_l + 4a_m + a_u)b_m + (a_m + 2a_u)b_u]. \quad (18)$$

System of fuzzy linear equation may be written in the form $\tilde{f}_{i1} \otimes \tilde{w}_1 \oplus \tilde{f}_{i2} \otimes \tilde{w}_2 \oplus .... \oplus \tilde{f}_{ik} \otimes \tilde{w}_k = \tilde{\lambda} \otimes \tilde{w}_i$; $i = 1,2,....,k$, where the sign $\oplus$ denotes a fuzzy summation.

Expected values of fuzzy products due to (18) are

$$EV(\tilde{f}_{ij} \otimes \tilde{w}_j) = \frac{1}{12}[(2f_{ij,l} + f_{ij,m})w_{j,l} + (f_{ij,l} + 4f_{ij,m} + f_{ij,u})w_{j,m} + (f_{ij,m} + 2f_{ij,u})w_{j,u}] \quad (19)$$

$$EV(\tilde{\lambda} \otimes \tilde{w}_j) = \frac{1}{12}[(2\lambda_l + \lambda_m)w_{j,l} + (\lambda_l + 4\lambda_m + \lambda_u)w_{j,m} + (\lambda_m + 2\lambda_u)w_{j,u}]. \quad (20)$$

Expected value of the sum of fuzzy numbers is equal to the sum of expected values of fuzzy numbers

$$\sum_{j=1}^{k} EV(\tilde{f}_{ij} \otimes \tilde{w}_j) = \tilde{\lambda} \otimes \tilde{w}_i, \quad i = 1,2,....,k.$$

Introducing in this equations expressions for expected values of the fuzzy products (19) and (20) obtains system of linear homogenous equations

$$\bar{\mathbf{F}}_l \mathbf{w}_l + \bar{\mathbf{F}}_m \mathbf{w}_m + \bar{\mathbf{F}}_u \mathbf{w}_u - \bar{\lambda}_l \mathbf{w}_l - \bar{\lambda}_m \mathbf{w}_m - \bar{\lambda}_u \mathbf{w}_u = 0 \quad (21)$$

where are

$$\bar{F}_l = 2F_l + F_m, \quad \bar{F}_m = F_l + 4F_m + F_u, \quad \bar{F}_u = F_m + 2F_u. \tag{22}$$

Since all values in these equations are nonnegative ones, this system of equations may be decomposed into three systems, that represent three eigenvalue problems

$$\bar{F}_l w_l - \bar{\lambda}_l w_l = 0, \quad \bar{F}_m w_m - \bar{\lambda}_m w_m = 0, \quad \bar{F}_u w_u - \bar{\lambda}_u w_u = 0. \tag{23}$$

Solving these three eigenvalue problems are obtained eigenvectors w_l, w_m and w_u and auxiliary eigenvalues $\bar{\lambda}_l, \bar{\lambda}_m$ and $\bar{\lambda}_u$ and then requested eigenvalues λ_l, λ_m and λ_u by solving linear equations $2\lambda_l + \lambda_m = \bar{\lambda}_l$, $\lambda_l + 4\lambda_m + \lambda_u = \bar{\lambda}_m$, $\lambda_m + 2\lambda_u = \bar{\lambda}_u$ To meet requirements

$$\bar{w}_l \le \bar{w}_m \le \bar{w}_u, \text{ for } \lambda_l \le \lambda_m \le \lambda_u, \tag{24}$$

calculated eigenvectors w_l, w_m and w_u should be normalized according to following formulas

$$\bar{w}_l = w\lambda_l/(s_l\lambda_m), \quad \bar{w}_m = w_m/s_m, \quad \bar{w}_u = w\lambda_u/(s_m\lambda_m); \quad s_l = \sum_{j=1}^{k} w_{j,l}, \quad s_m = \sum_{j=1}^{k} w_{j,m}, \quad s_u = \sum_{j=1}^{k} w_{j,u}. \tag{25}$$

3.2 Steps in the execution of Fuzzy AHP

Fuzzy AHP is carried out in several steps in a similar way as the procedure with nonfuzzy (crisp) numbers, that will be here briefly explained.

First step. Define the problem, overall goal that have to be attained, criteria and alternatives.

Second step. Define the hierarchy structure from the top level thru intermediate levels that contains criteria and subcriteria to the lowest level, which usually are related to the alternatives, as it shown in Fig. 1.

Third step. Formulate the pairwise comparison reciprocal fuzzy matrix $\tilde{C}$ for the criteria $C_1, C_2, \ldots, C_n$ by assessing priority values as fuzzy numbers $\tilde{c}_{ij} = (c_{ij,l}, c_{ij,m}, c_{ij,u})$ ($i=1,2,\ldots,n$; $j=1,2,\ldots,n$) using Saaty's fundamental comparison scale adjusted to fuzzy values according to (14). Express fuzzy matrix $\tilde{C}$ by three matrices C_l, C_m and C_u according to (13). Solve fuzzy eigenvalue problem $\tilde{C} \otimes \tilde{w} = \tilde{\lambda}\tilde{w}$, as it described in previous section, and determine the principal fuzzy eigenvalue $\tilde{\lambda} = (\lambda_l, \lambda_m, \lambda_u)$ and corresponding fuzzy eigenvectors $\tilde{w} = (w_l, w_m, w_u)$ and normalize these vectors by formulas (25) to obtain *fuzzy priority vectors of criteria* $\bar{\tilde{w}} = (\bar{w}_l, \bar{w}_m, \bar{w}_u)$. For the matrix C_m calculate consistency index CI and consistency ratio CR. If $CR \le 0.10$, accept assessed fuzzy elements of pairwaise matrix $\tilde{C}$ and obtained values of the eigenvalues and eigenvectors. If $CR>0.10$, improve consistency of the fuzzy matrix $\tilde{C}$ by changing some of its elements and repeat procedure until this condition is satisfied.

Fourth step. Formulate pairwise comparison matrices $\tilde{A}^{(j)}$ related to the criterion C_j ($j = 1,2,\ldots, n$)

$$\tilde{A}^{(j)} = \begin{array}{c|cccc} C_j & A_1 & A_2 & \cdots & A_m \\ \hline A_1 & 1 & \tilde{a}_{12}^{(j)} & \cdots & \tilde{a}_{1m}^{(j)} \\ A_2 & 1/\tilde{a}_{12}^{(j)} & 1 & \cdots & \tilde{a}_{2m}^{(j)} \\ \cdots & \cdot & \cdot & \cdots & \cdot \\ A_m & 1/\tilde{a}_{1m}^{(j)} & 1/\tilde{a}_{2m}^{(j)} & \cdots & 1 \end{array}, \quad j=1,2,\ldots; \text{ or } \tilde{A}^{(j)} = (A_l^{(j)}, A_m^{(j)}, A_u^{(j)}) \tag{26}$$

Solve fuzzy eigenvalue problem $\tilde{A}^{(j)} \otimes \tilde{p}^{(j)} = \tilde{\lambda}^{(j)} \otimes \tilde{p}^{(j)}$, $j = 1,2,\ldots,m$; to find fuzzy principal eigenvalues $\tilde{\lambda}_{max}^{(j)} = (\lambda_l^{(j)}, \lambda_m^{(j)}, \lambda_u^{(j)})$ and fuzzy eigenvectors $\tilde{p}^{(j)} = (p_l^{(j)}, p_m^{(j)}, p_u^{(n)})$, consistency indices $CI^{(j)}$ and consistency ratios $CR^{(j)}$ ($j = 1,2,\ldots, m$) for matrices $C_m^{(j)}$. If the consistency ratio $CR^{(j)} > 0.10$ change some of assessed values $\tilde{a}_{ij}$ to obtain satisfactory consistency of this matrix. Normalize vectors $\tilde{p}^{(j)} = (p_l^{(j)}, p_m^{(j)}, p_u^{(n)})$ by formulas (25) to obtain normalized local priority vectors $\bar{\tilde{p}}^{(j)} = (\bar{p}_l^{(j)}, \bar{p}_m^{(j)}, \bar{p}_u^{(j)})$. This procedure is the same as in the step 3.

Fifth step. Formulate local priority fuzzy matrix $\tilde{P} = (P_l, P_m, P_u)$, that contains normalized local priority vectors, where $P_l = [\bar{p}_l^{(1)}\ \bar{p}_l^{(2)} \ldots \bar{p}_l^{(n)}]$, $P_m = [\bar{p}_m^{(1)}\ \bar{p}_m^{(2)} \ldots \bar{p}_m^{(n)}]$, $P_u = [\bar{p}_u^{(1)}\ \bar{p}_u^{(2)} \ldots \bar{p}_u^{(n)}]$.

Multiply these matrices from the right by the priority vectors of criteria which are determined in the third step respectively

$$\bar{w}_l = [\bar{w}_{1,l}, \bar{w}_{2,l}, \ldots, \bar{w}_{n,l}]^T, \quad \bar{w}_m = [\bar{w}_{1,m}, \bar{w}_{2,m}, \ldots, \bar{w}_{n,m}]^T, \quad \bar{w}_u = [\bar{w}_{1,u}, \bar{w}_{2,u}, \ldots, \bar{w}_{n,u}]^T.$$

and obtain *vectors of global priorities* $\mathbf{g}_l, \mathbf{g}_m$ and $\mathbf{g}_u$

$$\mathbf{g}_l = \mathbf{P}_l \mathbf{w}_l = [g_{1,l}, g_{2,l}, \ldots, g_{m,l}]^T, \; \mathbf{g}_m = \mathbf{P}_m \mathbf{w}_m = [g_{1,m}, g_{2,m}, \ldots g_{m,m}]^T, \; \mathbf{g}_u = \mathbf{P}_u \mathbf{w}_u = [g_{1,u}, g_{2,u}, \ldots, g_{m,u}]^T. \quad (27)$$

These vectors constitute fuzzy matrix of global priorities $\tilde{\mathbf{G}} = [\mathbf{g}_l \, \mathbf{g}_m \, \mathbf{g}_u]$ of alternatives $A_1, A_2, \ldots, A_m$.

To every alternative $A_i (i = 1,2,\ldots,m)$ corresponds approximate triangular fuzzy number

$$\tilde{g}_i = (g_{i,l}, g_{i,m}, g_{iu}); \quad i = 1,2,\ldots,m. \quad (28)$$

Sixth step. Alternatives A_i are ranked in this step according to their global priorities that are expressed by triangular fuzzy numbers $\tilde{g}_i$. In the literature exist more proposals for ranking of fuzzy numbers. Here is used Lee and Le's (1998) method improved by Cheng (1992).

In this paper, comparison of fuzzy numbers is based on the probability measure of fuzzy events, which is introduced by Zadeh (1968). Fuzzy numbers are ranked according to the generalized fuzzy mean (expected value) and generalized fuzzy spread (standard deviation). For the triangular probability distribution of triangular fuzzy number as a fuzzy event, these values for the fuzzy number $\tilde{g}_i$ are calculated by formulas (Cheng, 1992):

- generalized fuzzy mean (expected value)

$$g_{i,e} = (g_{i,l} + 2g_{i,m} + g_{i,u})/4, \; i = 1,2,\ldots,m; \quad (29)$$

- generalized spread (standard deviation)

$$\sigma_i = [\frac{1}{80}(3g_{i,l}^2 + 4g_{i,m}^2 + 3g_{i,u}^2 - 4g_{i,l}g_{i,m} - 2g_{i,l}g_{i,m} - 4g_{i,m}g_{i,u})]^{1/2}, \quad i = 1,2,\ldots,m. \quad (30)$$

According to Lee and Li (1998), a fuzzy number with a higher mean value and at the same lower spread is ranked better. However, when higher mean value and at the same time higher spread or lower mean and at the same time lower spread it is not easy to compare the orderings clearly. Therefore, Cheng (1992) proposed to rank fuzzy numbers according to coefficient of variation CV_i

$$CV_i = \sigma_i / g_{e,i}, \quad i = 1,2,\ldots,m. \quad (31)$$

Fuzzy number or alternative with smaller CV_i is ranked better, and the best ranked alternative A^* is alternative A_i with minimal CV_i. For this ranking of fuzzy numbers may be used Chang's extend analysis method (Chang 1992). According to this procedure, authors have developed corresponding computer program in MATLAB that has been used to solve several problems of ranking alternatives in construction industry.

4. A CASE STUDY

Proposed method Fuzzy AHP is applied for choice of the optimal structural reinforced concrete system of one industrial hall 50.00x120,00 m. Three alternatives for the structural system are considered:

Alternative A_1 – Two chord reinforced concrete and steel girder supported by the reinforced concrete columns; Alternative A_2 - Prestressed concrete girder supported by the reinforced concrete columns; Alternative A_3 – Classical reinforced concrete structure.

The criteria used in this example are: C_1 - Summary costs of design and construction of the hall, C_2 - Costs of the annual maintenance, C_3 - Necessary time for the construction works in weeks, C_4 - Technological possibilities of the contractor to construct this hall in the chosen system.

According to the existing data authors have formulated fuzzy priority matrices:

Pairwise comparison fuzzy matrix $\tilde{\mathbf{C}}$ for the criteria

$$\tilde{\mathbf{C}} = \begin{bmatrix} 1 & 1.\tilde{5} & \tilde{2} & 2.\tilde{5} \\ 1.\tilde{5}^{-1} & 1 & \tilde{1} & 1.\tilde{5} \\ \tilde{2}^{-1} & \tilde{1}^{-1} & 1 & 1.\tilde{5} \\ 2.\tilde{5}^{-1} & 1.\tilde{5}^{-1} & 1.\tilde{5}^{-1} & \end{bmatrix}.$$

Pairwise comparison matrices $\tilde{\mathbf{A}}^{(j)}$ related to the criterion C_j (j=1,2,3,4) are

$$\mathbf{A}^{(1)} = \begin{bmatrix} 1 & 1.\tilde{5} & \tilde{2} \\ 1.\tilde{5}^{-1} & 1 & 1.\tilde{5} \\ \tilde{2}^{-1} & 1.\tilde{5}^{-1} & 1 \end{bmatrix} \mathbf{A}^{(2)} = \begin{bmatrix} 1 & 1.\tilde{3} & 1.\tilde{5} \\ 1.\tilde{3}^{-1} & 1 & 1.\tilde{2} \\ 1.\tilde{5}^{-1} & 1.\tilde{2}^{-1} & 1 \end{bmatrix}, \mathbf{A}^{(3)} = \begin{bmatrix} 1 & 1.\tilde{2} & 1.\tilde{5} \\ 1.\tilde{2}^{-1} & 1 & 1.\tilde{4} \\ 1.\tilde{5}^{-1} & 1.\tilde{4}^{-1} & 1 \end{bmatrix}, \mathbf{A}^{(4)} = \begin{bmatrix} 1 & 1.\tilde{3} & 1.\tilde{2} \\ 1.\tilde{3}^{-1} & 1 & 1.\tilde{2} \\ 1.\tilde{2}^{-1} & 1.\tilde{2}^{-1} & 1 \end{bmatrix}$$

Applying mentioned computer program, are obtained principal eigenvalues $\tilde{\lambda} = (\lambda_l, \lambda_m, \lambda_u)$, eigenvectors $\tilde{\bar{\mathbf{w}}} = (\bar{\mathbf{w}}_l, \bar{\mathbf{w}}_m, \bar{\mathbf{w}}_u)$ for the fuzzy matrix $\tilde{C} = (\mathbf{C}_l, \mathbf{C}_m, \mathbf{C}_u)$ and for local priorities eigenvalues and eigenvectors

$\tilde{\lambda}^{(j)} = (\bar{\lambda}_l^{(j)}, \bar{\lambda}_m^{(j)}, \bar{\lambda}_u^{(j)})$, $\tilde{p}^{(j)} = (\bar{p}_l^{(j)}, \bar{p}_m^{(j)}, \bar{p}_u^{(j)})$ for fuzzy matrices $\tilde{A}^{(j)}$ (j=1,2,...,5). Principal fuzzy eigenvalues and consistency ratios CR for matrices $\mathbf{C}_m$ and $\mathbf{A}_m^{(j)}$ are given in Table1. According to these values are obtained fuzzy vectors of global priorities $\mathbf{g}_l$, $\mathbf{g}_m$ and $\mathbf{g}_u$ and their components as fuzzy numbers $\tilde{g}_i = (g_{i,l}, g_{i,m}, g_{i,u})$. These values are given in Table 2. For these fuzzy numbers are determined generalized fuzzy means (expected values) $g_{i,e}$ and coefficients of variations V_i for alternatives A_i (i=1,2,3) using expressions (29) and (30). According to these values alternatives are ranked and results are given in Table 3

Table 1: Eigenvalues and consistency ratios

	$\tilde{C}$	$\tilde{A}^{(1)}$	$\tilde{A}^{(2)}$	$\tilde{A}^{(3)}$	$\tilde{A}^{(4)}$
λ_l	3.366	2.663	2.588	2.598	2.560
λ_m	4.008	3.002	3.002	3.002	3.001
λ_u	5.146	3.408	3.521	3.507	3.565
CR	0.003	0.004	0.005	0.002	0.004

Table 2: Vectors of global priorirties

Altern.	Vector $\mathbf{g}_l$ ($g_{i,l}$)	Vector $\mathbf{g}_m$ ($g_{i,m}$)	Vector $\mathbf{g}_u$ ($g_{i,u}$)	Expected value $g_{i,e}$	Stand. dev. σ_i (%)
A_1	0.308	0.421	0.621	0.443	11.41
A_2	0.106	0.195	0.442	0.235	11.71
A_3	0.120	0.245	0.575	0.297	11.78

Table 3: Ranks of alternatives

Rank	Alternative	Expect.val. $g_{i,e}$	Alternative	Coefficient V_i (%)
1	A_1	0.443	A_1	11.41
2	A_2	0.339	A_3	11.71
3	A_3	0.274	A_2	11.78

Alternative A_1 is the best ranked according to the expected value $g_{1,e}$ = 0.443 and coefficient of variation V_1 = 11.41%. This alternative has a discernible advantage over other alternatives. All consistency ratios for matrices $\mathbf{C}_m$ and $\mathbf{A}_m^{(j)}$ are CR<0.004<<0.10, so that these matrices are consistently assessed. This alternative is accepted and industrial hall is successfully completed. This problem has been solved by the authors using modified Fuzzy TOPSIS method and obtained similar results (Prascevic & Prascevic 2013).

5. CONCLUSION

The fundamental approach of AHP is to break down a "big" problem into several problems that are solved separately to determine their priority vectors. According to these values, final priority vector of the alternatives is calculated taking into account relationships between hierarchy levels. AHP method has been widely used for MCDM in the construction industry and project management. Proposed Fuzzy AHP method in comparison with AHP with the crisp data, gives more complete and realistic results, especially for decision criteria that have qualitative nature.

REFERENCES

[1] Buckley, J.J.(1985). Fuzzy hierarchical analysis, *Fuzzy Sets and Systems*, **17**, 233-247.

[2] Chang, D.-Y. (1992). Application of extent analysis method on fuzzy AHP. Europ. Journ. of Opert. Research, **95**, 649-655.

[3] Cheng, C.-H. (1992). A new approach for ranking fuzzy numbers by distance method", Fuzzy Sets and Systems, **95**, 307-317.

[4] Lee, E.S & Li, R.L. (1998). Comparison of fuzzy numbers based on the probability measure of fuzzy events, Comput. Math. Appl. **15**, 887-896.

[5] Prascevic, Z. & Prascevic, N. (2013). One Modification of Fuzzy TOPSIS method. Journ. of Modelling in Management, **8**, 82 –102.

[6] Saaty, T.L. (1980). The Analytic hierarchy Process, McGraw-Hill, New York,1980.

[7] Saaty T.L.(1990).Haw to make a decision: Analytic Hierarchy Process. European Journal of Operational Research, **48**, 9-26.

[8] Van Laarhoven, P.J.M., Pedrycz, W. (1983). A fuzzy extension of Saaty's priority theory, *Fuzzy Sets and Systems*, **11**, 229-241.

[9] Zadeh, L.Z.(1968). Probability measures of fuzzy events, Journ. of Math. Anal. and Applic. 23, 421-427.