DRM formulation for axisymmetric laser-material interactions

R. Gospavić[1], V. Popov[2], M. Srecković[3] & G. Todorović[1]
[1]*Faculty of Civil Engineering, Serbia and Montenegro*
[2]*Wessex Institute of Technology, UK*
[3]*Faculty of Electrical Engineering, Serbia and Montenegro*

Abstract

The modeling of laser-material interaction using the boundary element dual reciprocity method (BE-DRM) is presented. Thermal effects in the case of cylindrical geometry for mono as well as multi layer structures were considered. The different aspects of interaction up to the melting point of considered materials are presented. The effect of temperature dependence of the absorption coefficients on the process of laser heating was considered. The BEM formulation is based on the fundamental solution for the Laplace equation. The numerical results for spatial as well as temporal temperature distribution inside the material bulk are presented. Two cases were considered: a mono-layer and a multi layer case. In the case of a mono-layer structure DRM and DRM-MD approaches were used, and the numerical results were compared with the analytical ones. In the multi layer case only the DRM-MD approach was used.
Keywords: axisymmetric laser-material interaction, dual reciprocity method.

1 Introduction

The dual reciprocity method (DRM) was applied for laser-material interaction analysis. Laser beams have a number of applications in different areas of science, technology, and medicine. In the present work, thermal models of interaction in case of cylindrical geometry and mono as well as multi layer structures were considered. The spatial and temporal distributions of temperature field were considered. The numerical model of laser-material interaction described here is restricted only to heating effects of the targeted material without destructive and

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disintegration processes during interaction i.e. the incident intensity of laser radiation was considered to be equal to critical intensity.

In the present work the dual reciprocity method [1] is used to solve axisymmetric problems. The DRM has been used previously for axisymmetric problems, see for example [2,3,4]. The difference in this case is that the Laplace fundamental solution is used instead of the one for axisymmetric problems expressed in terms of Eliptic integrals. The present approach simplifies the DRM part and the construction of a suitable particular solution.

In order to estimate the accuracy of the numerical method, analytical results were compared to the results obtained using the boundary element DRM approach.

2 Mathematical model of the interaction

The heating process provoked by a laser beam during interaction was considered. It was assumed that absorption of the laser beam occurred in the thin surface layer of the bulk material.

The interaction with the material is modeled as an equivalent surface thermal source with appropriate spatial ant temporal distributions. The analysis is focused on cylindrical geometry and surface distributions of absorbed incoming laser beam fluxes, and accordingly the temperature field analysis was performed using the cylindrical coordinate system. Though this problem is a three-dimensional one, as there is axial symmetry, the temperature field is a function of the radial and axial coordinates only, i.e. the problem under consideration becomes a two dimensional one. In this work only mono and two layer structures, with ideal thermal contacts between adjacent layers, were considered, however the results can be applied to multi layer structures.

The geometry of the considered problem for a two-layer case is shown in Fig. 1. It was assumed that the spatial and temporal distributions of the laser beam intensity on the surface of the material specimens could be described by a product of two independent functions of the radial coordinate and time e.g. $q(r)$ and $\varphi(t)$, respectively.

It was also assumed that all the thermal parameters of the material of interest in the considered temperature range are constant and temperature independent. A linear temperature dependence of the material optical parameter, i.e. the absorption coefficient, was assumed [5]. The initial temperature inside the specimen is equal to the ambient temperature T_0. Heating of material, according to above assumptions, for a two layer cylindrical structure (Fig. 1.), with ideal thermal contact between layers, could be described by the following equations [5]:

$$\begin{aligned} &\frac{1}{a_1}\frac{\partial T_1(z,r,t)}{\partial t} = \Delta T_1(z,r,t), \quad 0 \le z \le h_1; \quad t \ge 0, \quad 0 \le r \le R \\ &\frac{1}{a_2}\frac{\partial T_2(z,r,t)}{\partial t} = \Delta T_2(z,r,t), \quad h_1 \le z \le h; \quad t \ge 0, \quad 0 \le r \le R \end{aligned} \tag{1}$$

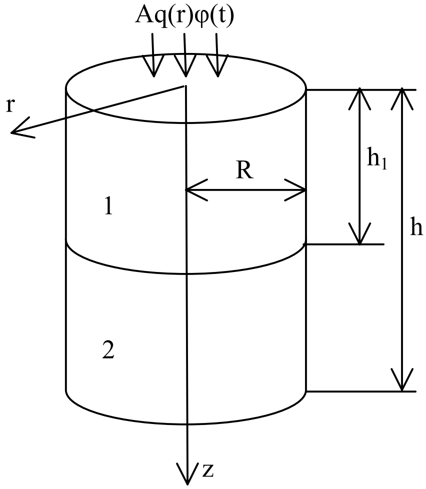

Figure 1: Geometry of the problem domain (R-radius of the structure; h_1-thickness of upper layer; h-height of whole structure; A-absorption coefficient).

Subscripts 1 and 2 correspond to the upper and to the lower layer, respectively. The corresponding boundary conditions are:

$$-\lambda_1 \frac{\partial T_1}{\partial z} = A(T)\,q(r)\,\varphi(t), \quad z = 0, \quad 0 \le r \le R;$$

$$-\lambda_1 \frac{\partial T_1}{\partial r} = \alpha_1 T_1, \quad r = R, \quad 0 \le z \le h_1$$

$$\begin{aligned} -\lambda_2 \frac{\partial T_2}{\partial r} &= \alpha_2 T_2, \quad r = R, \quad h_1 \le z \le h; \\ -\lambda_2 \frac{\partial T_2}{\partial z} &= \alpha_2 T_2, \quad z = h, \quad 0 \le r \le R \end{aligned} \tag{2}$$

$$T_1 = T_2, \quad \lambda_1 \frac{\partial T_1}{\partial z} = \lambda_2 \frac{\partial T_2}{\partial z}; \quad z = h_1, \quad 0 \le r \le R$$

$$\frac{\partial T_1}{\partial r} = 0; \quad r = 0, \quad 0 \le z \le h_1; \quad \frac{\partial T_2}{\partial r} = 0; \quad r = 0, \quad h_1 \le z \le h$$

where T_i is the temperature difference between the interior domain temperature and ambient one, λ is the coefficient of thermal conductivity, $a = \dfrac{\lambda}{\rho \cdot c}$ is the coefficient of thermal diffusivity, c is the specific heat, ρ is the material density, α is heat transfer coefficient which determines the rate of thermal losses on boundary surface, R and h are specimen's radius and length, respectively, $A(T)$ is absorption coefficient of the laser radiation by the material of the upper layer at temperature difference T.

The temperature dependence of the absorption coefficient is assumed to follow the following linear form

$$A\left(T\right) = A_0 + B \cdot T$$

where A_0 is of the absorption coefficient at ambient temperature T_0 and B is a constant whose value depends on the type of material [5]. For Al the above constants have the following numerical values [5]:

$$A_0 = 0.642; \quad B = -4.28 \cdot 10^{-4} \frac{1}{K}$$

The thermal losses, in axial and radial directions, were modeled by free thermal convection. Structures with three or more layers could also be described by the above model.

3 The boundary element formulation

For a mono-layer structures the governing equations (1) at n-th time step could be transformed for cylindrical coordinates into the following form:

$$\frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{a}\frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial T}{\partial r} = b \quad 0 \le r \le R; \quad 0 \le z \le h$$

$$\begin{aligned} b = b_1 - b_2; \quad b_1 &= \frac{1}{a}\frac{\partial T}{\partial t} \approx \frac{1}{a\Delta t}\left(T\left(r, z, \left(n+1\right)\Delta t\right) - T\left(r, z, n\Delta t\right)\right) \\ &= \frac{1}{a\Delta t}\left(T_{n+1} - T_n\right); \quad b_2 = \frac{1}{r}\frac{\partial u}{\partial r} \end{aligned} \tag{3}$$

where Δt is time step. Equation (3) is the main form of the equation which is solved in the present case. It is clear that the term with $1/r$ on the right hand side does not appear in the classical axisymmetric formulations which use fundamental solutions for axisymmetric problems. In the present case the Laplace fundamental solution is used and the term with $1/r$ is added to the non-homogeneous part of the Laplace equation. This term requires special care when $r \to 0$, as is explained further in the text.

By applying the Green's identity (3) can be transformed into the following integral form:

$$\chi(x)T(x) + \int_{\Gamma_y} q^*(x,y)T(y)d\Gamma_y - \int_{\Gamma_y} T^*(x,y)q(y)d\Gamma_y = -\int_{\Omega_y} T^*(x,y)b(y)d\Omega_y \tag{4}$$

where $x = (r_x, z_x)$, $y = (r_y, z_y)$, Ω is the problem domain Γ is the boundary of Ω, n is the direction of the normal to Γ, $q = \partial T/\partial n$ and $q^* = \partial T^*/\partial n$. The boundary conditions are given as:

$$q\big|_{z=0} = \frac{A\left(T\right)\cdot I\left(r,n\cdot\Delta t\right)}{\lambda}, \quad 0\le r\le R;\quad \frac{\partial T}{\partial r}\bigg|_{r=0} = 0,\ 0\le z\le h$$
$$q\big|_{r=R,z=H} = -\frac{\alpha\cdot T}{\lambda}, \quad r=R, 0\le z\le h \vee z=h, 0\le r\le R \tag{5}$$

4 The dual reciprocity formulation

To avoid domain integration on the right hand side in expression (4) the DRM approximation is applied [1] yielding:

$$\chi\left(x\right)T\left(x\right)+\int_{\Gamma_y}\left(T(y)q^*(x,y)-T^*(x,y)q(y)\right)d\Gamma_y = \tag{6}$$
$$\sum_{j=1}^{N+L}\alpha_j\left(\chi(x)\hat{T}\left(x,y_j\right)+\int_{\Gamma_y}\left(\hat{T}\left(y,y_j\right)q^*(x,y)-\hat{q}\left(y,y_j\right)T^*(x,y)\right)d\Gamma_y\right)$$

In this work the DRM approximation function f was the 1+R radial basis function.

The thermal flux through the elementary surface S which encloses elementary volume dV during infinitesimally time period dt, see Figure 2, is represented using the following expressions:

$$dV\cdot\rho\cdot c\cdot dT = -\oint_S \vec{q}\cdot d\vec{s}\cdot dt;\quad dT = \frac{\partial T}{\partial t}dt+\vec{v}\nabla T, (\vec{v}=0);\quad dV = \Delta r^2\pi\cdot dz$$
$$\oint_S \vec{q}\cdot d\vec{s} = q_r\cdot 2\Delta r\pi\cdot dz+\left(q_{z_2}-q_{z_1}\right)\Delta r^2\pi;\Rightarrow \frac{\rho\cdot c}{2}\frac{\partial T}{\partial t} = -\left(\frac{q_r}{\Delta r}+\frac{1}{2}\frac{q_{z_2}-q_{z_1}}{dz}\right)$$
$$q_r = -\lambda\frac{\partial T}{\partial r};\quad q_z = -\lambda\frac{\partial T}{\partial z}\Rightarrow \lim_{\Delta r\to 0}\frac{1}{\Delta r}\frac{\partial T}{\partial r} = \frac{1}{2}\cdot\left(\frac{1}{a}\frac{\partial T}{\partial t}-\frac{\partial^2 T}{\partial z^2}\right) \tag{7}$$

where a, ρ and c have same meaning as in relations 1 and 2, $\vec{v}$ is velocity of element dV, $q_{1,2}$ is thermal flux in axial direction at point z and $z+dz$ respectively, and q_r is thermal flux in radial direction on boundary surface S. After discretization of the boundary Γ, the unknown temperature T is interpolated on elements on the boundary, the boundary integrals are evaluated and using collocation technique equation (6) is transformed into a system of linear equations.

The nodal values $T(x_i)$, $\hat{T}\left(x_i\right)$, $q(x_i)$, $\hat{q}\left(x_i\right)$, $b(x_i)$, $b_1(x_i)$, $b_2(x_i)$ and the coefficients α_i could be expressed in matrix form as:

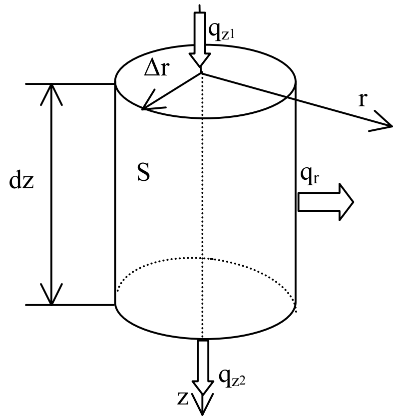

Figure 2: The thermal flux along z-axis.

$$
\begin{aligned}
&\mathbf{b} = \mathbf{F}\boldsymbol{\alpha} = \left[b(y_1) \cdots b(y_K) \right]_{K\times 1}, \quad \mathbf{F} = \left[f(y_i, y_j) \right]_{K\times K}, \\
&\boldsymbol{\alpha} = \left[\alpha_1 \cdots \alpha_K \right]_{K\times 1} \Rightarrow \boldsymbol{\alpha} = \mathbf{F}^{-1}\mathbf{b} \\
&\mathbf{T} = \left[T(x_1) \cdots T(x_K) \right]_{K\times 1}, \quad \mathbf{q} = \left[q(x_1) \cdots q(x_K) \right]_{K\times 1}, \\
&\hat{\mathbf{T}} = \left[\hat{T}(x_1) \cdots \hat{T}(x_K) \right]_{K\times 1}, \quad \hat{\mathbf{q}} = \left[\hat{q}(x_1) \cdots \hat{q}(x_K) \right]
\end{aligned} \tag{8}
$$

$$
\mathbf{b} = \mathbf{b_1} - \mathbf{b_2}; \quad \mathbf{b_1} = \frac{1}{a\Delta t}\left(\mathbf{T} - \mathbf{T_0} \right) \tag{9}
$$

$$
\begin{aligned}
&b_{2i} = \begin{cases} \sum\limits_{j,n,m} \delta_{ij} \dfrac{1}{r_i} \dfrac{\partial f_{jn}}{\partial r} f_{nm}^{-1} T_m, & r_i \neq 0 \\ \dfrac{1}{2a\Delta t}\left(T_i - T_{0i} \right) - \sum\limits_{j,n} \dfrac{1}{2} \dfrac{\partial^2 f_{ij}}{\partial z^2} \cdot f_{in}^{-1} T_n, & r_i = 0 \end{cases}; \\
&\mathbf{b_2} = \left[b_{21} \cdots b_{2k} \right]_{K\times 1}; \quad K = N + L
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
&\frac{\partial f_{jn}}{\partial r} = \left. \frac{\partial f(y, y_n)}{\partial r} \right|_{y=y_j}; \frac{\partial^2 f_{jn}}{\partial z^2} = \left. \frac{\partial^2 f(y, y_n)}{\partial z^2} \right|_{y=y_j}; \\
&f_{nm}^{-1} = \left[\mathbf{F}^{-1} \right]_{nm}; \quad \vec{y} = (r, z); \quad \vec{y}_n = (r_n, z_n)
\end{aligned} \tag{11}
$$

where $\mathbf{T_0}$ is a vector obtained in the previous time step, N is the number of boundary nodes, L is the number of internal nodes, and δ_{ij} is the Kronecker delta symbol.

Now the equation (6) can be expressed in the following matrix form:

$$\mathbf{Hu} - \mathbf{Gq} = (\mathbf{H}\hat{\mathbf{u}} - \mathbf{G}\hat{\mathbf{q}})\mathbf{F}^{-1}\mathbf{b} + \mathbf{I_0} \tag{12}$$

$$\begin{gathered}\mathbf{I_0} = \left[I_{0i}\right]; \quad I_{0i} = \frac{A_0}{\lambda}\int_0^{r_0} I\left(r_y, n\cdot\Delta t\right)\cdot u^*\left(\left|\vec{y} - \vec{y}_i\right|\right)\cdot dr_y; \\ \vec{y} = \left(r_y, 0\right); \vec{y}_i = \left(r_i, z_i\right); \; i = 1, \ldots, K\end{gathered} \tag{13}$$

where **H** and **G** are matrices whose matrix elements were evaluated from the contour integrals. The elements of the vector **q** over contour Γ could be expressed, according to the boundary conditions, by the elements of vector **u**. The elements of vector I_0 represent the equivalent thermal loads on upper surface of the specimens.

Sub-domain technique in the DRM, further referred to as DRM-MD [6] has been used in some examples in order to improve the accuracy.

5 Numerical results

The temperature field distributions in radial and axial direction, inside a mono-layer Al cylinder with radius 7 mm and length 5 mm, which were obtained using the DRM, the DRM-MD with four sub-domains and analytical solution [7] for t=1s are presented in Figures 3 and 4, respectively.

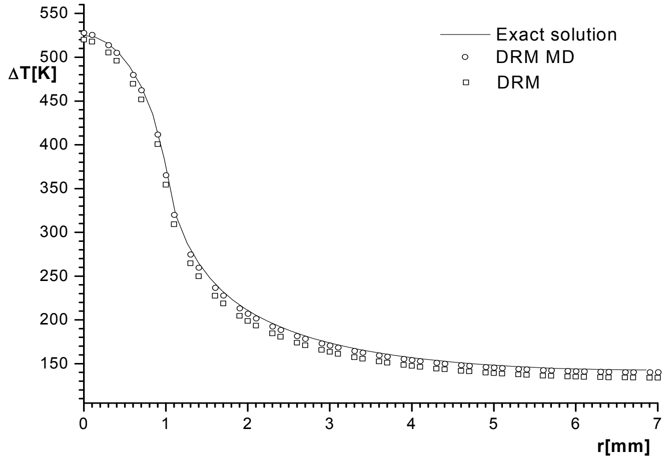

Figure 3: Temperature difference distribution along r-axis for the Al specimen on the upper surface (number of subdomains for DRM-MD=4).

The following properties of the incoming laser beam were considered: power-500W, radius of laser beam 1mm, top head profile and constant laser beam intensity with time duration of 1s.

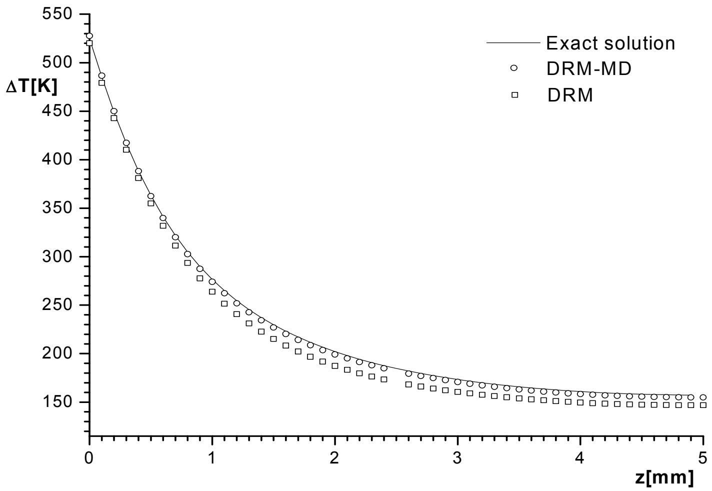

Figure 4: Temperature difference distribution along the z-axis inside the Al specimen obtained by using DRM, DRM-MD and analytical expression (number of subdomains for DRM-MD=4).

The DRM and the DRM-MD results compared to the analytical ones along radial direction for different number of boundary nodes are shown in Figure 5 for t=1s. It can be observed that the accuracy of the DRM-MD was higher than the one achieved using the DRM in all cases.

The distribution of temperature field at t=1 s, in axial directions, for the case of two layer cylindrical structures, is shown in Figure 6. The upper layer of the two-layer structure is made of Al and the lower layer is made of glass. The following dimensions of the structures were used: (i) Al-layer-0.5 mm, Glass layer- 4.5 mm; and (ii) Al-layer-0.7 mm, Glass layer- 4.3 mm thicknesses. In both cases the radius was 7 mm. The following properties of the laser beam were assumed: Power – 100W, radius of laser beam – 1mm, the laser beam has constant intensity with the top head profile and time duration of 1s. The presented results were obtained by using the DRM-MD procedure with nine sub domains. A linear temperature dependence of the absorption coefficient was assumed.

6 Conclusions

The boundary element dual reciprocity method (BE-DRM) was applied to the problem of interaction of laser-mono/multi layer structures with axial symmetry. The BEM formulation is based on the fundamental solution for the Laplace equation.

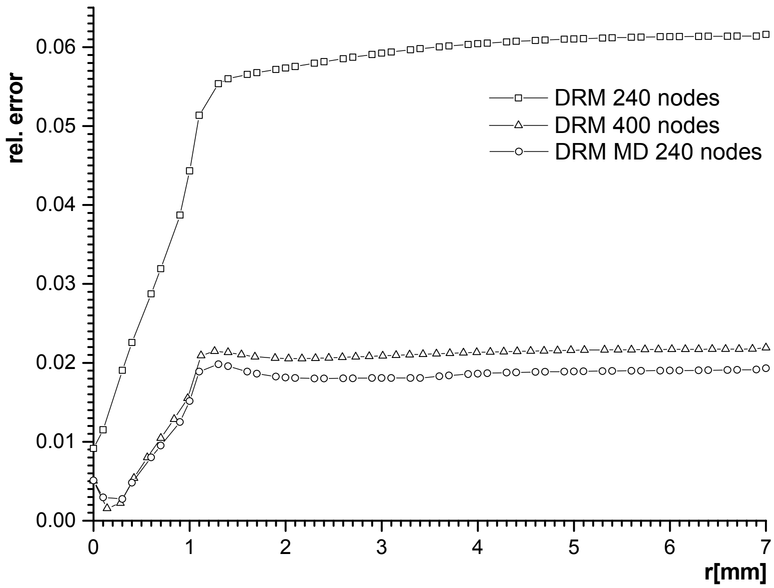

Figure 5: Relative error for the DRM and the DRM-MD along r direction on the upper surface for different number of boundary nodes (number of subdomains for DRM-MD=4).

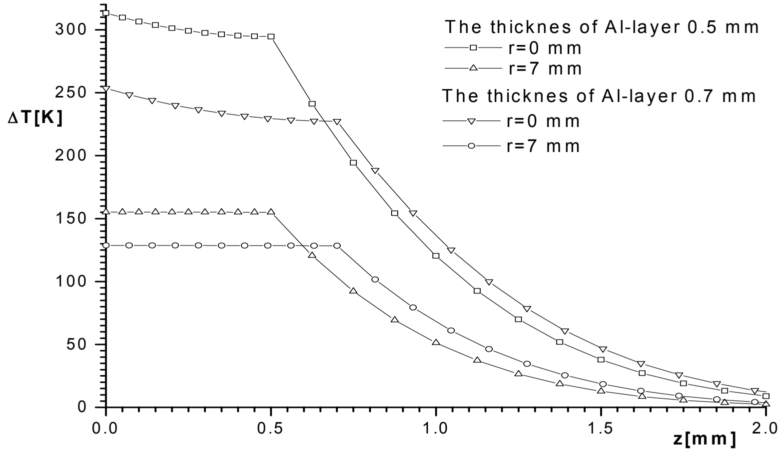

Figure 6: Temperature difference distribution along z-axis in case of two layer structures obtained by the DRM-MD with nine sub-domains at upper surface and at interfaces between layers.

The accuracy of the developed DRM formulation was first tested using analytical solution for a mono-layer case and then applied to a two-layer structure consisting of Al and glass. The results show that the formulation can provide accurate results for this type of problems. The results were compared for the DRM when the domain was kept as a single domain and when it was divided into sub-domains (DRM-MD). The sub-domain formulation showed increase in the accuracy. This behavior of the DRM formulation has already been reported in the past [8].

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