GEOMETRY, GRAPHICS AND DESIGN IN THE DIGITAL AGE

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Ivana Bajšanski
Marko Jovanović

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Editors

Ivana Bajšanski

Marko Jovanović

Graphic design

Jelena Pepić, Marko Jovanović, Ivana Bajšanski

Text formating

Jelena Kićanović, Miloš Obradović, Ivana Bajšanski, Marko Jovanović

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LEARNING WHILE PLAYING - THROUGIE PLATFORM FOR CREATING MODELS OF SPATIAL STRUCTURES

Marija Obradović*, Slobodan Mišić[2]

1University of Belgrade, Faculty of Civil Engineering (SERBIA), e-mail: marijao@grf.bg.ac.rs
2 University of Arts in Belgrade, Faculty of Applied Arts (SERBIA), e-mail: slobodan.misic@fpu.bg.ac.rs

Abstract

The paper shows a way to create a set of tools that allows the construction of different models of spatial deltahedral structures as a result of the game. It is a set of accessories with which participants (school children, students) through play, experimenting with different layouts and combinations of elements of the whole, create spatial forms from uniform elements: sticks of length "*a*" and ring-shaped joint connections. The game uses a STEM learning approach and the "learning by doing" method to provide knowledge about the spatial relationships of elements and static properties of the structure, in the process of playing. The process of forming structures via mentioned connecting elements uses the same principle of connecting supports and nodes as in the real space truss. A similar method is used by some other gaming platforms, such as Geomag, Magna-tiles, Picasso, or Blockaroo. Unlike them, our solution does not use magnets, but prestressing.

Keywords: space structure, deltahedron, strut, node, game, education.

1 INTRODUCTION

Every locomoting living being must understand the characteristics of the space in which it moves in order to maneuver through it and to survive in it [1]. Spatial imagination and, according to some sources, also spatial intelligence ([2], [3]) develops in the early childhood years. Spatial intelligence, also referred to as spatial ability, involves the manipulation of information presented in a visual, diagrammatic, or symbolic form as opposed to a language-based verbal modality [2], [4]. It is "the ability to generate, retain, retrieve, and transform well-structured visual images" [4]. Since spatial intelligence has been the subject of study by psychologists, pedagogues, medicine and related sciences, it is known that by spatial ability we mean different skills of the mind and body: from physical orientation to visualization, mental rotation and perspective taking [4].

Play has a significant role in this matter, and it has been recognized not only in scientific world, but also in the world of commercial products that aim to develop children's motoric skills, spatial imagination, creativity and logic. A whole range of different products can be found under the heading "building toys", which are intended for a certain type of game: OBJECT PLAY [5], [6], [7], [8].

1.1 Play as a way of learning about space

What is play, anyway? No matter how the concept and the activity it represents may seem trivial and easy to explain, it turns out that the notion of play is challenging to define, so some authors [9] are of the opinion that it is not possible to define what play is, while others [10], [11] consider that play is "a free and voluntary activity that is done for its own sake and the pleasure of it". Thus, if we look for a definition, e.g. in the Oxford Dictionary, it looks like this: "engage in activity for enjoyment and recreation rather than a serious or practical purpose." Not surprisingly, because even in scientific circles, play has long been placed outside the domain of learning and treated as an exclusively children's activity. Everything indicates that play is still primarily understood as an optional activity in free time, ignoring its evidently beneficial effects and its necessity for the development of their mental, physical, social, and artistic skills [5], [12], [13], and the well-being of the human brain, or even a physiological need, such as sleep [6].

However, many researchers from the fields of psychology, pedagogy and even philosophy believe that play should be taken seriously [5], [10], [13]. In the last couple of decades (since the end of the 70s of the 20th century), one can notice the valorization of the qualities found in the very process and experience of the game, as well as its potential. This also applies to older age groups, not only to children. Adults also tend to play also, and the playfulness can be recognized in various activities, from methods of creation as an artistic process, to application in modern ways of developing products and services, to courses and hobbies [14]. Therefore, play has an obviously greater and more significant role in the process of imagination, creativity and connection of processes, situations and cause-and-effect relationships, than one might think at first glance.

Play, from the authors' perspective, can be described as a fun experiment in which an idea is tested for future implementation. What makes play attractive and fun is not the outcome of the play itself, but rather the process, which does not lead to a "once and for all" defined result. It allows imagination, inventiveness, the influx of new ideas in the process itself, some trials and errors, joy at the success of attempts and in the correction of mistakes. Finally, it gives a victorious feeling when the result of the game turns out to be successful.

According to the theory that can be found in several different sources [15], [7], [8], there are several forms of play: Locomotor play, social play, parallel play, object play, language play and pretend play. Some sources mention celebratory and ritual play or, according to Dr. Stewart Brown [5], there are: Attunement Play, Body Play & Movement, Object Play, Social Play, Imaginative & Pretend Play, Storytelling-Narrative Play, Creative Play [8]. For this study, we are interested in the possibilities of OBJECT PLAY, i.e. playing with objects – gadgets and toys, and its role in developing spatial abilities for perceiving, figuring out, designing and creating forms given by a certain set of elements. Also, the kind of play we strive for should also enable research, self-expression, insight into the behaviour of certain materials and familiarization with the properties of both the material and the construction itself, as a synergistic result.

What is the purpose of this research? To connect play, learning about shapes and about basic principles of statics that make structures rigid and stable, as well as to show that through the physical creation of models (mock ups), rather than through virtual 3D modeling, successful solutions can be reached in terms of design, construction and creativity.

2 3D STRUCTURES AND THEIR POTENTIAL FOR PLAY - CREATING POLYHEDRAL FORMS FROM IDENTICAL ELEMENTS

Since our goal in this paper is to demonstrate the possibility of forming statically stable structures created exclusively from uniform elements - rods and joint connections (i.e. nodal connections), we know from statics that the only stable flat figure that we can form from rods is triangle. This leads us to structures made primarily or exclusively of triangles. These are polyhedral structures that have only triangles, more precisely: equilateral triangles in their composition. They are called – deltahedra (delta: Greek capital letter D has the shape of a triangle) [16], [17]. Since they consist of rigid faces, deltahedra can be formed as statically stable and rigid, but primarily depending on their joint connections. The tighter, more stable and more precisely defined the connections, the greater the rigidity of the structure.

Another interesting feature of deltahedra is that they can only have an even number of faces [18], which can be of great help if we want to form a solid or a surface that encloses the convergent space.

So, for our game, we will focus on deltahedra - polyhedra that are made exclusively of equilateral triangles. However, since this strict criterion makes the task a bit difficult, as it requires greater knowledge of topology, trigonometry and discrete geometry, we will extend the task to structures with a deltahedral lateral surface with one n-sided (non-triangular) basis allowed, i.e. polygon that will be the foundation of the structure itself.

2.1 Deltahedra and their geometry

Let us mention some of the known deltahedra that can appear as solutions – results of the game. The eight convex deltahedra should be mentioned: 3 of the 5 Platonic solids, as regular deltahedra: tetrahedron, octahedron and icosahedron (Fig. 1, top row), then 5 Johnson solids [19]: triangular bipyramid (J12), pentagonal bipyramid (J13), snub disphenoid (J84), triaugmented triangular prism (J51), gyroelongated square bipyramid (J17) (Fig. 1, middle and bottom row).

There is an infinite number of concave, i.e. non-convex deltahedra. Among them we can mention: Stella octangula (Fig. 1, bottom row, framed), "boat" with 8 triangles, excavated dodecahedron, toroidal deltahedron of 8 octahedra, etc.

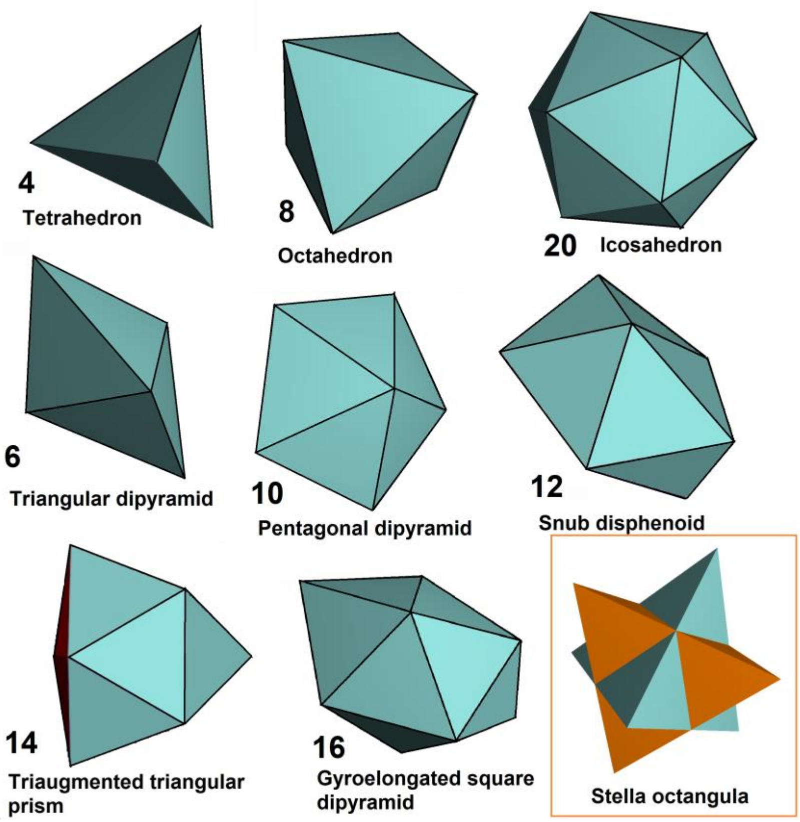


Fig. 1. Eight convex deltahedra and an example of a concave one (Stella octangula)

We can also point out to certain members of the family of concave polyhedra of the second and higher sorts ([20], [21], [22], [23], [24], [25]), especially bipyramids of the second sort (CbP-II-n), augmented bicupolas of the second sort (CbC-II-4 and CbC-II-5), and toroidal deltahedral structures created by double incavation of CP-II-(6-9) in bicupolas CC-II-(6-9) [23]. Of course, there is an unlimited number of free, non-convex deltahedral forms. Such breadth and freedom in number and form of results is exactly the polygon for playing combinatorics and exploring shapes so to find: in which various ways triangles can be assembled to provide a statically stable 3D structure i.e. that can stand independently, without additional supportive elements.

3 3D MODELING - VIRTUAL AND PHYSICAL

3D modelling through graphical software has, with all its advantages (low cost, precision, immersiveness, ability to connect with other media, possibility of 3D printing, etc.) also some disadvantages. It requires prior knowledge of work on computers, which, along with learning the software itself, training and gaining experience can consume a lot of time. Then, it requires additional special skills: mandatory knowledge of space representation, 2D projections, descriptive geometry and mathematics, and it also requires excellent navigation in virtual 3D space. Besides, it creates the illusion that pervasion of two solids' spaces is feasible, while is not in the physical world. Lastly, it lacks the immediate experience of an object in real space that makes the created construction actualized and be tangible.

Although contemporary software (e.g. Kangaroo for Grasshopper, Tomohiro Tachi's Origamizer [26], and several others) are moving towards fully compensating these shortcomings and being able to simulate the real play of shapes in physical space also in virtual one, the instant insight into viability, feasibility, durability and other physical characteristics of the structure is still lacking, if prior detailed calculations are not done.

On the other hand, a physical model requires more resources (if we hope for appropriate results). Workshops, laboratories, tools, cutting machines, materials, consumables, etc. are necessary for such an activity. Of course, there is the question of financing numerous new attempts. Exercise, training and modeling in real space has its disadvantages too, in terms of physical material and space requirements, all of which create costs. In addition, the material is often disposable.

In the meantime, playing a game in the real space with real and tangible elements allows a direct insight into the result of the experiment (game), and even into durability of the construction itself (e.g. how much load it can bear). Hence, this type of empirical knowledge and learning by experience (learning-by-doing method), allows the user of the game to adopt the principles that the game aims to to promote more quickly and comprehensively.

So, we translate a physical model into a handicraft that integrates the game as an experiment, using the STEM principle in learning (an approach to learning and development that integrates the areas of science, technology, engineering and mathematics), and the Learning-by doing [27] principle at the same time. Thereby, all the elements of the model itself are treated as re-usable building blocks of a play set, the game becomes possible, and the experiments are unlimited. This is the very reason that toys are developed, so they can be assembled and disassembled, used multiple times, and their parts treated as an accessory, a tool-kit that can be used in creation of different shapes, many times.

LEGO sets uses plastic building blocks, HABA - wooden pieces, while GEOMAG sets use magnetic sticks and meal balls supplemented with plates for stiffening unstable polygons (squares and pentagons). Magna-tiles and Picasso Tiles also uses magnetic plates, Blockaroo – magnetic foam, and so on. We notice that magnets have become an increasingly common way to connect elements, because they allow freedom in the formation of dihedral angles or angles in a nodal connections.

For example, if we wanted to make building blocks toys with solid sticks, solving the joint connections emerges as a problem. This would imply that such a connection could be either fixed (meaning that the angles within the nodal connection must be predefined) or flexible (which means that the rods could be adjusted relative to the node, i.e. the angles are not predefined. In the first case, the game is limited to assigned solutions, which decreases the freedom of investigation. In the second case, the problem arises: how to solve the way of connecting and tying the rod to the node. Magnet-based games solve this through magnetic force; however, such a method dictates smaller dimensions of the elements (which is fine for younger children, but as a substitute for a design experiment, it has its own limitations), or a significantly more expensive toy.

The movement and rotation of rods within a solid joint connection is not a problem in 3D modeling, but in the physical world this problem exists and a way to solve it needs to be found.

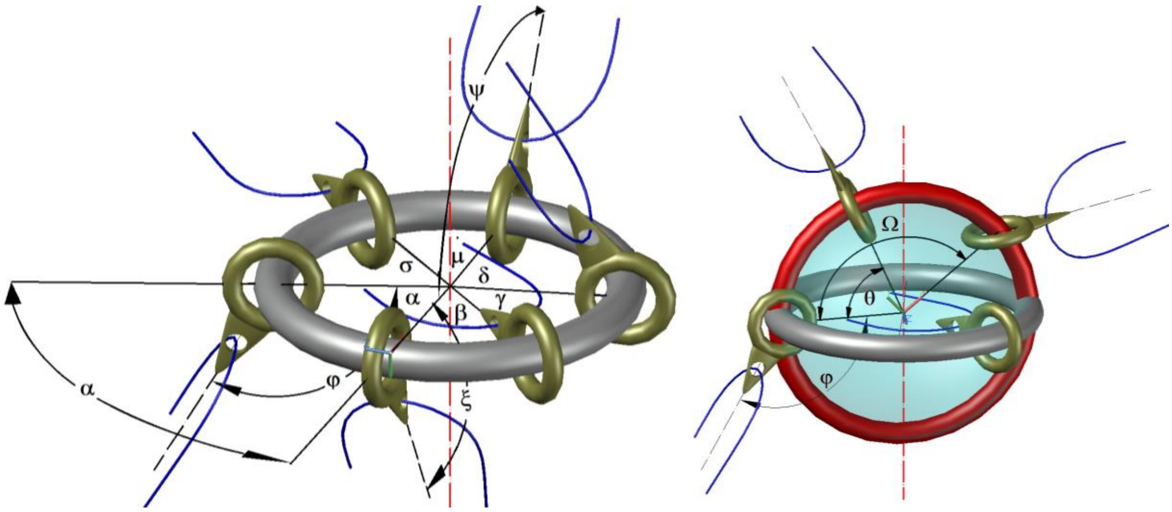
4 THROUGIE PLATFORM

Our solution uses geometric knowledge: first of all on the stability of the triangle as a polygon, on the stability of deltahedral structures, and then on the properties of the torus as a shape that allows the 3D rotation of the clasp (the joint of the rod and the node) for any desired angle.

The name THROUGIE is an acronym of the words: **T**ube - **H**ook - **R**ing: **O**riginal, **U**niform, **G**eometric, **I**nstructive **E**quipment, which all briefly describe the elements of the tool kit, as well as the platform itself. THROUGIE consists of 4 basic elements: tubular rod (rigid pipe), hook (clasp), toric nodal joint (ring link), and elastic band. These elements can be found in wholesale or retail, so this tool kit can be almost homemade, or more precisely: “home-found”.

In our accessories, the main component elements are: tubular rods and ring links (joint connections), with the fact that we used the knowledge of the formation of real world spatial prestressed structures to connect these elements. Therefore, in order to connect the rod and the joint, we use a tube (a paper straw) to play the role of a rod, we pass an elastic band (rubber band) inside it and then attach one clasp to each end. As joints, we use metal ring links to which we attach the endpoints of the rods – the clasps. Given that in such structures there are a large number of different angles between the rods themselves, the most convenient shape for the nodal connection is a torus, because it ensures the mobility of the "loops" on the clasps in all directions from 0 to 2π in the plane of the torus radius, and also from 0 to 2π in the normal planes of the tube radius, Fig. 2 a). Since the link is torus-shaped, it allows the clasp's ring-like end to move radially by 2π around the link, and also allows each clasp to rotate by (almost) 2π during that movement, depending on the position of the rod itself in the construction. We will use this feature to derive both simple (e. g. Plato solids) and very complex forms of structure (free form deltahedra, concave polyhedra of the second and higher sorts).

For the workshop, we used: for rods – "frappe" paper straws, l=218mm and Φ=8mm, key clasps and lobster clasps 25x9mm, metal rings Φ=15mm and rubber bands Φ=55mm, Fig. 3.



a) *b)*

Fig. 2. Connection between the clasp and the ring: a) the clasp moves and rotates around the torus link, b) the clasp cannot move around the sphere, or in a plane that is outside the equator of the torus

Unfortunately, we have to admit that the torus doesn't work flawlessly (like a sphere, Fig. 2 b), when there are more complex nodes with rods that need to be centered in between the other rods in the connection, so such a connection should be predefined and solved in advance.

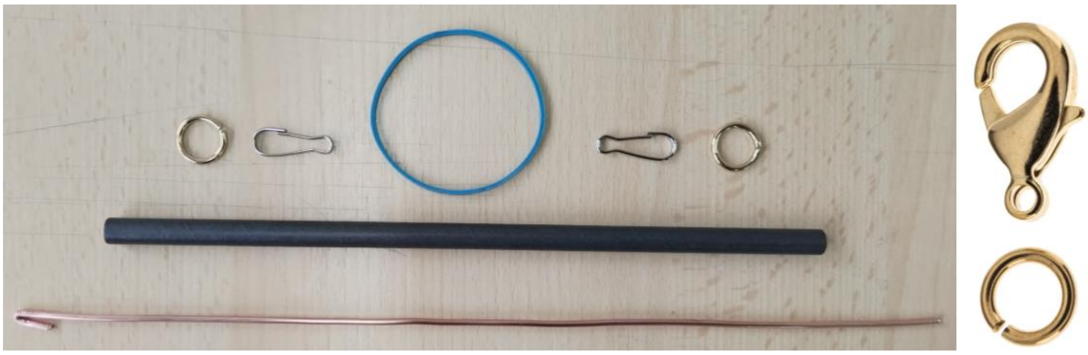


Fig. 3. Basic elements of the platform: (left) ring link, key clasp, rubber band, paper straw for frappe, wire hook for threading the rubber band through the straw, (right): the lobster clasp and the ring link

The advantage of the THROUGIE platform compared to some other similar toys is that the dimensions of its unit element - rod can be significantly longer (depends on the dimensions of adopted tubular rods), which gives the construction more comprehensibility and a fuller experience of space. Also, one of the key advantages of the platform is that it can be improvised from materials at hand, which encourages creativity and resourcefulness. The procedure for forming the structure model is relatively simple and quick, which allows easy assembly and disassembly even for school children. Students can use this tool kit in the workshop to make diverse deltahedral structures, even transformable ones.

The THROUGIE tool kit, although intended for educational purposes, can also be useful for engineers to test their ideas or experiment with the shapes that the elements of the kit can form.

5 THE PROCEDURE AND WAY OF ASSEMBLING THE STRUCTURE - THE RULES OF THE GAME

We start from the fact that in order to connect the rod and the joint, we need to pass an elastic band (rubber band) through the tube of the rod - which is done using a handy tool: a long hook. In order for the band to be taut from end to end of the rod, first we attach a clasp to one side, then thread the band through the tube with a hook, and lastly, attach another clasp to the band at the opposite side of the tube. The clasps should be of such dimensions that the diameter of the annular part of the clasp is larger than the diameter of the rod tube. Thus, both clasps will remain outside the tube and wait to be connected with the joint - the link.

Next, we connect the rods into joint connections by hooking the clasps to the torus link. In this way, we can form the vertices of the structure:

directly, based on a picture, photo or projections (which requires a bit more geometric knowledge and spatial ability),

using the grid of the structure, which should be made first, and then just assembled (with a minimum number of steps and snaps), or intuitively,

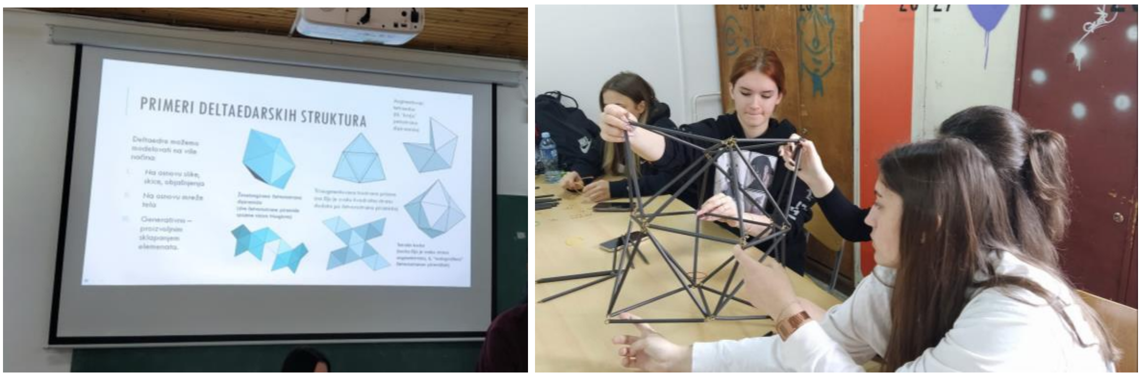
randomly until a satisfactory result is obtained (a procedure similar to generative design).

In the workshop, held on February 18, 2023 at the Faculty of Applied Arts in Belgrade, we divided the students into 3 groups.

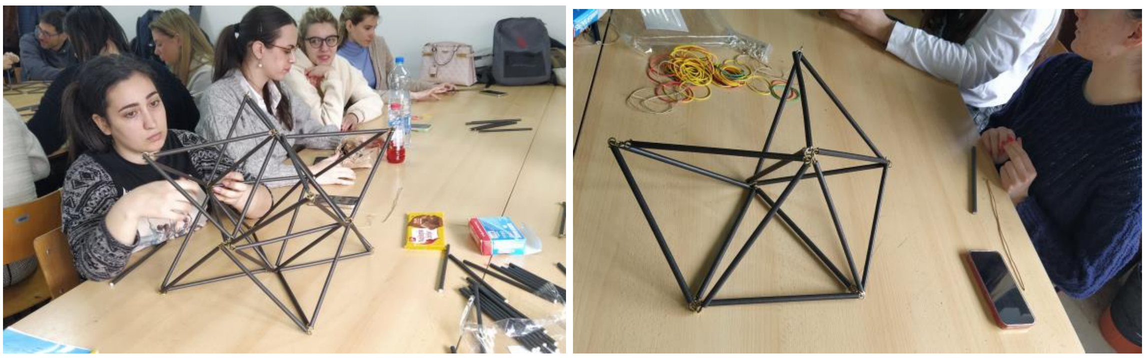
Group A made the structures according to the picture: regular convex and concave deltahedra (Fig 4a and 4b);

Group B made structures according to the network: Concave pyramids of the IV sort, *CP IV-20B* ($C^*Q^*O_2$) [28] (Fig 5);

Group C made arbitrary, free form structures (Fig 6).



a)



b)

Fig. 4. Group A: a) creates closed deltahedral structures based on the image - 2D representation. Pictured is a hexaugmented cube; b) creates closed deltahedral structures based on the image - 2D representation. The picture shows: Stella octangula and scarce pentagonal dipyramid

Groups A and C worked with lobster clasps, and group B with key clasps.

It turned out that the application of key clasps is more suitable for the idea implemented with group B, because they rests more firmly on the ring link and make the nodal connection more stable, while the lobster clasps, due to their looser connection with the links, were not able to maintain deltahedral structures rigid if they were not composed of octahedron-tetrahedron (OCTET) elements.

Also, we previously experimented with another type of connection: just with a rubber band tied directly to a rubber ring (Fig. 7 a), which gave an even tighter and more stable joint connection. This is because there was no free rotation of the clasps around the rings and it was still possible to move the rods in accordance with the required position. However, the disadvantage was in the difficult disassembly of the structure.

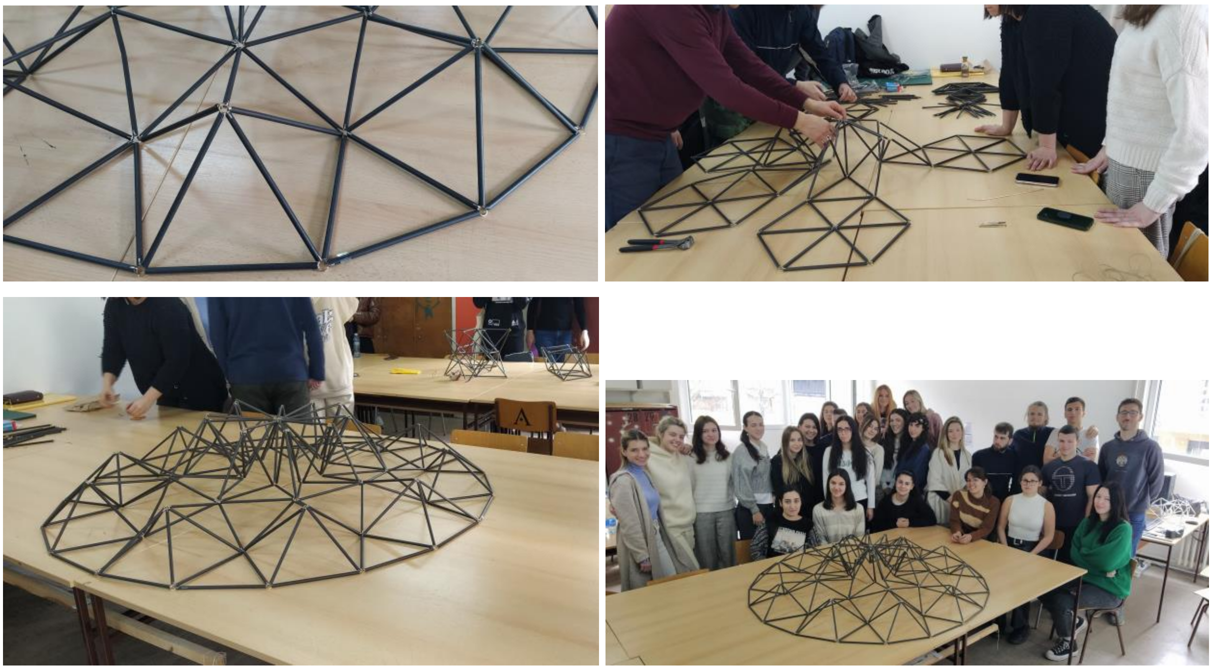


Fig 5. Group B: creates deltahedral lateral surface of CP IV-20B ($C^*Q^*O_2$) based on the scheme - 2D plan. The picture shows phases of creating the structure

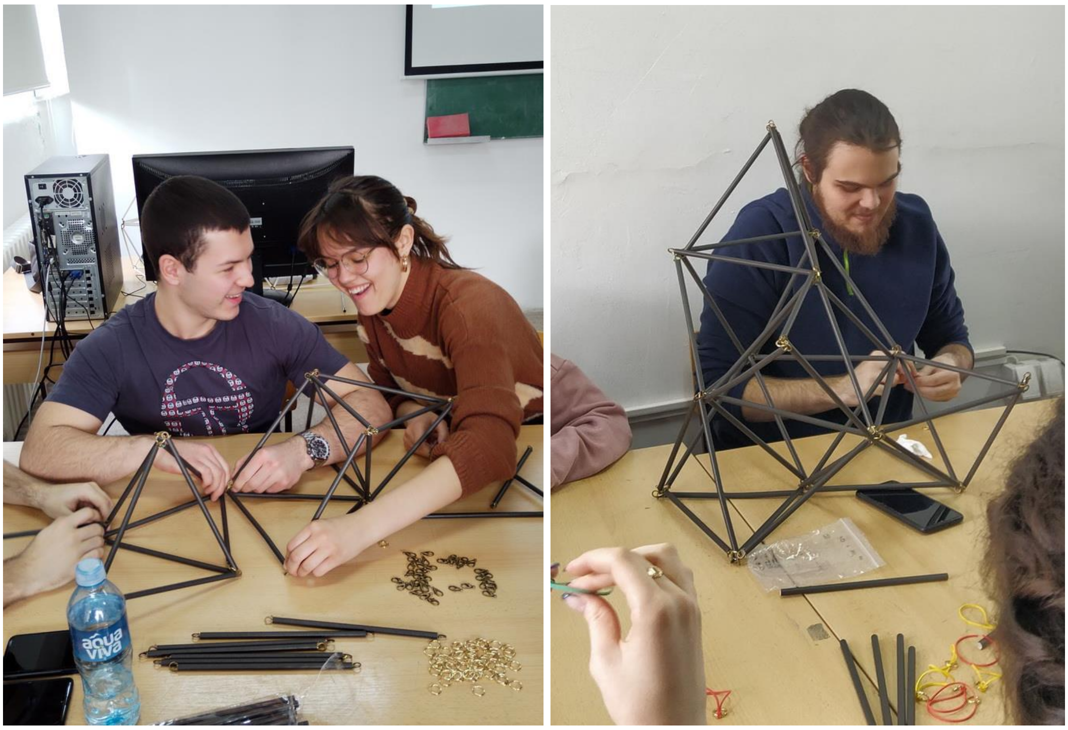


Fig 6 a). Group C: creates free-form deltahedral structure without any pre-defined plan. The picture shows phases of creating the structure

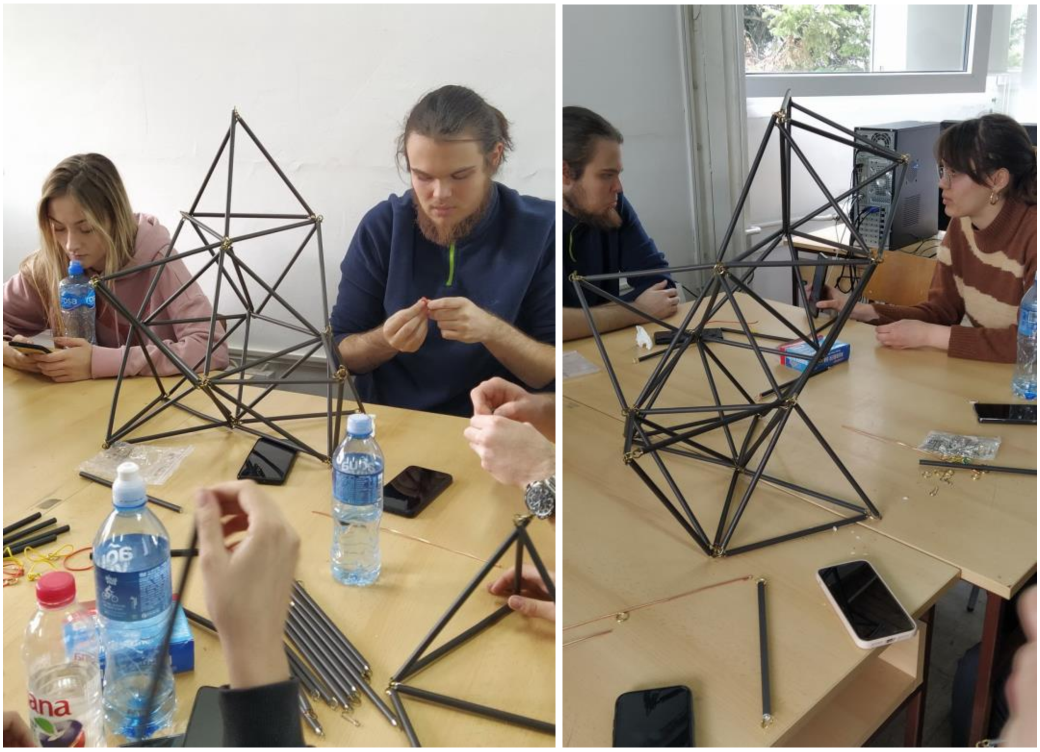


Fig 6 b). Group C: creates free-form deltahedral structure without any pre-defined plan. The picture shows free development of the structure by adding tetrahedra to its triangular faces

6 A STEP FURTHER – TRANSFORMABLE STRUCTURES

What is particularly interesting to engineers nowadays, when it comes to polyhedral structures, space truss or folding structures is the possibility of their transformation, i.e. changes in the shape of the structure itself and (or) the position of its elements, for the same number of elements and the same sequence of their joints.

Transformable structures as mechanically conditioned systems capable of changing the spatial position of their elements: rods, joints or entire plates, and thus their geometric shape are becoming increasingly accepted solutions in contemporary architecture and design. They can adapt their shape to changes in the function of the space and to various changeable external and internal influences [29], [30], [31], thus becoming a flexible framework, as opposed to the traditional notion of a permanently given shape of the structure [32].

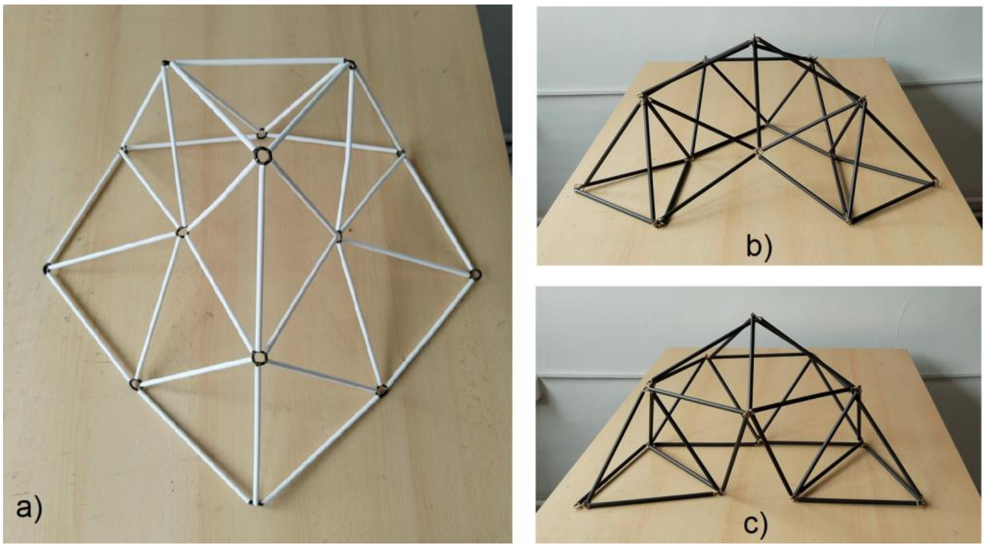


Fig 7. Transformable deltaheral structures: a) with rubber rings joint connections, b) and c) with lobster clasps

Why is structure transformability interesting for our game? A deltahedron lateral surface can change its shape, if it is not closed in a convergent structure, i.e., if it has at least one n>3 polygon. The larger the number n, the greater is the variety of shapes of the non-triangular face, so the greater the freedom of the structure's transformability. By defining the position of the structure's vertices, we define its final shape. In the papers [20], [21], [22], [23], [24] and [25] it was shown that there is a large number of structures with two variants: major (M) and minor (m), depending on the way the structure is assembled, i.e. whether certain vertices are recessed or protruding. This feature can be considered a transformable feature of such structures, whereby their base polygon remains the same and unchanged in each variant. Such structures, when the given base polygon n (3<n<10) is fixed, become completely rigid thanks to their deltahedral lateral surface [23], [25].

7 DISCUSSION

Existing building toys enable play that includes a STEM approach and develops creativity and spatial abilities in children. However, although they can also be used as experimental accessories for empirical research of structures, many of them do not meet such criterion, mainly due to:

- the dimensions of the elements,
- the way they are connected (which does not correspond to the actual joining of elements of spatial structures), and
- the design of toys, which, mainly because of the bright colors intended for children, are associated with an activity that has primarily the role of leisure, and not serious research. (As insignificant as this may seem, psychologically it turns out to be important.)

7.1 Advantages

The platform that we propose in the paper consists of ready-made elements that can be found in retail, or can be used or re-used, as recycled items. **THROUGIE** is created by connecting tubular rods with toric rings via clasps that tighten rubber bands. In order to get a rigid structure, one of the rules of the game is that the faces of the structure must be triangles, with the possible exception of one.

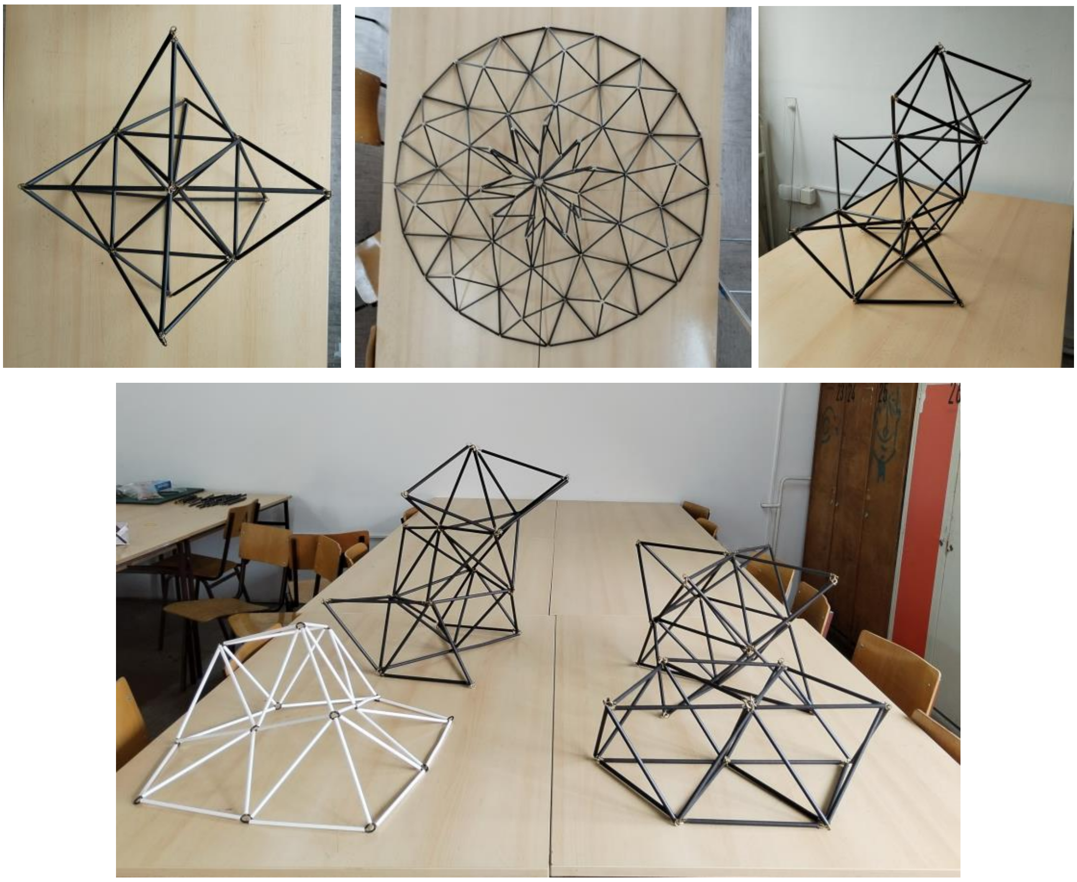


Fig 8. The resulting deltahedral structures from the workshop

Through the workshop, we showed that this method works and gives the expected results (see Fig. 4-8). This way of playing as an experiment can help in the development of creativity, not only of children but also of designers. Its advantage is that it instantly confirms static stability, sustainability and durability in real, physical space.

As an advantage of this way of exploring structural forms over virtual one, it is important to emphasize that often in 3D software, in order to create the desired structure, we need to geometrically define the positions of its vertices. By creating physical models in real space, we can actually form a construction that "stands" both geometrically and statically, without knowing the exact positions of its vertices. We obtain them empirically. Finally, the advantage of this kind of modeling over the digital one is in practicing motor skills, dexterity, feasibility, maintaining the contact with concrete physical forces and conditions (tension, pressure, gravity, balance, flexibility, endurance, etc.), to mention only some, because without these direct experiences, we lose the idea of eventual future behavior of real constructions in real space.

7.2 Disadvantages

During the workshop, a couple of issues appeared as disadvantages of this platform: a) joining rods in a joint connections that require spherical rotation of the clasps (which asks for more spatial thinking and greater spatial abilities, and often predicting the arrangement of the rods in advance); b) sloppy connection of the rods and nodes causes the joint connections to be loose and not keep the structure in a stable position.

Conclusion

The THROUGIE platform certainly met the expectations and gave the desired results, which has been proven in practice through completed assignments at the workshop. However, it requires some additional refinements and improvements in order to secure and strengthen the nodal connections.

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