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REVIEW OF METHODS FOR DETERMINING THE GRAVITATIONAL EFFECT OF TOPOGRAPHIC MASSES

Abstract

The text discusses methods used in geodesy and geophysics to determine the gravitational effect of topographic masses. Three main methods — utilizing the formulas of the right parallelepiped, spherical tesseroid, and flat tesseroid — are applied within Bjerhammar's reduction scheme. The correction for the terrain's gravitational effect is evaluated on a digital terrain model. The research concludes that, overall, the three methods are practically equivalent, even in rough topographic conditions. However, in one test of Bouguer anomalies, formulas involving tesseroids and flat tesseroids show a slightly lower root mean square (RMS) compared to those using standard right parallelepiped formulas.

Keywords (Style Abstract Title)

Gravitational effect of topographic masses, Modeling, Terrain correction, Right parallelepiped, Spherical tesseroid, Flat tesseroid

1. INTRODUCTION

The text discusses the constant analysis and modeling of the gravitational effect of topographic masses. Standard methods for modeling this effect include Bouguer reduction, Helmert reduction, and isostatic reductions based on Eire-Heiskanen and Pratt-Hayford models [1]. In recent years, Residual Terrain Correction (RTC) has been devised as a method for geodetic applications to assess gravity fields and geoids [2].

In all these approaches, the gravitational effect of topographic masses is calculated using a Digital Terrain Model (DTM) at a given resolution, assumed to represent the actual shape of the Earth's surface. The topography is discretized in DTM resolution, and the gravitational effect of each element of the topographic masses is calculated, with different formulas applicable. Typically, individual terrain elements are modeled as right parallelepipeds [2] or as spherical or ellipsoidal tesseroids [3]. This study describes these two approaches, as well as the one introduced by Tsoulis [4], where the lower and upper sides of the tesseroid are flat surfaces (flat tesseroid). This method is used for terrain corrections within the framework of Bouguer reduction [18].

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Chapters 2 and 3 contain formulas for calculating the gravitational effect of the right parallelepiped, spherical tesseroid, and flat tesseroid. Bouguer reduction is computed using these three different approaches, and this will be presented in Chapter 3.

2. TERRAIN CORRECTION

The text discusses the essential concept of terrain correction in the context of calculating Bouguer's gravitational anomaly. When measuring gravity at a specific point P on Earth's surface, the gravitational effect of topographic masses significantly influences the readings. To enable accurate geophysical analyses, like determining Moho depth or crustal mass anomalies, the gravitational effect of the terrain is removed. This is commonly achieved through Bouguer reduction.

The observed gravity value, g(P), is adjusted by subtracting the gravity effect of a hypothetical Bouguer plate located at point P. This plate is treated as an infinite horizontal surface with constant density (ρ), and its gravitational effect is computed using the well-known formula A_B=2πGρHP, where G represents the universal gravitational constant. The refinement of this topographic signal reduction is further achieved by incorporating terrain correction (TC). TC considers the masses above or below the Bouguer plate at height HP, always yielding a positive quantity added to the observed gravity to account for the terrain effect. Therefore, the corrected gravity at point P is expressed as: g(P)- A_B+TC(P).

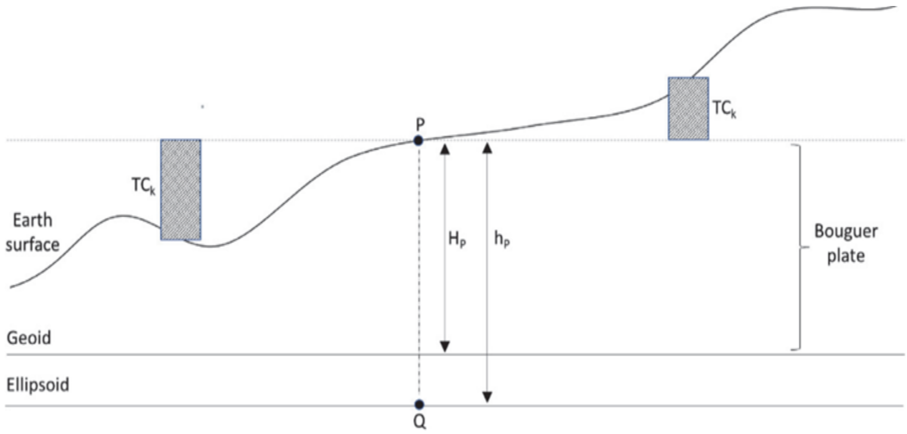

Figure 1 Plate and terrain correction (H_P = orthometric height, h_P = ellipsoidal height) [18]

The effect of terrain correction (TC) and plate density (ρ) on the computation of Bouguer's gravitational anomaly. It emphasizes that residual effects from masses with different densities can still affect reduced gravity values. However, the method is presumed effective in eliminating the significant topographic effect from observed gravity values g(P).

Next, the reduced gravity data undergo adjustment to the geoid, considering the gravity gradient (∂γ/∂h) approximated by the normal gravity gradient of free air. The adjustment formula is expressed as $g(P) \; - A_B + TC(P) - \frac{\partial g}{\partial h} H_P$.

The text further introduces the concept of normal gravity at a specific point on the ellipsoid (point Q), subtracting it to obtain the standard Bouguer anomaly $\Delta g_B = g(P) - A_B + TC(P) - \frac{\partial g}{\partial h} H_P - \gamma(Q)$.

While the effect of the Bouguer plate can be readily computed, the terrain correction effect poses greater complexity. In a planar approximation, it is formulated as a numerical integral, often facilitated by methods like Fast Fourier Transform (FFT).

Alternatively, TC can be determined through a quadratic integral using the Digital Terrain Model (DTM) within a specified area (S), with a radius dependent on topographic roughness. The TC effect is approximated as a sum of individual TC_k integrals, where each TC_k represents the gravitational effect of a specific DTM element.

3. METHODS OF DETERMINING THE GRAVITATIONAL INFLUENCE OF TOPOGRAPHIC MASSES

The proposal and utilization of various mathematical models for calculating TC_k. The focus of this work involves the examination and comparison of three methods for determining the gravitational effect of topographic masses: the formula for the gravitational effect of the right parallelepiped, spherical tesseroid, and flat tesseroid.

In the subsequent subsections, a more detailed description will be provided for each method.

3.1. RIGHT PARALLELEPIPED

In the context of calculating the gravitational effect of topographic masses, a right parallelepiped is a geometric model representing a portion of Earth's topography. It's a three-dimensional figure with rectangular faces and right angles between them. This shape serves as a simplified representation of the local topography. The gravitational effect of each right parallelepiped is computed based on its dimensions (length, width, height) and the density of the material it represents. The calculation involves considering the mass enclosed within the parallelepiped and its distance from the specific point where the gravitational effect is being assessed. Overall, the use of right parallelepipeds provides a practical approach to estimate the gravitational influence of complex topographic features.

Given the gravitational potential V of body B of constant density ρ as an integral:

$$V(P) = G\rho \iiint \frac{1}{l} dv(Q)$$

$$l = |r(P) - r(Q)|$$

$r(P)$ − the position vector of the point to be calculated

$r(Q)$ − integration point position vector

the gravitational effect of the right parallelepiped can be calculated assuming that the calculation point P is in the Cartesian reference system, formula [2]:

$$\frac{\partial V}{\partial z} = G\rho \left|\left|\left| x\,ln(x+y) + y\,ln(x+r) - z\,arctan\left(\frac{xy}{zr}\right)\right|_{x_2}^{x_1}\right|_{y_2}^{y_1}\right|_{z_2}^{z_2}$$

Where: $-\sqrt{x\ ^2 + y\ ^2 + z\ ^2}$, ($x_1, x_2, y_1, y_2, z_1, z_2$) are the sides of the parallelepiped.

3.2. SPHERICAL TESSEROID

UNIPOL, a method for investigating the gravitational effects of spherical tesseroids, is introduced in the text. This approach is built upon the insight that a closed formula for the Newtonian integral emerges when the observation point P aligns with the polar axis, causing the tesseroids to align with sectors of the spherical zonal belt of a spherical cap.

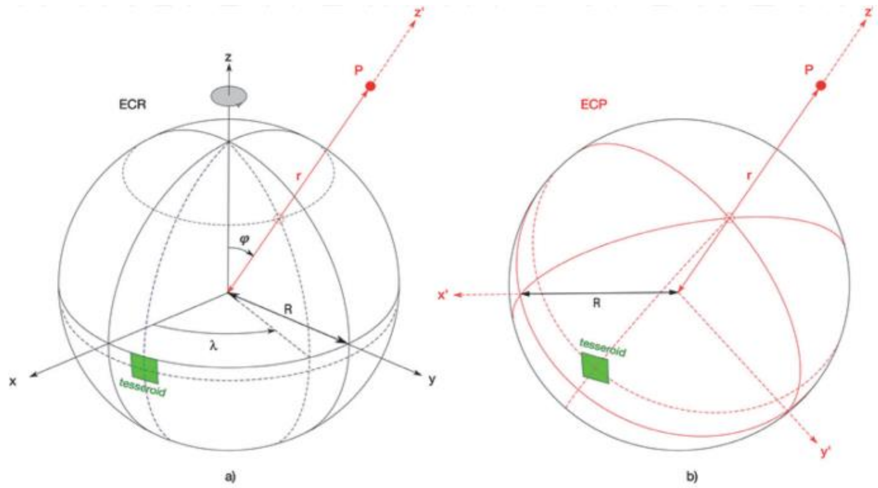

Figure 2 Geometry is used to calculate the contribution of a spherical tesseroid (green rectangle) to the gravitational acceleration at point P outside the spherically symmetric Earth. a) Representation in the rotational reference frame (ECR). b) Representation in the P-rotational reference frame (ECP) at the center of the Earth, defined with respect to the observation point P. R represents the mean radius of the spherical Earth; φ and λ denote geographical latitude and longitude, respectively. [18]

When the observation point P deviates from the polar axis, UNIPOL maps each tesseroid, defined in a rotational reference frame centered on Earth (ECR Tesseroid), to a sector of the spherical zonal belt of a spherical cap centered on point P on Earth. This mapping involves two procedures based on the tesseroid's spherical distance from the observation point.

The text provides a complex formula expressing the gravitational contribution of a spherical tesseroid with height h, incorporating parameters such as the Earth's radius (R), geographical coordinates (φ and λ), among others.

In summary, UNIPOL is detailed as a method for calculating the gravitational effects of spherical tesseroids, accommodating various scenarios and parameters.

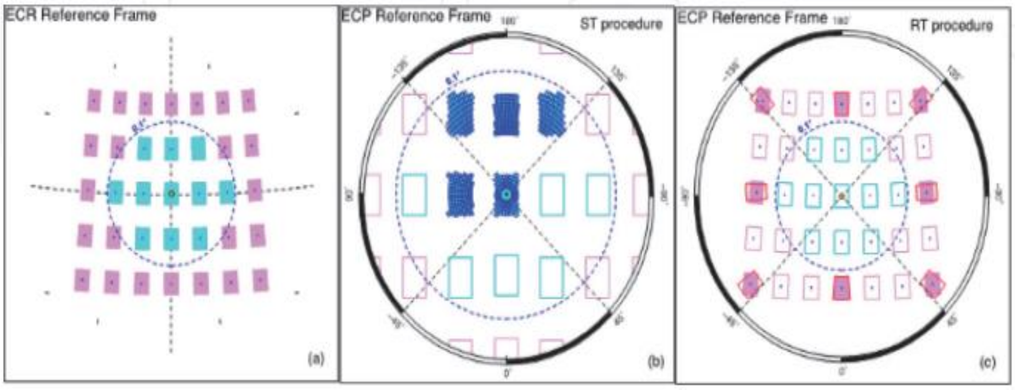

Figure 3 Scheme of the UNIPOL approach. a) Set of tesseroids in the rotational (ECR) reference frame. The yellow circle represents the observation point. The blue dashed circle around the observation point indicates the region where tesseroids are located at a distance ≤0.1° from the observation point, requiring the ST procedure. Cyan and pink colors are used to mark tesseroids requiring the application of both ST (cyan) and RT (pink) procedures. b) Tesseroids mapped in the Earth-centered P-rotational (ECP) reference frame and the decomposition of some into sectors of a spherical belt (light blue tesseroids) in the local ECP reference frame. c) Tesseroids transformed into the reference frame of the R-rotation (ECP) ECP (pink areas) and the reorientation of some in the local ECP reference frame (red sectors). Adapted from Marotta and Barzaghi. [11][18]

The second procedure (RT procedure) is based on the rotation of the ECRTesseroid (pink areas in Figure 3c) around its center and its resizing to form a sector of a spherical belt (red lines in Figure 3c) that develops along the local ECP meridian. The resized tesseroid has ECP-longitudinal dimensions to maintain the same surface area as the original ECRTesseroid.

3.3. FLAT TESSEROID

The text introduces the Flat Tesseroid (FT) approximation, a polyhedron obtained by flattening tesseroids. This polyhedron approximates both right and spherical prisms. Under the FT assumption, a linear integral algorithm is applied for calculating the gravitational effect of topographic masses. The algorithm involves applying Gauss's divergence theorem and using analytical formulas, which include transcendental expressions. The approach requires managing relative positions of points on each face of the polyhedron, handled algorithmically using vector calculus tools. The text concludes with a brief description of implementing this approach for terrain residual correction using a set of calculation points and mesh points representing a Digital Elevation Model (DEM).

The key formulas include:

Linear Integral Algorithm:

$$V_{x_i}(P) = G\rho \iiint_B \frac{\partial}{\partial x_i}\left(\frac{1}{l}\right) dv(Q); i = 1, 2,3$$

Gravitational Potential Components:

$$V_{x_i}(P) = G\rho \left[\sum_{q=1}^{m} \quad \sigma_{pq} h_{pq} LN_{pq} + h_p \sum_{q=1}^{m} \quad \sigma_{pq} AN_{pq} + sin\left(g_{A_P}\right)\right]$$

Transcendental Expressions:

$$LN_{pq} = ln\frac{s_{2pq} + l_{2pq}}{s_{1pq} + l_{1pq}}$$

$$AN_{pq} = arctan\frac{h_p s_{2pq}}{h_{pq} l_{2pq}} - arctan\frac{h_p s_{1pq}}{h_{pq} l_{1pq}}$$

These formulas involve parameters like G (gravitational constant), ρ (density), σ_{pq} (segmental density), h_{pq} (distance between P' and segment q), and others. The approach necessitates managing relative positions and applying transcendental expressions for accurate calculations.

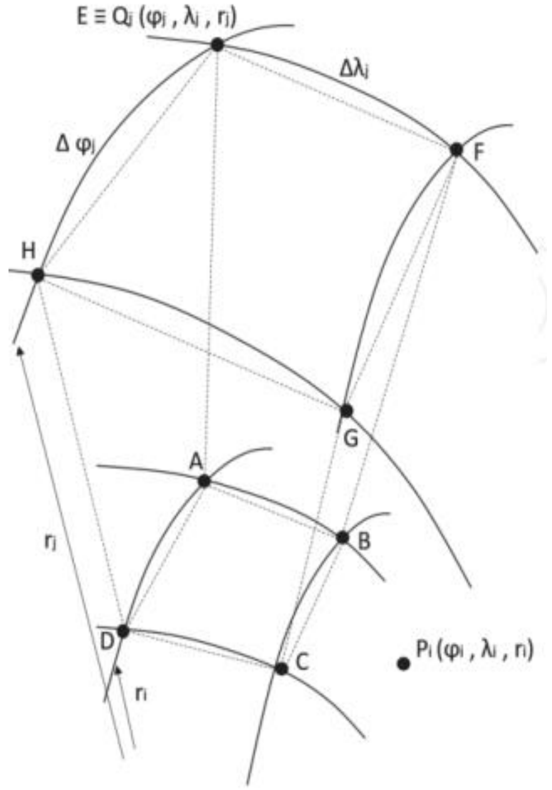

Figure 4 Sketch of constructing the vertex of a polyhedron based on the DMT point Qj and the computational point Pi [18]"

The vertices of a polyhedron, situated at both the terrain height and the level of the computational point P_i, are determined for the calculation of the Bourgeois plate. The computation of first-order derivatives in the gravitational direction corresponds to a terrain correction applied to P_i, involving a contribution from Q_j. This correction is obtained through the "polyhedron.f" code, provided by the author [4], utilizing a linear integral approach. Input parameters include the relative Cartesian coordinates of polyhedron vertices with respect to the calculation point and their topological relationships. These coordinates are transformed by changing the reference system to a local one, where P_i serves as the origin, the z-axis aligns with the direction of r_i, and the x and y axes align with local east and north directions, respectively.

Topological relationships, defining the outward normal direction of the polyhedron's six faces, are established by a matrix indicating the order of vertices in a counterclockwise direction when viewed from the outside. The output is the absolute value of the calculated Vr. The procedure involves two nested loops over all m DMT points Q_j and n computational points P_i. Within the loop over computational points, different local reference systems are defined, causing slight changes in the x and y axes but not the z-axis. This ensures consistency between various Vr values calculated for the same DMT point Q_j with respect to different computational points P_i.

3.4. CONSIDERATIONS OF THE GRAVITATIONAL EFFECT OF THE TOPOGRAPHIC MASSES OF THE DESCRIBED METHODS

Three different models adopted in calculating TC take into account the varied geometry of a terrain element. The Tesseroid formula (9) considers the radial component of the convergence of vertical sides of each terrain element and provides a spherical approximation for the top and bottom of this element. A flat tesseroid has the same geometry in the radial component, but its upper and lower parts have flat surfaces. Therefore, its geometry is, in principle, less accurate than the tesseroid geometry.

The calculation based on the right parallelepiped neglects the radial convergence of vertical edges. This means that this model describes the topography geometry of each element in a way that does not adhere properly to the two main characteristics of the given DTM element. However, since calculating TC effects requires considering only the differences between the calculation point's height and the DTM height, the above differences between the geometries of the elements used in TC calculations should have a limited effect on the calculated values.

4. CONCLUSION

Based on the research of works and reviewed numerical tests conducted using these three different methods for determining the gravitational effect of topographic masses, no significant differences among the methods were discovered. The statistics of values obtained by modeling in different ways the shapes of discretized topographic elements are practically equivalen [18]. The differences between TC effects and Bouguer anomalies calculated with a parallelepiped, spherical tesseroid, and flat tesseroid amount to maximum values of about 1 and 3 mGal, respectively. There are other sources of errors (e.g., density heterogeneities, DTM, and inconsistency in hights of gravity points, that may have an effect on the computation of gravitational effects greater than 3 mGal. However, if we consider the values themselves in another Bouger calculation test, spherical tesseroid and flat tesseroid models perform slightly better when comparing RMS values, i.e., spherical tesseroid and Bouguer anomalies based on the flat tesseroid are smoother.

The concept applied in flat tesseroid modeling can adapt the computation of gravitational effects of topographic masses when shaping the topography according to a model of a triangulated irregular network [17]. This way, a more detailed evaluation of the gravitational effects of topographic masses, especially in the vicinity of calculation points, will be possible by better modeling the terrain slope.

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