RECENZIRANI ČLANKI | PEER-REVIEWED ARTICLES

MODELIRANJE ODZIVA STEBRA MOSTA NA POSKUSNO OBREMENITEV Z UPORABO METODE PODPROSTORA

MODELLING BEHAVIOUR OF BRIDGE PYLON FOR TEST LOAD USING SUBSPACE METHOD

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IZVLEČEK

V delu je prikazan postopek analize in modeliranja vedenja konstrukcije z metodo podprostora, ki se uporablja pri identifikaciji sistema. Metoda v geodetski praksi ni pogosta. Z njo se ocenjujejo parametri dinamičnega modela prostora stanja. Za oblikovanje dinamičnega modela sistema je bil uporabljen model napake popravkov. Prav tako je dokazano, da se dva stebra istega mostu vedeta različno zaradi položaja sonca. Celoten postopek identifikacije sistema je prikazan z modeliranjem vedenja stebra mosta Sloboda na Donavi v Novem Sadu. Merjenja so bila izvedena med poskusno obremenitvijo mostu septembra 2005. Vhodni signali, ki so bili analizirani, so dolgodobni signali: sila v kablih stebra in temperatura. Izhodni signali so premikanje vrha stebra. Zahteva strokovnjaka gradbene stroke je bila, da se odkrijejo vsi premiki, večji od 10 milimetrov. Položaj vrha stebra je bil določen s statično GNSS-metodo, opazovanja pa so trajala 25 minut na epoho.

ABSTRACT

This paper presents the procedure of structure behaviour analysis and modelling applying the subspace method, which has been used in system identification but not in geodetic practice. This method enables the state space dynamic model parameters to be estimated. The equation error model has been used to form the dynamic system model. In addition, two pylons of the same bridge have been demonstrated to behave differently due to the position of the Sun. The entire system identification procedure has been demonstrated in modelling behaviour of the "Sloboda" bridge over the Danube pylon in direction of Novi Sad. The measurements were compiled during the bridge test load conducted in September 2005. The input signals subject to the analysis are the long-periodic signals: namely, the force in pylon cables and temperature. The output signals are pylon top displacements. The civil engineering experts' requirements called for identification of all displacements exceeding 10 mm. The pylon top position was determined by static GPS along with the 25- minute measurements per epoch.

KLJUČNE BESEDE

identifikacija dinamičnega sistema, metoda podprostora, prostor stanja, steber

KEY WORDS

dynamic systems identification, subspace method, state space, pylon

1 INTRODUCTION

Deformation analysis carried out at the beginning of the 1970s dealt with the geometrical interpretation of deformation (model congruence). Some of the methods used are: Hannover (Ambrožič, 2001; Mihailović and Aleksić, 1994), Fredericton (Vrečko and Ambrožič, 2013), Karlsruhe (Ambrožič, 2004; Mihailović and Aleksić, 1994), Delft (Marjetič et al., 2012; Mihailović and Aleksić, 1994).

As of the beginning of the 1990s, geodetic experts began dealing with the physical interpretation of deformation structures and soil. The physical interpretation of the structures' deformation processes established the mathematical relation between the input and output signals. Mathematical relation can be established based on natural laws of the continuum mechanics or by means of system identification. The surveyors have been more familiar with the system identification given that the dynamic process is modelled on the basis of measurements. To successfully model the process applying the system identification, the following questions need to be answered first:

- Is there a cross-correlation between input and output signals (Pelzer, 1978),
- Is there an autocorrelation between deformations for different epochs (autocorrelation within the time series) (Pelzer, 1978),
- Is there a trend or a seasonal effect (Pelzer, 1978),
- Which type and order of the model to adopt based on the answers to the previous questions? The model can be presented in the form of transfer function or the state-space model (Milovanović, 2012),
- Which model validation procedure should be applied? Simulation and prediction of the system behaviour based on the adopted model represent mandatory procedures of model validation (Milovanović, 2012).

The first paper in the field of geodetic deformation analysis where deformations were observed as result of a process interpreting was published by Pelzer (Pelzer, 1978). In mid-1990s the International Federation of Surveyors formed an Ad-Hoc Committee of Working Group 6.1 to define models and terminology for the analysis of geodetic monitoring observations (Modeling and Terminology for the Analysis of Geodetic Monitoring Observations, 2002). Up until that period the surveyors have been working intensively on the dynamic models of the deformation processes. Most commonly used was the state-space model (Kalman filter) (Eichhorn, 2006; Kuhlmann, 2003; Mastelić-Ivić and Kahmen, 2001). In papers published by Chatzi and Smyth (2009), and Gulal (2013), the state-space model based on the natural laws has been used, i.e. the so-called "white-box", where the system structure and model parameters' values are already known.

However, the theoretical presumptions and the structure behavior model defined by the designers are yet to be verified in practice. The verification can be carried out solely on the basis of the measurements performed and system modeling without adopting the presumptions. This paper aims to demonstrate the modeling procedure based only on measurements using the subspace method, which has not been used in geodetic practice to date. Moreover, it shows how the system structure can be defined on the basis of a graphic presentation of the input and output signals. Instead of solving the system of differential equations, the more numerically stable system of integral equations has been used.

The modeling procedure applying the subspace method was illustrated in the example of modeling the behavior of the pylon of the "Sloboda" bridge on the Novi Sad river bank. Measurements were conducted during the bridge test load carried out by the Faculty of Civil Engineering, University of Belgrade, on 17 September 2005. The model structure was defined based on the visual and static analysis. The "grey box" modelling procedure was applied to obtain the state-space model of the pylon behaviour. This modelling procedure implies that model structure is known while the model parameters' values are unknown. The system was modelled by a first order differential equation with constant coefficients. The input signals are forces in the cables anchored at 36 m, 46 m, and 56 m respectively, and temperature. The force was induced to the cables by placing the trucks with known loads. The value of the force occurring in the cable due to the load has been computed applying the finite element method. Temperature was measured only at the bottom of the pylon. The output signals are pylon top coordinates. The pylon top coordinates have been determined in the coordinate system defined by the longitudinal axis of the bridge (Y-axis), while X-axis is vertical to the Y-axis in the Danube flow direction. Displacements along the Y-axis are caused by the force in the cables, while displacements along the X-axis are caused by temperature changes. The analysis of the input signals' effect on the displacement has been carried out in the time domain (autocorrelation and cross-correlation). The estimation of the parameters has been conducted using the least squares method.

2 SYSTEM IDENTIFICATION

The system identification includes the following procedures:

- Designing the experiment – defining the input signals analysed for their effect on the output signal, methods and periods of data collection;
- Data analysis – visual analysis of time series, data detrending and the analysis in time and frequency domains;
- Selecting the model type – transfer function or state-space model;
- Determining the model order;
- Estimating the model parameters;
- Model validation.

The paper addresses the following: the complex signal forms analysed in the experiment, the state-space model error equations, continuous system discretization and the discrete system movements through the state space.

2.1 Complex form of signals

To successfully perform data analysis and draw conclusions on the system order one must be familiar with the complex signal form. This subchapter further elaborates on the complex form of the measured input (temperature and force in the cables) and output signals occurring during the bridge test load.

a) Step function

Step function is most commonly used for the analysis of dynamic systems, given that the system response (output signal) is fully described by this function. In testing of structures against the test load the input signal takes the form of this function. The graph of the step function corresponding to amplitude b with the break at the point $t = 0$ has been shown in Figure 1, with its analytical expression being

$$f(t) = bh(t), \tag{1}$$

where $h(t)$ is unit step (Heaviside) function

$$h(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases}. \tag{2}$$

The complex form of the step function of a continuous system is obtained applying the Laplace transformation:

$$F(s) = \int_0^\infty bh(t)e^{-st}dt = -\frac{b}{s}e^{-st}\Big|_0^\infty = \frac{b}{s}. \tag{3}$$

In the discrete time system $t \geq 0$ there is a set of signals with amplitude b, and thus Z transformation is

$$G(z) = \sum_{t=0}^{\infty} bz^{-t} = \frac{b}{1 - z^{-1}} = \frac{bz}{z - 1}. \tag{4}$$

b) Slope function

The slope function $f(t) = ath(t)$ shown in Figure 2 graph describes linear changes. This function, for example, describes the temperature changes during the day and the water pressure fluctuations in the reservoir. The complex function form in the continuous system is obtained applying the Laplace transformation:

$$F(s) = \Lambda[ath(t)] = \int_0^\infty ate^{-st}dt = \frac{a}{s^2}, \tag{5}$$

whereas the complex form in the discrete system is obtained applying the Z transformation :

$$G(z) = Z[ath(t)] = \frac{aTz}{(z - 1)^2}. \tag{6}$$

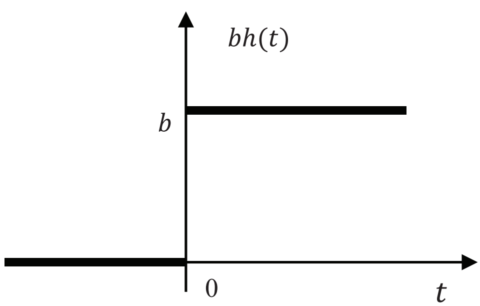

Figure 1: Step function for $t = 0$ and amplitude b (Stojić, 1999).

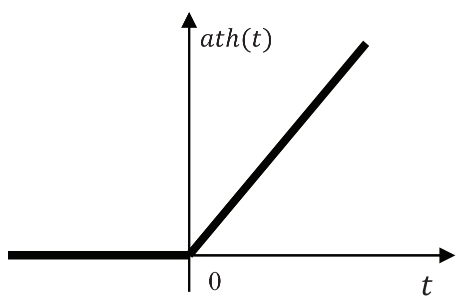

Figure2: Slope function (Stojić, 1999).

2.2 System identification using equation error model structure

Differential equations of the corresponding order are used to describe all processes. The majority of processes are described by the differential equations of first or second order with constant coefficients. The processes occurring in the structures and in the surrounding ground are commonly described by differential equations of first or second order. The procedure of system identification using the equation error has been explained in the example of the dynamic system with a single input and output signal, since such a dynamic system has been applied in the example concerned. The equation error model presupposes that the differential equation order is known. The differential equation order is determined based on the system response.

Generally, the differential equation is of the n-th order (the highest order of the output signal derivative $c(t)$) and includes the r-th derivative of the input signal $u(t)$ on the right side of the equation (7). The order of the input signal derivative r must be $r \leq n$, so that the system state variables could be selected. This paper explores the example of the first order differential equation where the right side of the formula (7) excludes the input signal derivatives. Differential equation used in the example takes the following form:

$$a_1\dot{c}(t) + a_0 c(t) = u(t). \tag{7}$$

Parameters a_1 and a_0 are unknown and they form a vector of unknown parameters $\mathbf{q} = [a_1 \;\; a_0]^{\mathrm{T}}$. The estimated values of the parameters a_1 and a_0 are $\hat{a}_1$ and $\hat{a}_0$. Substituting the true parameters' values with the estimated values in the equation (7) results in the equation error $e(t)$.

$$\hat{a}_1\dot{c}(t) + \hat{a}_0 c(t) - u(t) = e(t). \tag{8}$$

In this case $n = 1$ and $r = 0$. Given that the numerical integration is more stable than numerical differentiation, the equation (7) will be integrated one time because $n = 1$,

$$a_1 c(t) + a_0 \int_{t=0}^{p} c(t)dt = \int_{t=0}^{p} u(t)dt, \tag{9}$$

where p- is the number of measurement epochs. Numerical value of the integrals is obtained applying the Newton–Cotes quadrature rules

$$\int_{x_0}^{x_1} f(x)dx = \frac{h}{2}(|f_0| + |f_1|), \tag{10}$$

where:

$h = \dfrac{1}{p}$ - step,

f_0 and f_1 - value of the function at points x_0 and x_1.

Let us denote that $c_1(t)$ and $u_1(t)$ the integrals of $c(t)$ and $u(t)$. $c(t)$ and $u(t)$ are the values of the measured output and input signals in the discrete moments t. The system of integral equations is then:

$$\begin{bmatrix} c(t_1) & c_1(t_1) \\ c(t_2) & c_1(t_2) \\ \vdots & \\ c(t_2) & c_1(t_2) \end{bmatrix} \mathbf{q} = \begin{bmatrix} u_1(t_1) \\ u_1(t_2) \\ \cdots \\ u_1(t_{20}) \end{bmatrix} + \begin{bmatrix} e_1(t_1) \\ e_1(t_2) \\ \cdots \\ e_1(t_{20}) \end{bmatrix}. \tag{11}$$

Estimation of the unknown parameters $\mathbf{q}$ is obtained applying the least squares method. The system is described by the first order differential equation without having to select the system variable states. Differential equation (7) describes a continuous system. The equation of the stationary continuous system in the matrix form is:

$$\dot{\mathbf{c}}(t) = \mathbf{Ac}(t) + \mathbf{Bu}(t) \tag{12}$$

where:

$\mathbf{A}_{nxn}$ is dynamic coefficient matrix of the system of first order differential equations,

$\mathbf{B}_{nxm}$ is input coupling matrix,

m is a number of input signals.

Measurements are discrete and the continuous system (12) must be discretized. In discrete systems we are interested in the state of the system at the moment of sampling $t = 0, T, 2T, \ldots$. When the equation (12) is integrated in the interval $kT \leq t < (k+1)T$ and following integration t is substituted by $(k+1)T$ the discrete system equation in a matrix form is obtained:

$$\mathbf{c}[(k+1)T] = \mathbf{E}(T)\mathbf{c}(kT) + \mathbf{F}(T)\mathbf{u}(kT), \tag{13}$$

$$\mathbf{E}(T) = \mathbf{\Phi}[(k+1)T - kT] = \mathbf{\Phi}(T) = e^{\mathbf{A}T}, \tag{14}$$

$$\mathbf{F}(T) = \int_{kT}^{(k+1)T} \mathbf{\Phi}[(k+1)T - \tau]\,\mathbf{B}d\tau = (e^{\mathbf{A}T} - \mathbf{I})\mathbf{A}^{-1}\mathbf{B}, \tag{15}$$

where:

$\mathbf{E}(T)$ - are matrix coefficients of the discrete model system parameters,

$\mathbf{F}(T)$ - are matrix coefficients of the discrete model input signals,

$\mathbf{\Phi}(T)$ - is fundamental matrix of system,

T - is the sampling period.

2.3 Subspace method and validation criteria

The state-space model consists of two systems of equations: state-space and system output. The state-space equation is similar to the equation (13) in the matrix form. The system output equation establishes the link between the state-space parameters and outputs.

The state-space model is shown in the following system of matrix equations:

— the matrix equation of state of the discrete system

$$\mathbf{x}[(k+1)T] = \mathbf{Ex}(kT) + \mathbf{Fu}(kT) + \mathbf{w}(kT), \tag{16}$$

where:

$\mathbf{x}(kT)$ - is vector of system parameters at the moment kT,

$\mathbf{w}(kT)$ - is vector of white noise at the moment kT (presumably not correlated)

— the matrix equation of the system output is

$$\mathbf{y}(kT) = \mathbf{Dx}(kT) + \mathbf{v}(kT), \tag{17}$$

where:

$\mathbf{D}$ - is matrix of observation equations,

$\mathbf{v}(kT)$ - is vector of sensor errors (presumably not correlated).

The basic assumptions for the subspace method identification techniques are that:

— the input signal vector is known - the time series $\mathbf{v}(kT)$, $k = 1, \ldots, N$,
— the output signal vector is known – the time series $\mathbf{y}(kT)$, $k = 1, \ldots, N$,
— the system parameters' vector is known (estimated) $\hat{\mathbf{x}}(kT)$, $k = 1, \ldots, N$.

If these assumptions are met a linear regression state-space model can be formed:

$$\mathbf{Y}(kT) = \begin{bmatrix} \mathbf{x}[(k+1)T] \\ \mathbf{y}(kT) \end{bmatrix}, \boldsymbol{\theta} = \begin{bmatrix} \mathbf{E} & \mathbf{F} \\ \mathbf{D} & \mathbf{0} \end{bmatrix}, \boldsymbol{\mu}(kT) = \begin{bmatrix} \hat{\mathbf{x}}(kT) \\ \mathbf{u}(kT) \end{bmatrix}, \mathbf{S}(kT) = \begin{bmatrix} \mathbf{w}(kT) \\ \mathbf{v}(kT) \end{bmatrix}$$
$$\mathbf{Y}(k) = \boldsymbol{\theta}\boldsymbol{\mu}(k) + \mathbf{S}(k). \tag{18}$$

In equation (18) coefficients of a block matrix $\boldsymbol{\theta}$ are unknown. Their estimation is obtained using the least squares method.

There are several validation criteria. The detailed overview of the model validation criteria is further elaborated in the book published by Ljung (1987). Model validation in this paper was undertaken based on the following two criteria:

— $Fit[\%] = \left[1 - \frac{|c-\hat{c}|}{|c-\bar{c}|}\right]$ fitting criteria of the measured output signals using simulation is obtained as per the following formula: where: c - is measured output, $\hat{c}$ - is simulated model output and $\bar{c}$ - is measured output average,
— residual analysis:
    — witnesses – autocorrelation function must be within the confidence interval, residuals are not auto-correlated,
    — independence – residuals are not correlated with the input signal.

3 EXAMPLE

As already implied in the introduction, the first order error differential equation has been applied to model the behaviour of the "Sloboda" bridge pillar in direction of Novi Sad against the test load. In its introductory part, this chapter elaborates on the structure of the bridge, the bridge testing and pylon top geodetic monitoring procedure, and closes with the system identification procedure.

3.1 Bridge construction and examination programs

The "Sloboda" bridge was constructed over the Danube River in the period between 1976 and 1981 to connect Novi Sad and Sremski Karlovci. The longitudinal section of the bridge with stabilization disposition is shown in Figure 3. Structure of the bridge was partially damaged in 1999, however the bridge reconstruction ensued in the period between 2003– 2005, followed by a test examination, providing data used for the pylon behaviour modelling. The bridge structure was examined for static and dynamic loads, according to the legislation effective in the Republic of Serbia in September 2005. The examination program covered measurement of global and local deformations, including dynamic characteristics of the structure.

The test load was effected using 16 heavy load trucks. Each of the loaded trucks weighed approximately 42 tons. The trucks were weighed prior to the testing. Loading the trucks with the known load activated the corresponding forces in the cables (respective values presented in Table 1). Static load was carried out in phases from 1 to 7, where each of the phases included different load distribution and values (distribution of the loaded trucks by phases is shown in Figure 3). Detailed procedure of testing the bridge against the test load is shown in the Bridge Test of the Sloboda Bridge in Novi Sad – Final Report, Part 1- Static Load, 2005, and the work of Milovanović et al. (2011).

Theoretical-numerical analysis

A finite element model with beam elements is used for the analysis of the main bridge subject to the test load. The grider and pylons are discretized with beam elements. The pylon is discretized by a mesh with 6 m beam elements. The geometric data including cross sectional properties is taken from the DSD (Deforming Spatial Domain) design. The cables are modelled with truss elements taking into account sag effect by introducing effective cable modulus. The main cables properties (number of strands and total area, weight per unit of length) are taken from the DSD design. The conducted procedure enabled identification of displacements, bending moments and axial forces in the cables caused by the test loads.

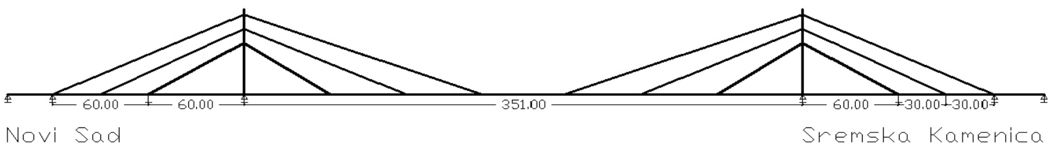

Figure 3: Longitudinal section of the bridge with stabilization disposition.

The pylon is 60 m high, with two cables being affixed at 36 m, 46 m and 56 m heights, one on each side of the pylon. The forces occurring in the cables act along the cable, and therefore have been designed in direction of the longitudinal axis of the bridge. For each pair of cables affixed at the same height the sum of the projected forces has been computed. It has been agreed that the forces in the cables are positive in direction of the river bank, and negative towards the central part of the bridge. The negative value of the sum of the forces, presented in Table 1, denotes that the pylon is leaning towards the centre of the bridge, i.e. inner cables are more stressed.

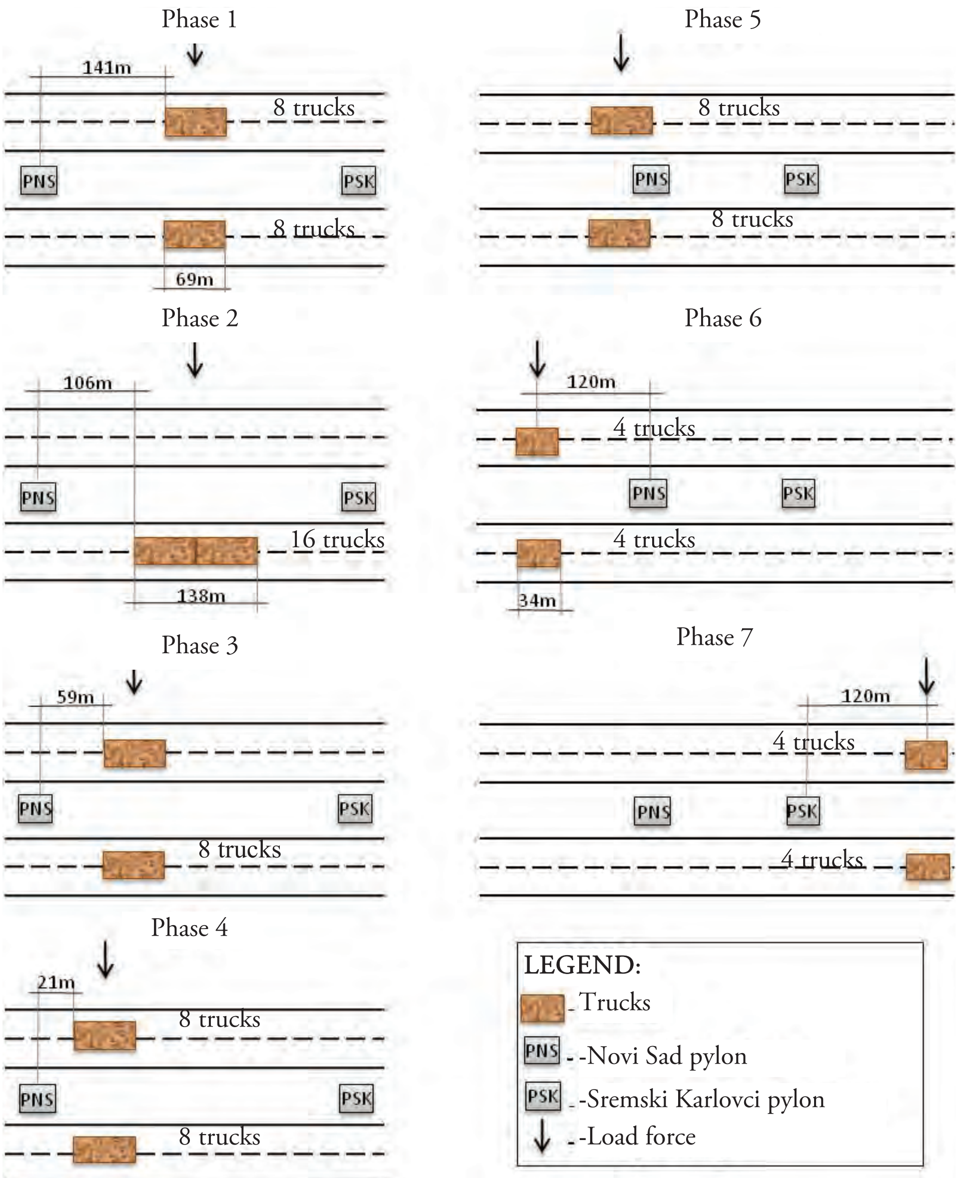

Figure 4: Load distribution as per phases.

In the testing phase 5 the load was evenly distributed in front of the pylon towards Novi Sad, so there was no torsion. The structure deflection in this phase occurs behind the cable anchored at the heights towards the river bank and in front of the pylon, however, it does not result in any significant stress in cables. In phases 6 and 7 the behaviour of the access ramp towards the bridge from direction of Sremska Kamenica was tested. The ramp is an independent segment and thus does not affect the bridge structure behaviour. These epochs have been included in modelling in order to investigate the temperature influence in more detail. It should be hereby noted that the temperature was measured on the driveway next to the pylon.

Table 1: Observations schedule with input parameter values for the Novi Sad pylon.

Epoch number	Time	Force in cables at: 56 m F56 [MN]	46 m F46 [MN]	36 m F36 [MN]	Temperature °C	Load phase
1	6^h00^{min} - 6^h25^{min}	0	0	0	17	Unload
2	6^h25^{min} - 6^h50^{min}	0	0	0	17	Unload
3	6^h50^{min} - 7^h15^{min}	0	0	0	17	Unload
4	7^h25^{min} - 7^h50^{min}	-0.426	-0.414	1.137	18	1
5	7^h50^{min} - 8^h15^{min}	-0.426	-0.414	1.137	18	1
6	8^h15^{min} - 8^h40^{min}	-0.426	-0.414	1.137	19	1
7	8^h40^{min} - 9^h05^{min}	-0.426	-0.414	1.137	20	1
8	9^h15^{min} - 9^h40^{min}	0	0	0	20	Unload
9	9^h40^{min} - 10^h05^{min}	0	0	0	21	Unload
10	10^h20^{min} - 10^h45^{min}	-0.130	-0.647	0.951	22	2
11	10^h45^{min} - 11^h10^{min}	-0.130	-0.647	0.951	23	2
12	11^h15^{min} - 11^h40^{min}	0	0	0	24	Unload
13	11^h40^{min} - 12^h05^{min}	0	0	0	25	Unload
14	12^h05^{min} - 12^h30^{min}	1.318	-1.413	-0.413	27	3
15	12^h40^{min} - 13^h05^{min}	1.263	-0.759	-1.050	28	4
16	13^h15^{min} - 13^h40^{min}	0	0	0	28	5
17	13^h50^{min} - 14^h15^{min}	0	0	0	28	6
18	14^h15^{min} - 14^h40^{min}	0	0	0	28	7
19	14^h40^{min} - 15^h05^{min}	0	0	0	29	Unload
20	15^h05^{min} - 15^h30^{min}	0	0	0	29	Unload

Measurement of global deformation of pylons

The request of the civil engineering experts was to "safely" identify (with test power $1-\beta = 0.80$ and the level of significance $\alpha = 0.05$)all displacements greater than 10 mm. Due to this request the static GPS method, with the coordinate estimated accuracy of few millimetres had to be applied given that the accuracy of RTK (Real Time Kinematic) method is 15 mm to 35 mm (Nickitopoulou, Protopsalti and Stiros, 2006). The static method applied allowed measuring of the long-period signals only. The input signals are forces in the cables and temperature, and the output signals are the pylon top coordinates. The wind effect has not been taken into consideration since the day when the measurements were taken was windless.

Survey control network consisted of four points: one point at top of each bridge pylon and one point on both river banks of the Danube River, as shown in the network sketch (Figure 5). Examination of the pylon top displacement with test load was carried out using a static GPS method with a sampling rate 30 s. Two Trimble 4600LS receivers were placed on the pylon tops, and one HIPER and LEGACY receiver on the network points (geodetic pillars) on the Danube River banks, towards Novi Sad and towards

Sremski Karlovci. Measurement time and values of input signals referring to the pylon closer to Novi Sad are shown in Table 1. The designed 25-minute duration of each measurement epoch allowed obtaining fixed solutions of GPS measurement vectors. This minimum duration of measurement epoch enabled estimating the temperature influence, since the input step signal (load) is identical for the phase concerned.

After vector processing and obtaining fixed solutions, 2D adjustment was performed in a local coordinate system, where Y-axis was defined along the longitudinal axis of the bridge, and X-axis along the Danube flow. A commercial software, Trimble Total Control, was used for vector processing and network adjustment.

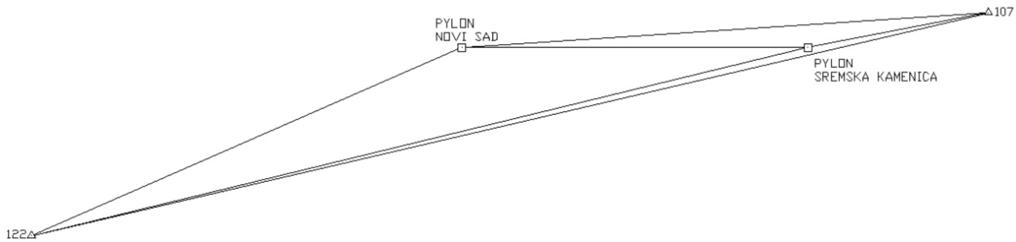

Figure 5: Survey control network sketch.

All measurement epochs (20 in total) were adjusted together. The pylon tops' coordinates were estimated for each epoch separately, while coordinates of geodetic pillars on the river banks were considered unchanged for the entire observation period, given that the induced load did not affect geodetic pillars. This procedure of adjustment produced:

- The number of measurements of measured variables is 101, with 5 vectors for each measurement epoch (1 between pylon tops and 4 between vectors of the river bank geodetic pillars and pylon tops) and 1 vector between the geodetic pillars for the entire observation period (20 x 5 + 1);
- The total number of adjusted points is 42, 20 for each pylon top and 2 for geodetic pillar points (2 x 20 + 2).

Estimation of standard deviation of unknown parameters from adjustment is

- 1.1 mm to 2.2 mm along the Y-axis;
- 1.2 mm to 3.9 mm along the X-axis.

3.2 Analysis of input and output signals for the Novi Sad pylon

Based on visual data analysis it becomes evident that the forces in the cables are step signals and that the system response along the Y-axis is the step response (Figure 6). Temperature is the input signal taking the form of a slope function, and the system response along the X-axis (Figure 7) is also taking the slope function. Time series data has not been detrended, because it is necessary to preserve the influence of the physical effect value. However, the first value in the time series (value of the measurement in the first epoch) was subtracted from all other values of the time series data.

The signal analysis may be carried out in time and frequency domains. The analysis presented herein was conducted in the time domain. The frequency domain analysis would require a greater number of measurement epochs. The time domain analysis implies computing the autocorrelation coefficient for a concrete lag within the time series, and checking whether it falls outside the confidence interval. Between the time series, the cross-correlation coefficient between the input and output signals is computed to determine whether the coefficient falls outside the confidence interval. When the autocorrelation for the specific lag (e.g. k) falls outside the interval, it signifies that the present signal value (at T moment) is influenced by all steps preceding the step k (T-1, T-2,..., T-k). The same principle applies when establishing links between the input and output signals, i.e. it needs to be determined how many preceding input signal values affect the present output signal. The information obtained serves as a basis for drawing conclusions about the model structure.

The signal analysis in the time domain has shown that displacements along the Y-axis depend on the force occurring in cables anchored at the heights of 46 m and 56 m, while displacements along the X-axis caused by the force in the cable anchored at the height of 36 m manifested to a lesser extent with the predominant influence of temperature (Table 2).

The civil engineering theory considers the pylon as an elastic console. Displacements caused by the effect of forces operating at certain distance from the anchorage point, affect displacements of the loose end of the pylon vertically against the pylon axis. The reasons for the lacking influence of the sum of forces at the height of 36 m may be explained by the opposite direction of the force in the first and second test load phases in comparison to the remaining two forces, given that such effect, as presented in the theory, depends on the distance of its effect from the console anchorage. The dominant influence of temperature on displacements vertically against the bridge axis is entirely clear, taking into account that the pylon is anchored by cables along the bridge axis and that acting forces dominate displacements in that very direction. Temperature influences the behavior of steel structures to a significant extent, and therefore is the only effect to could have manifested along the X-axis.

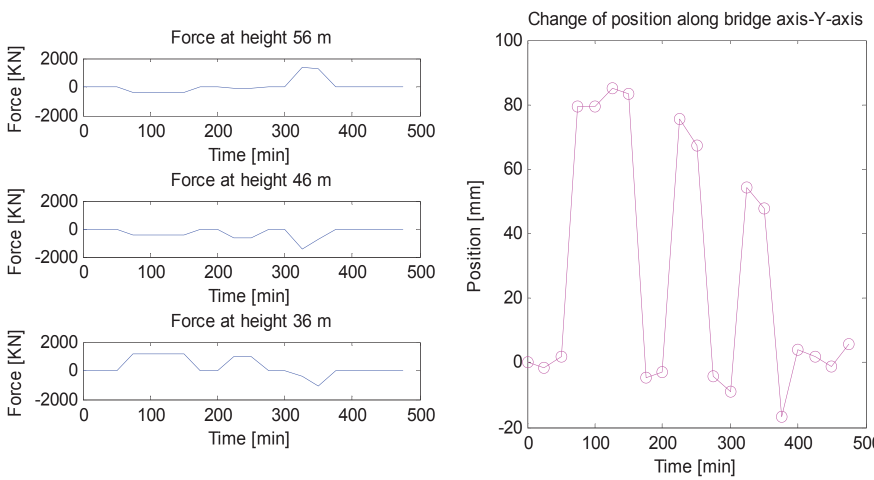

Figure 6: Input force signals and output signal along the Y-axis.

Cross-correlation between the displacements along the X-axis and sum of forces in the cables anchored at 36 m of height is for step 1 and significance level $\alpha = 0.05$ outside the confidence interval, while for $\alpha = 0.01$ it falls within the confidence interval. The significance of this influence is questionable.

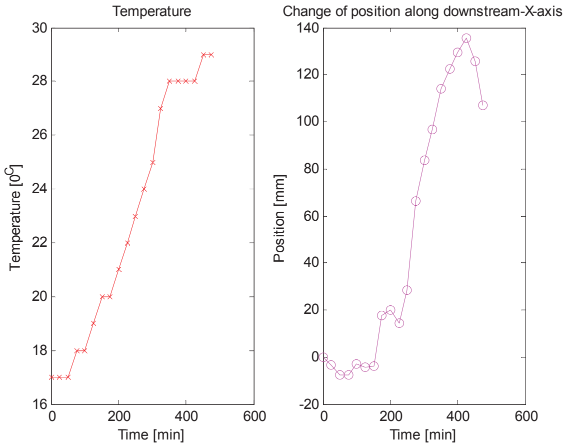

Figure 7: Input temperature signal and output signal along the X-axis.

Table 2: Autocorrelation and cross-correlation of signals.

Correlation	Lag
Auto Y coordinate	*None*
Auto X coordinate	*1*
Cross Y and X	*None*
Cross Y and F56	*4*
Cross Y and F46	*2*
Cross Y and F36	*None*
Cross Y and tem	*None*
Cross X and F56	*1*
Cross X and F46	*None*
Cross X and F36	*1*
Cross X and tem	3

The simplest approximation in civil engineering is to observe the structure as a rigid object influenced by the sum of forces at its centre. Moreover, the equation error structure implies that there is only one input (testing) signal. For this reason it was taken that the displacement along the Y-axis was affected only by the sum of forces occurring in the cables anchored at the heights of 46 m and 56 m, and the displacements along the X-axis was affected only by temperature. Both output signals have been described by the first order differential equation (8). It was demonstrated that in the 14th measurement epoch (the third load phase) the displacement along the Y-axis was also affected by the third force, the force in the cable anchored at the height of 36 m. Introduction of this change in the input signal produced a much better model.

3.3 Model parameters estimation and model validation for the Novi Sad pylon

Integral equation for displacements along the Y-axis at p moment is:

$$a_{0Y} y(t) + a_{1Y} \int_{t=0}^{p} y(t) dt = \int_{t=0}^{p} [F56(t) + F46(t)] dt,$$

except for the 14th epoch where the right part of the equation is

$$\int_{t=0}^{p=14} [F56(t) + F46(t) + F36(t)]dt.$$

Integral equation for displacements along the X-axis at *p* moment is:

$$a_{0X}x(t) + a_{1X}\int_{t=0}^{p} x(t)dt = \int_{t=0}^{p} temperature(t)dt.$$

The estimation of parameters was carried out by solving the system of equations (11) applying the least square method based on all 20 measurement epochs. Model validation was performed by comparing the measured data against the simulated data obtained from the estimated model based on the fitting criterion and the residual analysis. Simulation was conducted applying the equation (13).

a) *Model displacements along the Y- axis - the input signal is the sum of forces in the cables that are anchored at heights 46 m and 56 m*

Parameters estimation with parameter standard deviation is:

Parameter	Parameter estimation	Parameter standard deviation
a_{0Y}	$-9.16\dfrac{KN}{mm}$	$0.25\dfrac{KN}{mm}$
a_{1Y}	$-0.19\dfrac{KN \times epoch}{mm}$	$0.10\dfrac{KN \times epoch}{mm}$

Parameters' significance test proved both parameters significant. Fitting criterion is 66.06% (Figure 8). Residuals are witnesses and are independent of the input signal (Figure 9).

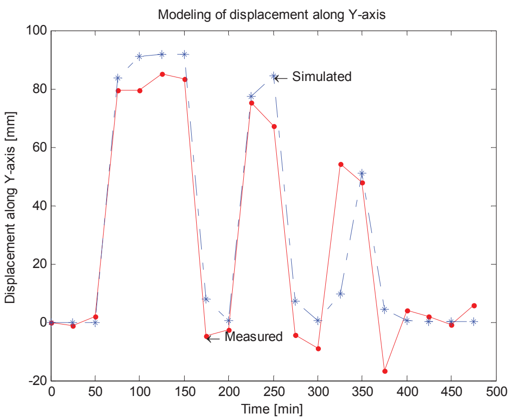

Figure 8: Measured and simulated output signal along the Y-axis (input signals F56 and F46).

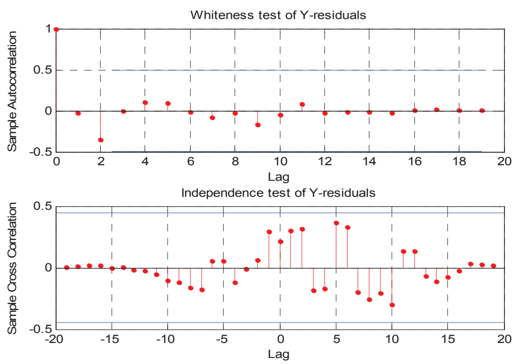

Figure 9: Analysis of residuals.

b) Model displacements along the Y- axis - the input signal is the sum of forces in the cables that are anchored at heights 46 m and 56 m and at 14 measurement epoch (third phase load) the input signal is the sum of forces in the cables that are anchored at heights 46 m and 56 m

Parameters' estimation with parameter standard deviation is:

Parameter	Parameter estimation	Parameter standard deviation
a_{0Y}	$-9.68\,\frac{KN}{mm}$	$0.17\,\frac{KN}{mm}$
a_{1Y}	$-0.13\,\frac{KN \times epoch}{mm}$	$0.07\,\frac{KN \times epoch}{mm}$

Parameters' significance test proved both parameters significant. Fitting criterion is 82.86% (Figure 10). Residuals are witnesses and are independent of the input signal.

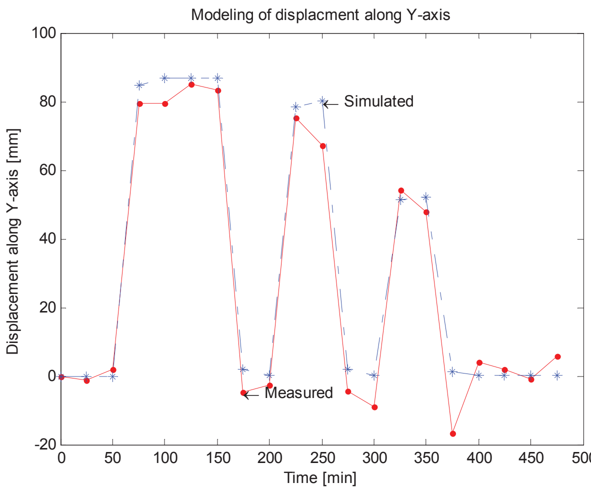

Figure 10: Measured and simulated output signal along the Y-axis (input signals F56 and F46, except 14th measurement epoch).

c) Model displacements along the X axis of the input signal is temperature

Parameters' estimation with parameter standard deviation is:

Parameter	Parameter estimation	Parameter standard deviation
a_{0Y}	$0.077\,\frac{°C}{mm}$	$0.0080\,\frac{°C}{mm}$
a_{1Y}	$0.014\,\frac{°C \times epoch}{mm}$	$0.0021\,\frac{°C \times epoch}{mm}$

Parameters significance test proved both parameters significant. Fitting criterion is 70.69% (Figure 11). Residuals are witnesses and are independent of the input signal.

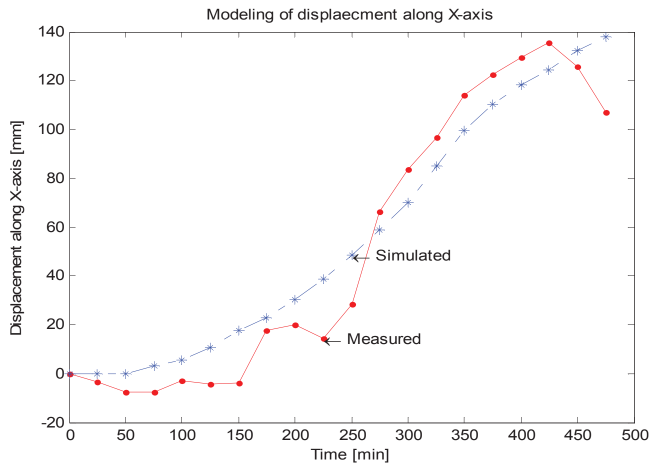

Figure 11: Measured and simulated output signal along the X-axis.

4 DISCUSSION

This paper explores application of the pylon behaviour modelling procedure by system identification using the grey box model. This model assumes that the model structure is known, while the model parameters' values are unknown. Signals affecting the present value of the output signal were determined by means of a time domain analysis and the parameters' significance test. Civil engineering experts are known to describe the console displacements, provided that only load forces are in effect, using a second order differential equation. If besides the forces temperature is also taken as an input signal, the console displacement is described by a fourth order differential equation. In this paper, pylon behaviour is described by a differential equation of first order since the pylon top displacement graph, based on a series of measurements, corresponds in its form to the first order system. Therefore, it becomes evident that the static GPS method cannot be used to register changes occurring immediately after the load application. Whether the process is to be described by a differential equation of first or second order is to be determined based on the displacement graph right after the application of load.

Milanović at al. (2011) in their paper consider the pylon as a deformation object. Transfer function for nonlinear process was used to describe the pylon top behaviour. Under the transfer function each significant input is considered individually. In this paper the simplest method of structure approximation, i.e. a rigid object, has been applied. In rigid objects forces operate in their sum at the structure centre. Both approaches to pylon observation resulted in almost identical percentages of the model fitting (transfer function for nonlinear process: Y-axis 83.5% and X-axis 80.5%).

The pylon top displacements caused by the influence of temperature are greater than those affected by the force in cables. This is because the pylon is made of steel and the influence of temperature is dominant in steel structures. Moreover, the cables affixed symmetrically on both sides of the pylon do not allow for greater displacement in direction of the cables.

In this paper only the pylon behaviour at the Novi Sad side has been modelled. The other behaviour of the other pylon towards Sremski Karlovci has not been considered since it behaves differently. The displacement along the Y-axis is affected also by temperature, and the displacements along the X-axis are affected also by the forces in cables (Table 3). Figure 12 shows the forces in cables and displacements along the Y-axis, while Figure 13 illustrates temperature changes and displacements along the X-axis for this pylon.

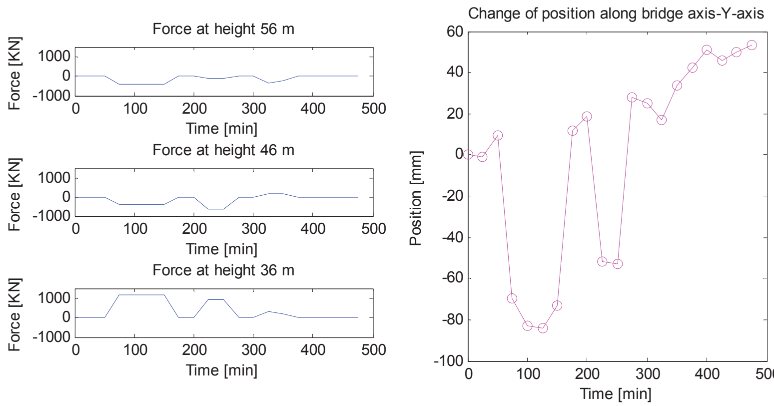

Figure 12: Input force signals and output signal along the Y-axis for the Sremska Kamenica pylon.

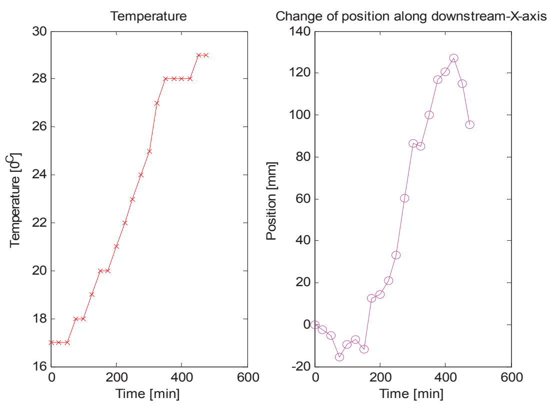

Figure 13: Input temperature signal and output signal along the X-axis for the Sremska Kamenica pylon.

Table 2: Autocorrelation and cross-correlation of signals for the Sremska Kamenica pylon.

Correlation	**Lag**
Auto Y coordinate	*1*
Auto X coordinate	*1*
Cross Y and X	*2*
Cross Y and F56	*1*
Cross Y and F46	*1*
Cross Y and F36	*1*
Cross Y and tem	*2*
Cross X and F56	*None*
Cross X and F46	*1*
Cross X and F36	*1*
Cross X and tem	*3*

Different behaviour of pylon pillars can be explained by the position of the Sun in relation to the bridge. Error equation applied in this paper is used only when a single input signal is concerned, which was not the case with the pylon towards Sremski Karlovci. For system identification the form of the applied error equation must be modified or the input signals modelled first by function, and only then the form of the differential equation solution may be identified, and the differential equation solution parameters estimated applying the system identification. This paper has demonstrated system modelling by the first order differential equation, however, the same modelling procedure needs to be applied regardless of the equation order. Likewise, it has been demonstrated that parts of the same object (pylons) behave differently due to the position of the Sun and that it has to be taken into consideration with each object.

5 CONCLUSIONS

Transfer function describes the linear stationary system and is particularly suitable for describing the system transition period. The structure transition period is the time of ground consolidation. Describing the system by the state space model features the following advantages compared to the transfer function:

- System analysis and designing are simpler,
- Process selection period need not be the same throughout the observed period,
- It can also be applied to certain cases of nonlinear or non-stationary systems,
- The concept of state space is based on the cause-effect relation of the classical mechanics, thus contributing to a simpler physical interpretation,
- In deformation analysis the system is described by coordinates, velocities and accelerations. System velocity and acceleration can be obtained by numerical differentiation.

The authors of this paper have attempted to model the system using Euler and Tustin methods, which imply solving the system of differential equations. Differential equations are unstable in numerical terms. Stable system is obtained by solving integral equations, which have been applied in this paper. Successful structure behaviour modelling requires an in-depth analysis of input and output signals in collaboration with the civil engineering experts. It is necessary to collect all information available to model the process successfully.

Acknowledgment

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