

# FIBER KONAČNI ELEMENT U NELINEARNOJ ANALIZI KVADRATNIH SPREGNUTIH CFT STUBOVA

## FIBER FINITE ELEMENT IN NONLINEAR ANALYSIS OF SQUARE CFT COLUMNS

Nikola BLAGOJEVIĆ  
Svetlana M. KOSTIĆ  
Saša STOŠIĆ

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### 1 UVOD

Čelične cevi ispunjene betonom, to jest CFT (*Concrete Filled Tubes - CFT*) stubovi predstavljaju jedan od tipova spregnutih stubova. Pri opisivanju njihovog ponašanja, neophodno je imati u vidu različite nelinearne uticaje koji ih karakterišu. U ovom radu predložen je numerički model za nelinearnu analizu kvadratnih CFT stubova, baziran na primeni fiber konačnog elementa raspodeljene plastičnosti. Prema dosadašnjim istraživanjima, ovaj konačni element pokazao se kao dosta pouzdan – kako prilikom modeliranja čeličnih i betonskih, tako i u slučaju spregnutih konstrukcija [1], [2]. Ovde su analizirani njegova primena i izbor parametara modela prilikom opisivanja nelinearnog ponašanja uzoraka izloženih delovanju statičkog opterećenja, pri različitoj vitkosti uzoraka, različitom odnosu  $D/t$  (odnos ukupne dimenzije čeličnog profila  $[D]$  i debljini čeličnog profila  $[t]$ ), kao i uzoraka izloženih različitim tipovima statičkog naprezanja. Tačnost modela proverena je upoređivanjem sa eksperimentalnim podacima dostupnim u literaturi.

### 1 INTRODUCTION

Steel tubes filled with concrete, also known as CFT (*Concrete Filled Tubes - CFT*) columns represent one type of composite columns. To model their behaviour appropriately, it is necessary to take into account different nonlinear effects. In this work, a numerical model based on distributed plasticity fiber element is presented. According to previous studies, fiber element has shown great reliability in modelling steel, concrete and composite structures [1], [2]. The application of the previously defined numerical model and determination the model parameters is presented. The ability to model nonlinear behaviour of samples exposed to static load with different slenderness,  $D/t$  ratio (where  $D$  is the total dimension of a cross section and  $t$  is the thickness of steel tube) and different loading conditions has been analyzed. Accuracy of the model has been verified by comparing numerical results with experiments found in literature.

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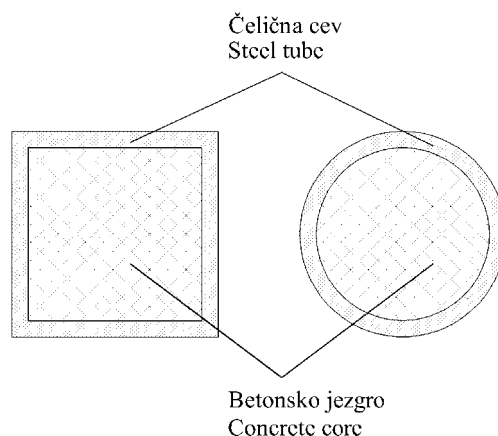
Nikola Blagojević, mast. inž. građ.  
Građevinski fakultet Univerziteta u Beogradu, Bulevar kralja Aleksandra 73, 11000 Beograd,  
e-mail: nblagojevic@grf.bg.ac.rs  
Doc. dr Svetlana Kostić, dipl. inž. građ.  
Građevinski fakultet Univerziteta u Beogradu, Bulevar kralja Aleksandra 73, 11000 Beograd,  
e-mail: svetlana@grf.bg.ac.rs  
Doc. dr Saša Stošić, dipl. inž. građ.  
Građevinski fakultet Univerziteta u Beogradu, Bulevar kralja Aleksandra 73, 11000 Beograd, e-mail: sasa@grf.bg.ac.rs

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Nikola Blagojevic, teaching assistant, MSc.  
Faculty of Civil Engineering, University of Belgrade, Bulevar kralja Aleksandra 73, Belgrade,  
e-mail: nblagojevic@grf.bg.ac.rs  
Svetlana Kostic, assistant professor, PhD.  
Faculty of Civil Engineering, University of Belgrade, Bulevar kralja Aleksandra 73, Belgrade,  
e-mail: svetlana@grf.bg.ac.rs  
Sasa Stosic, assistant professor, PhD.  
Faculty of Civil Engineering, University of Belgrade, Bulevar kralja Aleksandra 73, Belgrade, e-mail: sasa@grf.bg.ac.rs

## 2 OPIS CFT STUBOVA

U novijoj inženjerskoj praksi, CFT stubovi se sve više koriste budući da poseduju brojne prednosti u poređenju s čisto čeličnim i čisto betonskim stubovima [3], [4]. Usled toga što se čelični profil nalazi na spoljašnjoj ivici preseka (Slika 1), CFT stubovi imaju povoljne geometrijske karakteristike, veliku krutost, kao i nosivost na savijanje. Beton – koji može biti armiran ili nearmiran - ispunjava centralni deo preseka, i time povećava nosivost preseka na pritisak i sprečava izbočavanje čelične cevi „ka unutra”. Visoka duktilnost CFT stubova čini ih posebno interesantnim za primenu u seizmički aktivnim područjima [5].



Slika 1. Tipični poprečni presecci CFT stubova  
Figure 1. Typical cross sections of CFT columns

## 3 PONAŠANJE CFT STUBOVA

Dosadašnja eksperimentalna istraživanja [5]–[7] potvrdila su da je ponašanje CFT stubova nelinearno usled brojnih efekata kao što su: pojava pukotina u betonu, utezanje betona, viskozne deformacije betona (tečenje i skupljanje), tečenje čeličnog dela preseka, prisustvo rezidualnih napona u čeliku, izbočavanje čeličnog dela preseka, i tako dalje.

CFT stubovi najčešće su dominantno izloženi naponima pritiska. Pri dilatacijama u čeliku i betonu većim od 1‰, dolazi do pojave pukotina u betonu, povećanja njegove zapremine i sadejstva ova dva materijala [8]. Tada čelični presek deluje poput uzengija, uteže beton i time - zbog triaksijalnog stanja napona koje se javlja u betonu - povećava se čvrstoća betona na pritisak. S obzirom na oblik poprečnog preseka, efekat utezanja betona izraženiji je kod kružnih poprečnih preseka nego kod pravougaonih i kvadratnih [8]. Međutim, eksperimentalna istraživanja pokazala su da u određenim slučajevima ovaj efekat ne dolazi do izražaja. Naime, u slučaju vitkih CFT stubova sa odnosom  $L/D$  većim od 15, dolazi do izvijanja stuba, tj. gubitka stabilnosti, pre nego što dilatacije u betonu dostignu vrednost pri kojoj se pojavljuju pukotine i porast zapremine betona [9]. Stoga, budući da nema sadejstva betona i čelika, nema ni efekta utezanja betona. Takođe, pokazano je da je povećanje čvrstoće betona usled efekta utezanja zanemarljivo i kod ekscentrično opterećenih kružnih i pravougaonih stubova ukoliko je

## 2 DESCRIPTIONS OF CFT COLUMNS

In recent engineering practice, the use of CFT columns is increasing due to numerous advantages relative to steel or reinforced concrete columns [3], [4]. Since steel tube forms the exterior of the cross section (Figure 1), it provides CFT columns with large moment of inertia, leading to high stiffness and flexural capacity. The concrete, that can also be reinforced, fills the core of the cross section increasing compressive strength of the CFT column and also delaying local buckling of the steel tube by forcing all buckling modes outward. The use of CFT columns is of great interests in seismically active areas due to their high ductility[5].

## 3 BEHAVIOUR OF CFT COLUMNS

Previous studies[5]–[7] have confirmed that the behaviour of CFT columns is nonlinear due to numerous effects such as: cracking of the concrete, confining effect in concrete, viscous deformations (creep and shrinkage), yielding of the steel tube, residual stresses, buckling of the steel tube, etc.

CFT columns are predominantly exposed to compressive loading. The confining effects take place at an axial strain of approximately 1‰ when micro cracking in concrete start to occur and lateral expansion rate of concrete increases approaching lateral expansion rate of steel [8]. At that point, the steel tube acts as stirrups and confines concrete resulting in an increase in compressive strength due to triaxial stresses in concrete. Circular steel tubes provide higher degree of confinement than flat sides of rectangular tubes, hence, the increase of compressive strength due to confining effect is more evident in circular than in rectangular cross sections [8]. However, experiments have shown that confining effect is negligible under certain conditions. In slender CFT columns, when  $L/D$  ratio is higher than 15, loss of stability can appear even before axial strain in concrete achieves value when confining effects occur. In these cases increase in ductility and compressive strength of concrete due to confinement effect should be neglected [9]. In addition, when square or circular CFT columns are loaded eccentrically, it is shown that for  $e/D$  ( $e$  being force eccentricity) ratio

odnos  $e/D$  ( $e$  – ekscentricitet sile) veći od 0.125 [7].

U ovom radu, nelinearno ponašanje čelika i betona (sastavnih materijala CFT stuba) uzeto je u obzir primenom odgovarajućih nelinearnih konstitutivnih modela za ove materijale. Efekat utezanja betona buhvaćen je modifikovanjem konstitutivnih relacija za beton. Uticaji koji su zanemareni u prikazanoj numeričkoj analizi jesu: viskozne deformacije betona (tečenje i skupljanje betona), lokalno izbočavanje čeličnog dela preseka i rezidualni naponi u čeličnom delu preseka. Naime, budući da čelična cev obavlja betonski deo preseka, ceo sistem je zatvoren i nema značajnijih promena vlažnosti uzrokovanih uticajima spoljašnje sredine. Stoga, kao što su potvrdila i eksperimentalna istraživanja [10], uticaj skupljanja i tečenja uglavnom je zanemarljiv. Takođe, zbog betona koji ispunjava CFT stub, lokalno izbočavanje čeličnog dela preseka izraženo je samo kod preseka s malom debljinom čeličnog profila. Kao granica za  $D/t$ , kod pravougaonih preseka obično se usvaja vrednost  $\sqrt{3E/f_y}$  [10] i – ukoliko je odnos  $D/t$

manji od pomenute granice - do lokalnog izbočavanja dolazi tek nakon što se čelični deo preseka plastifikuje. Stubovi razmatrani u ovom radu zadovoljavaju datu granicu, pa lokalno izbočavanje nije razmatrano. Uticaj rezidualnih napona je zanemaren i predmet je naknadne studije.

#### 4 NUMERIČKI MODEL

Nasuprot brojnim eksperimentalnim istraživanjima CFT stubova, do sada je razvijeno relativno malo numeričkih modela kojima bi se moglo opisati njihovo ponašanje. Za detaljno modeliranje veza CFT stubova s gredama obično se koriste 3D solid konačni elementi, što je - s obzirom na malu kompjutersku efikasnost ovakvih modela - neprihvatljivo prilikom modeliranja čitavih konstrukcija. Stoga, u modeliranju celih konstrukcija i dalje se prvenstveno koriste linijski konačni elementi.

U ovom radu je za modeliranje kvadratnih CFT stubova korišćen fiber konačni element raspodeljene plastičnosti, formulisan prema metodi sila [2], koji se pokazao kao vrlo efikasan i pouzdan u modeliranju čeličnih i betonskih, kao i spregnutih konstrukcija [1], [2]. Kod ovog elementa, posmatra se određeni broj poprečnih preseka duž ose elementa. Vršni se diskretizacija svakog od ovih poprečnih preseka na određeni broj vlakana (fiber-a). Svakom vlaknu poprečnog preseka dodeljuje se odgovarajuća konstitutivna relacija kojom se opisuje ponašanje materijala koje dato vlakno predstavlja (Slika 2). Integracijom po čitavom poprečnom preseku, dobijaju se sile u preseku (aksijalna sila i momenti savijanja) i matrica krutosti preseka (1), (2).

$$K_S = \sum_{i=1}^N E_{ii} \begin{bmatrix} 1 & -y_i & z_i \\ -y_i & y_i^2 & -y_i \cdot z_i \\ z_i & -y_i \cdot z_i & z_i^2 \end{bmatrix} \cdot A_i \quad (1)$$

higher than 0.125 increase in concrete strength since the confinement effect can be neglected[7].

In this work, nonlinear material models are used to take into account nonlinear behaviour of steel and concrete, parts of a CFT column cross-section. Confinement effect is taken into account using modified constitutive models for concrete. Viscous deformations of concrete, buckling of steel tube and residual stresses in steel are neglected. In CFT columns, the effect of viscous deformations has much smaller influence than in reinforced concrete columns. The steel tube serves as an enclosed environment, so conditions remain ideally humid minimizing the effects of creep and shrinkage [10]. Also, due to concrete core, local buckling of steel tube is postponed. Local buckling should be taken into account in CFT columns with small thickness of the steel tube. In rectangular cross sections, if  $D/t$  ratio is smaller than  $\sqrt{3E/f_y}$  local buckling will not occur before steel yields [10]. All CFT columns considered in this paper have  $D/t$  ratios smaller than  $\sqrt{3E/f_y}$ , and the effect of

local buckling was not included in the numerical model. Residual stresses in steel tube are neglected, and are a subject of the future study.

#### 4 NUMERICAL MODEL

Contrary to numerous experimental research of CFT column, relatively few numerical models that can model their behaviour have been developed so far. For detailed modelling of connections between CFT columns and beam, 3D solid models can be used. However, when modelling whole structures, this model is unacceptable due to its low numerical efficiency. Therefore, when modelling larger structures, frame finite elements are still primarily used.

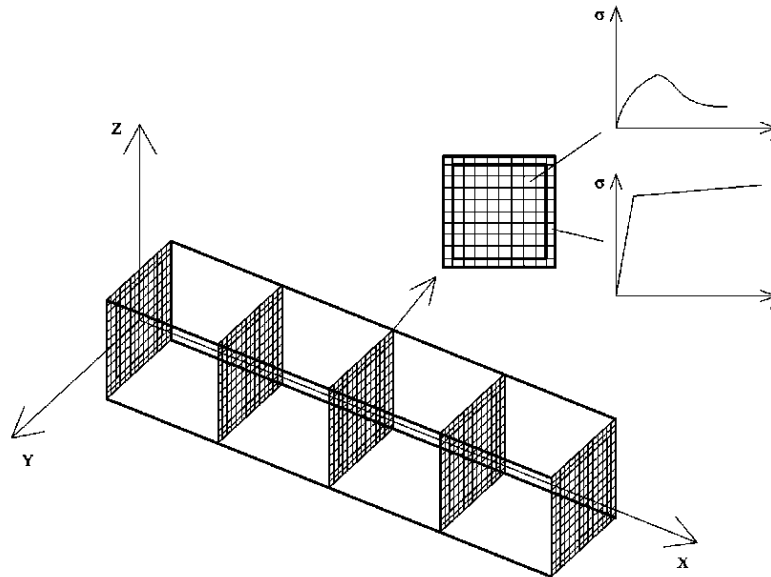
In this work, modelling of square CFT columns is done using distributed plasticity force based fiber beam/column elements [2], that were proven as very efficient and reliable when used for modelling steel, reinforced concrete and composite structures [1], [2].

In this element, a certain number of cross sections are being monitored along the element axis. Also, cross section discretization is done for each of the monitored cross-sections; they are discretized into a number of integration points (fibers). A corresponding uniaxial material model is assigned to each of the integration points (fibers) (Figure 2). The section response (the section stiffness matrix  $K_S$  and the section resisting forces) is obtained through integration over the cross section (1), (2).

$$s = \begin{bmatrix} N \\ M_z \\ M_y \end{bmatrix} = \sum_{i=1}^N \begin{bmatrix} 1 \\ -y_i \\ z_i \end{bmatrix} \cdot \sigma_i \cdot A_i \quad (2)$$

gde je  $K_s$  – matrica krutosti poprečnog preseka, a  $s$  – sile u preseku.

where  $K_s$  - stiffness matrix of the cross section, and  $s$  – section forces.



Slika 2. Konačni element s pet tačaka integracije duž ose  
Figure 2. Finite element with five points of integration along the axis

Fiber konačni element korišćen u ovom radu definisan je metodom sila. Prednost ovako formulisanog fiber elemenata jeste to što je nelinearno ponašanje CFT stuba moguće aproksimirati manjim brojem konačnih elemenata (jedan element ili dva elementa po dužini stuba), s nekoliko tačaka integracije duž ose elementa. Takođe, uticaj raspodeljenog opterećenja lako se može uzeti u obzir [11].

Geometrijske nelinearnosti su uzete u obzir primenom korotacione formulacije [12].

Kod fiber elemenata, naponi se dobijaju iz dilatacija pomoću prethodno definisanih konstitutivnih veza koje su jednoosne. Uticaj višeosnih stanja napona ima se u vidu indirektno, korekcijom relacija za jednoosna stanja napona. Korišćeni su modeli materijala dostupni u programu *OpenSees* [13], u kom je urađena numerička analiza.

Za modeliranje čeličnog dela preseka korišćeni su sledeći modeli materijala: bilinearni model s kinematičkim ojačanjem i *Giuffré-Menegotto-Pinto* model sa izotropnim ojačanjem. Za modeliranje betonskog dela preseka, korišćeni su *Kent-Scott-Park* model, *Popov* model i *Chang-Mander*-ov model sa uzimanjem u obzir čvrstoće betona na zatezanje i bez toga. Na osnovu detaljne parametarske analize [14], zaključeno je da se najbolje poklapanje sa eksperimentima dobija u modelima sa *Giuffré-Menegotto-Pinto* modelom materijala sa izotropnim ojačanjem (Slika 3) za čelični deo preseka i *Chang-Mander* modelom materijala (Slika 4) za betonski deo preseka. Pored toga, parametarska analiza pokazala je i to da je - u slučaju monotonomog

Force based formulated fiber beam/column elements are used in this work. The main advantage of the force formulation is that nonlinear behaviour of CFT column can be approximated by less number of finite elements (one or two elements per length of the column) with few integration points along the axis of the finite element. In addition, distributed element loading can easily be included into the formulation [11].

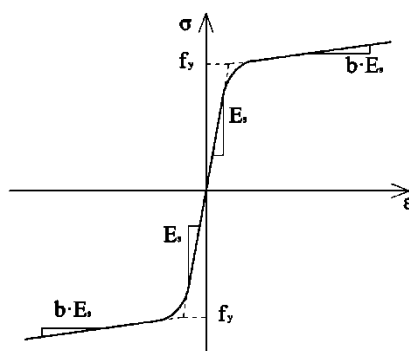
Nonlinear geometry is taken into account using the corotational beam formulation [12].

Stresses in fiber elements are obtained from strains, using previously defined constitutive relations. All stress-strain models are uniaxial, while space stress conditions are approximated indirectly, using the modified uniaxial stress-strain relations. The computer program "*OpenSees*" [13] is used for all numerical simulations and stress-strain material models available in this program are used for numerical models.

Behaviour of steel in the cross section was modelled using: bilinear model with kinematic hardening and *Giuffré-Menegotto-Pinto* model with isotropic hardening. For the concrete part of the cross section *Kent-Scott-Park's* model, *Popov's* model and *Chang-Mander's* model were used. In addition, the results are compared when the tensile strength of concrete is included in the material model and when it is neglected. Detailed parametric study is conducted [14] and it is concluded that *Giuffré-Menegotto-Pinto's* model with isotropic hardening for steel (Figure 3) and *Chang-Mander's* model (Figure 4) for concrete are the most appropriate material models in the numerical modelling.

statičkog opterećenja - uticaj čvrstoće betona na zatezanje zanemarljiv pri određivanju granične nosivosti. Naime, u svim testovima, razlika numerički dobijene granične sile - ukoliko je modelirana čvrstoća betona na zatezanje i numerički dobijene granične sile kada je čvrstoća betona na zatezanje zanemarena - bila je manja od 0.25%. Stoga, u narednoj analizi, prilikom provere numeričkog modela, ova čvrstoća je zanemarena. U slučaju cikličnog ili dinamičkog opterećenja, trebalo bi uraditi dodatne analize. Stav većine istraživača jeste da se ova čvrstoća može zanemariti pri određivanju granične nosivosti CFT stubova, nezavisno od vrste opterećenja [15], [16].

Osnovni parametri *Giuffré-Menegotto-Pinto* modela (Slika 3) jesu granica razvlačenja  $f_y$ , modul elastičnosti  $E_s$  i parametar  $b$  koji predstavlja odnos početnog modula elastičnosti i ojačanja. Zakrivljenje na delu između elastičnog i plastičnog dela krive, kao i histerezisno ponašanje čelika definisano je parametrima  $R_0$ ,  $cR_1$ ,  $cR_2$  čije vrednosti su takođe određene parametarskom analizom.



Slika 3. *Giuffré-Menegotto-Pinto* model čelika  
Figure 3. *Giuffré-Menegotto-Pinto* steel model

Osnovni parametri *Chang-Mander* modela betona (Slika 4) jesu: čvrstoća utegnutog betona na pritisak  $f'_c$ , dilatacija pri maksimalnoj čvrstoći betona na pritisak  $\epsilon'_{c0}$ , modul elastičnosti betona u elastičnoj oblasti  $E_c$ , čvrstoća betona na zatezanje  $f_t$ , dilatacija pri maksimalnoj čvrstoći betona na zatezanje  $\epsilon_{ct}$ , bezdimenzionalni parametar koji definiše krivu modela u oblasti zatezanja  $x_p$ , bezdimenzionalni parametar koji definiše krivu modela u oblasti pritiska  $x_n$ , parametar koji definiše nelinearnu opadajuću krivu modela  $r$ .

Vrednosti navedenih parametara sračunate su na sledeći način. Čvrstoća utegnutog betona na pritisak usvojena je prema preporukama autora eksperimenata [6], [7]. Dilatacija pri maksimalnoj čvrstoći betona sračunata je prema izrazu [17]:

$$\epsilon'_{c0} = \epsilon_{c0} \cdot (1 + 5(C - 1)) \quad (3)$$

gde je  $C$  koeficijent koji zavisi od vrste betona i efekta utezanja. Za eksperimente u kojima postoje podaci o koeficijentu  $C$  usvojena je data vrednost, dok je u testovima u kojima ovi podaci ne postoje, vrednost koeficijenta  $C$  sračunata kao [5]:

Furthermore, it is shown that when samples are exposed to monotonic static load, tensile strength of concrete has little influence on numerical results. In all the samples, difference between ultimate forces, when tensile strength is included in the model and when it is neglected, was smaller than 0.25%. Therefore, in the following evaluation study, tensile strength of concrete is neglected. In the case of cyclic or dynamic loading, this should be further analyzed, although most researchers agree that tensile strength can be neglected when calculating ultimate forces for CFT columns [15], [16] irrespective of loading conditions.

Parameters for defining *Giuffré-Menegotto-Pinto* model (Figure 3) are yield strength  $f_y$ , modulus of elasticity  $E_s$  and strain-hardening ratio  $b$ . Transition curve from elastic to plastic branch and hysteretic behaviour of the model is defined using parameters  $R_0$ ,  $cR_1$ ,  $cR_2$ . Values of these parameters are obtained through a parametric analysis.

Parameters for defining *Chang-Mander's* model (Figure 4) are: confined compressive strength  $f'_c$ , concrete strain at maximum compressive strength  $\epsilon'_{c0}$ , initial elastic modulus  $E_c$ , tensile strength  $f_t$ , tensile strain at maximum tensile strength  $\epsilon_{ct}$ , non-dimensional parameter that defines the strain at which the straight line descent begins in compression  $x_n$ , non-dimensional parameter that defines the strain at which the straight line descent begins in tension  $x_p$  and parameter that controls the nonlinear descending branch of the concrete model  $r$ .

Parameter values are calculated using expression available in the literature. Confined compressive strength is adopted in accordance with the recommendations given in [6], [7]. Concrete strain at maximum compressive strength is given by the following formula [17]:

where coefficient  $C$  depends on confinement effect and type of concrete. In experiments where coefficient  $C$  is given, that value is used, while in experiments where this value is not reported, it is calculated as [5]:

$$C = \left( 1 + \frac{A}{1 + \left( \frac{D/t}{B} \right)^4} \right) \quad (4)$$

$$A = 1.335e^{-\left[ \frac{f_c}{24.4} \right]}$$

$$B = 47.49 + \frac{207}{f_c}$$

Dilatacija  $\varepsilon_{c0}$  određena je prema izrazu [17]

Strain  $\varepsilon_{c0}$  is given by the equation [17]:

$$\varepsilon_{c0} = \frac{(C \cdot f_c)^{0.25}}{1150} \quad (5)$$

Za vrednost modula elastičnosti betona za uzorke u kojima postoji eksperimentalno dobijena vrednost, korišćena je data vrednost, dok je za uzorke za koje ne postoje podaci o eksperimentalno dobijenoj vrednosti – vrednost modula elastičnosti sračunata prema sledećem izrazu [18]:

In experiments where value for the modulus of elasticity of concrete, obtained experimentally, exists, that value is used, while in experiments where this value is not reported, the modulus of elasticity of concrete is calculated from the following expression [18]:

$$E_c = 8200 \cdot (C \cdot f_c)^{0.375} \quad (6)$$

Parametar koji definiše nelinearnu opadajuću krivu modela  $r$  definisan je izrazom [13]:

Parameter that controls the nonlinear descending branch of the concrete model  $r$  is defined [13]:

$$r = \frac{n}{n-1} \quad (7)$$

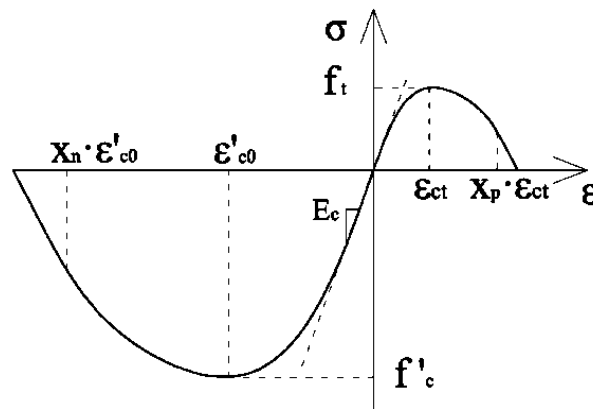
gde je:

Where:

$$n = \frac{E_c \cdot \varepsilon'_{c0}}{C \cdot f_c} \quad (8)$$

Parametri  $x_n$  i  $x_p$  imaju konstatne vrednosti koje za utegnuti beton unutar čelične cevi iznose:  $x_n=30$ ,  $x_p=2$  [13].

Parameters  $x_n$  and  $x_p$  are constant, and for confined concrete are:  $x_n=30$ ,  $x_p=2$  [13].



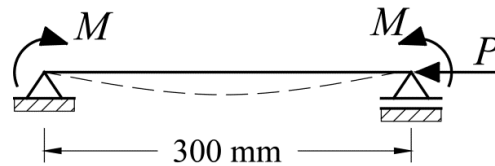
Slika 4. Chang-Mander-ov model betona  
Figure 4. Chang-Mander concrete model

## 5 PROVERA NUMERIČKOG MODELA

Tačnost opisanog numeričkog modela proverena je upoređivanjem numeričkih rezultata sa eksperimentalnim rezultatima dostupnim u literaturi. Eksperimenti korišćeni za proveru modela jesu eksperimenti *Tomii* i *Sakina* [6] i eksperimenti *Bridge*-a [7] na uzorcima izloženim monotono rastućem statičkom opterećenju.

### 5.1 Eksperimenti Tomii i Sakina [6]

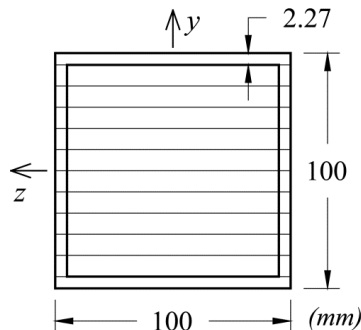
Eksperimenti *Tomii* i *Sakina* [6] vršeni su na prostoj gredi, izloženoj konstantnoj aksijalnoj sili i monotono rastućim momentima na krajevima (Slika 5). Praćene su deformacije grede, tj. obrtanje krajeva grede. Uzorci su kvadratnog poprečnog preseka s različitim debljinama čelične cevi, a geometrijski podaci i vrednost aksijalne sile  $P$  u odnosu na aksijalnu nosivost preseka pri pritisku  $P_0$  dati su u Tabeli 1.



Slika 5. Testovi Tomii i Sakina - dispozicija  
Figure 5. Tomii and Sakino experiment disposition

Tabela 1. Geometrijske karakteristike i karakteristike materijala uzoraka  
Table 1. Dimensions and material properties of samples

Test	Čelična cev (mm) $D \times B \times t$ Steel tube (mm) $D \times B \times t$	$D/t$	$f'_c$ [MPa]	$f_y$ [MPa]	$P$ [kN]	$P/P_0$
II-2	100 x 100 x 2.27	44	25.9	339	93.66	0.18
II-3	100 x 100 x 2.20	44	25.9	339	140.50	0.26
II-5	100 x 100 x 2.22	44	25.9	289	235.15	0.48
II-6	100 x 100 x 2.22	44	25.9	289	281.98	0.57
IV-3	100 x 100 x 4.25	24	22.4	288	188.32	0.29
IV-5	100 x 100 x 4.25	24	23.8	285	318.85	0.48
IV-6	100 x 100 x 4.26	24	23.8	288	381.62	0.57



Slika 6. Diskretizacija poprečnog preseka u testovima Tomii i Sakina  
Figure 6. Cross-section discretization for Tomii and Sakino experiments

## 5 NUMERICAL MODEL EVALUATION

Accuracy of the previously described numerical model is evaluated by comparing numerical with experimental results. *Tomii* and *Sakino* experiments [6] and *Bridge* experiments [7] are used for validating the numerical model. Samples were exposed to non-proportional and proportional monotonically increasing static loading.

### 5.1 Tomii and Sakino experiment [6]

Experiments conducted by *Tomii* and *Sakino* [6] were done on a simply supported beam exposed to constant axial compressive force and monotonically increasing end moments (Figure 5). End rotations of samples were monitored. Samples have square cross-sections and varying steel tube thickness. Details about dimensions of samples, values of axial force and ratio of  $P/P_0$ , where  $P_0$  is axial compressive capacity of the cross-section and can be found in Table 1.

Greda je modelirana s jednim fiber konačnim elementom sa pet tačaka integracije duž ose elementa. Budući da je u ovoj grupi testova poprečni presek izložen savijanju oko jedne ose, vršena je diskretizacija samo u pravcu  $y$  ose i betonsko jezgro je diskretizovano s deset slojeva (vlakana) u pravcu  $y$  ose, dok je čelična cev diskretizovana s deset slojeva po visini betonskog dela preseka i jednim slojem po debljini čelične cevi (Slika 6). Dalje povećanje broja vlakana nije uticalo na rezultate analize. Kao što je već rečeno, geometrijske nelinearnosti uzete su u obzir primenom korotacione formulacije [12].

Parametri *Chang-Mander*-ovog modela betona, sračunati prema izrazima (3-8) uz vrednost  $C=1.2$ , prikazani su u Tabeli 2.

Beam was modelled using one fiber element with 5 integration points along the axis of the element. Considering that samples in this experiment are exposed to uniaxial bending, discretization has been done only in the  $y$  direction. Concrete core is discretized with 10 fibers along the  $y$  axis, while steel tube is discretized with 10 layers through the height of the concrete core and one layer per tube thickness (Figure 6). Further increase in the number of fibres has no influence on the results. Geometrical nonlinearities are taken into account using the corotational formulation [12].

Parameters for *Chang-Mander's* model for concrete shown in Table 2 are calculated using previously defined expressions (3-8), with  $C=1.2$ .

Tabela 2. Parametri konstitutivnog modela betona

Table 2. Concrete model parameters

Test	$\epsilon'_{co}$	$E_c$ [MPa]	$r$
II-2	0.00392	27784.42	1.31
II-3	0.00392	27784.42	1.31
II-5	0.00392	27784.42	1.31
II-6	0.00392	27784.42	1.31
IV-3	0.00378	26312.18	1.29
IV-5	0.00384	26917.22	1.30
IV-6	0.00384	26917.22	1.30

Za parametre modela čelika  $E_y$ ,  $f_y$  i  $b$  korišćeni su eksperimentalno dobijeni rezultati [6], a za  $R_0$ ,  $cR_1$ ,  $cR_2$  vrednosti određene parametarskom analizom (Tabela 3):  $R_0=15$ ,  $cR_1=0.925$  i  $cR_2=0.15$ .

For some of steel model parameters (Table 3) experimentally obtained values are used [6] (for  $E_y$ ,  $f_y$  and  $b$ ), while for other parameters values are obtained through a detailed parametric study:  $R_0=15$ ,  $cR_1=0.925$  and  $cR_2=0.15$ .

Tabela 3. Parametri konstitutivnog modela čelika

Table 3. Steel model parameters

Test	$f_y$ [Mpa]	$E_y$ [MPa]	$b$
II-2	339	217385	0.007
II-3	339	217385	0.007
II-5	289	215745	0.007
II-6	289	215745	0.007
IV-3	288	225553	0.010
IV-5	285	225553	0.010
IV-6	288	215743	0.010

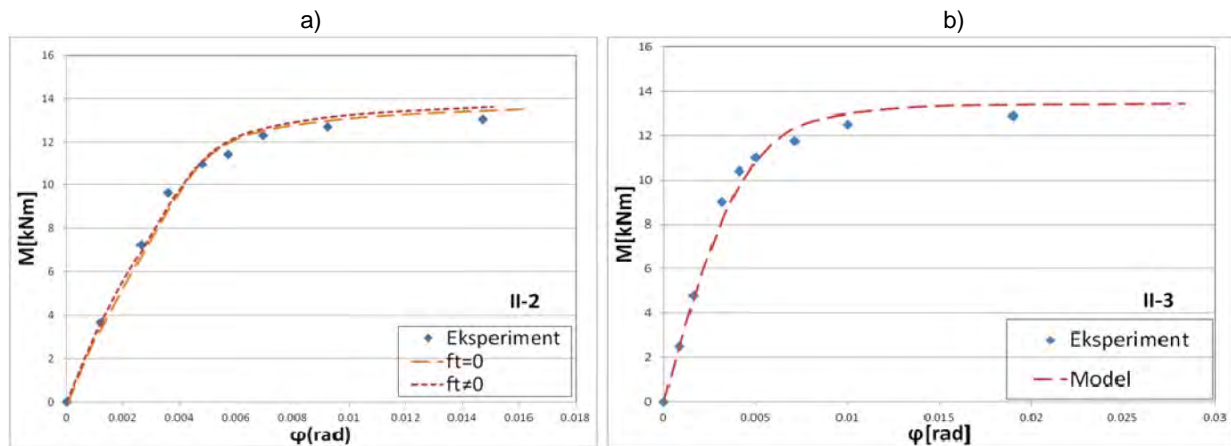
Slike 7-10 prikazuju poređenje numerički i eksperimentalno dobijenih relacija moment ( $M$ ) – rotacija kraja grede ( $\varphi$ ). Samo kao ilustracija prethodno navedenog zaključka u vezi sa zanemarivanjem čvrstoće betona na zatezanje ( $f_t$ ), za uzorak II-2, prikazani su

Figures 7-10 show comparison between end moment ( $M$ ) – end rotation ( $\varphi$ ) relations obtained using previously defined numerical model and experimental results. For sample II-2, two diagrams are shown to illustrate the small effect of the modelling of the tensile strength of

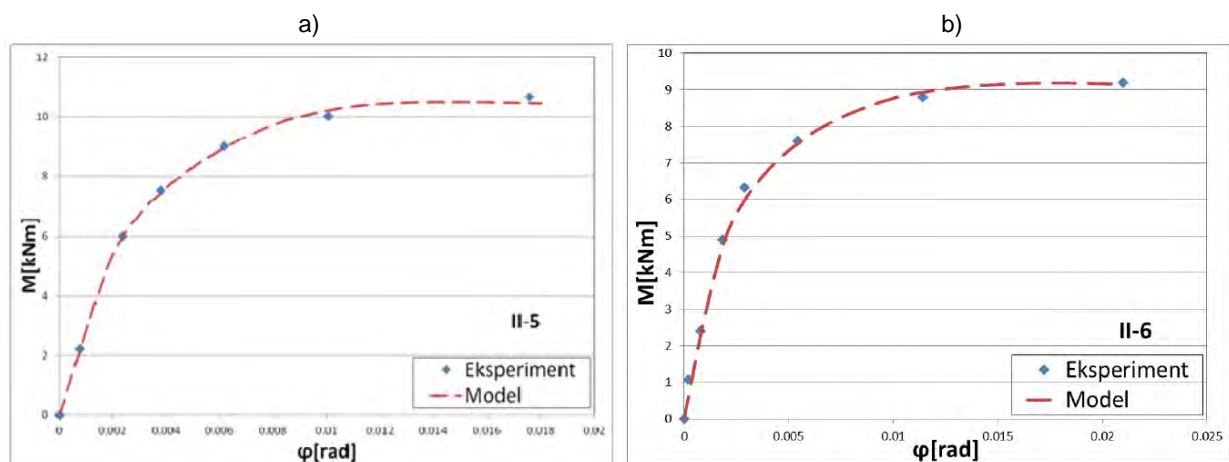


numerički rezultati modela sa uzimanjem u obzir čvrstoće betona na zatezanje i bez toga. Kod ostalih uzoraka, prikazani su samo numerički rezultati dobijeni bez uzimanja u obzir ove čvrstoće ( $f_t=0$ ). Vrednosti graničnih momenata nosivosti uzoraka dati su u Tabeli 4. Sračunata srednja vrednost i standardna devijacija za ovu grupu testova potvrđuju visok stepen tačnosti modela za sve nivoe aksijalne sile.

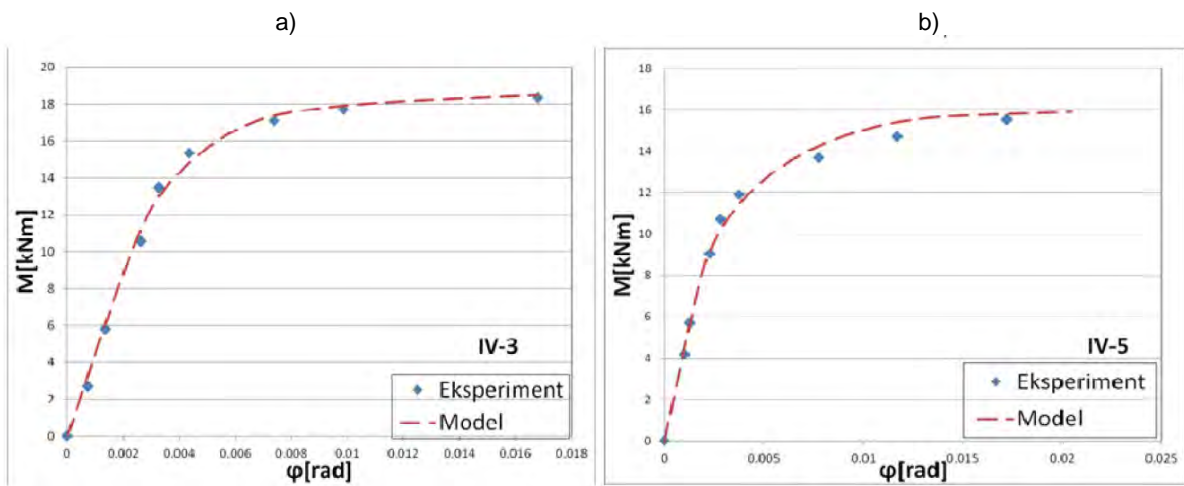
concrete on ultimate capacity. One is when tensile strength is included in the concrete model ( $f_t \neq 0$ ), and the other one is when it is neglected ( $f_t = 0$ ). For other samples, numerical results presented below are obtained not taking into account tensile strength of concrete. Ultimate moment capacities of samples are shown in Table 4. Calculated average and standard deviation for this group of test show high level of accuracy of the numerical model. Also, it is evident that numerical model is very good at describing nonlinear behaviour of CFT columns exposed to constant axial force and monotonically increasing end moments for all  $P/P_0$  ratios.



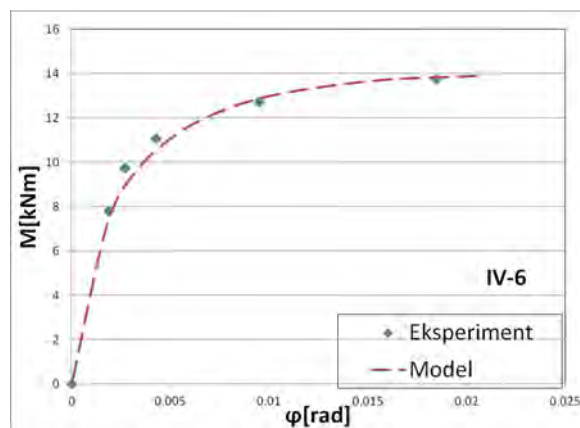
Slika 7. Moment  $M$  – rotacija  $\varphi$  dijagram a) test II-2 i b) test II-3  
 Figure 7. End moment  $M$  - end rotation  $\varphi$  diagram a) test II-2 and b) test II-3



Slika 8. Moment  $M$  – rotacija  $\varphi$  dijagram a) test II-5 i b) test II-6  
 Figure 8. End moment  $M$  - end rotation  $\varphi$  diagram a) test II-5 and b) test II-6



Slika 9. Moment  $M$  – rotacija  $\varphi$  dijagram a) test IV-3 i b) test IV-5  
 Figure 9. End moment  $M$  - end rotation  $\varphi$  diagram a) test IV-3 and b) test IV-5



Slika 10. Moment  $M$  – rotacija  $\varphi$  dijagram, test IV-6  
 Figure 10. End moment  $M$  - end rotation  $\varphi$  diagram test IV-6

Tabela 4. Poređenje numeričkih i eksperimentalnih rezultata  
 Table 4. Numerical and experimental results

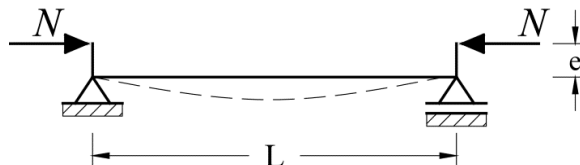
Test	$M_{u,exp}$ [kNm]	$M_{u,num}$ [kNm]	$M_{u,exp}/M_{u,num}$
II-2	13.03	13.3	0.98
II-3	12.9	13.25	0.97
II-5	10.66	10.43	1.02
II-6	9.2	9.15	1.01
IV-3	18.4	18.4	1.00
IV-5	15.51	15.68	0.99
IV-6	13.73	13.8	0.99
Srednja vrednost Average			0.99
St. dev			0.03

## 5.2 Eksperimenti Bridge-a [7]

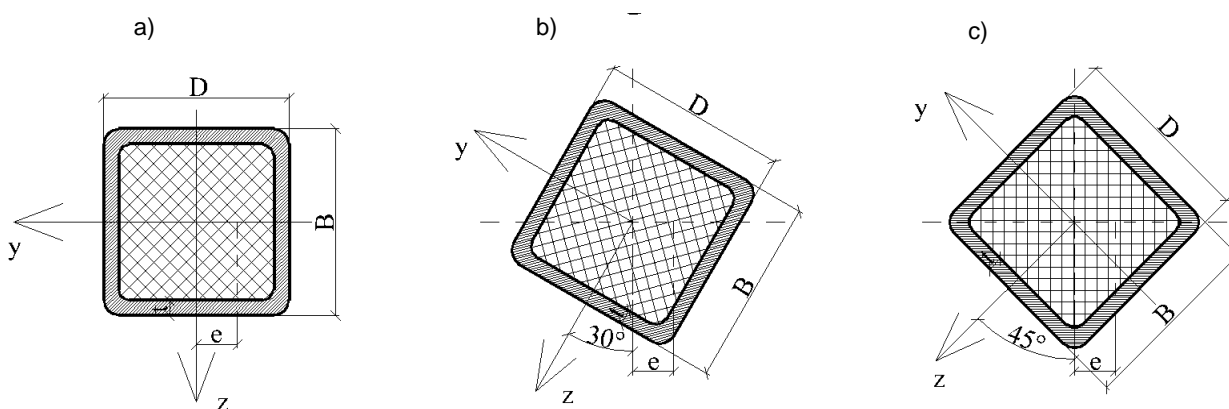
Eksperimenti *Bridge-a* [7] vršeni su na prostoj gredi izloženoj ekscentričnoj monotono rastućoj sili pritiska  $N$  (Slika 11). U ovoj grupi testova, ispitivan je uticaj vitkosti, ekscentriciteta sile  $N$  i kosog savijanja na nelinearno ponašanje CFT stubova.

## 5.2 Bridge experiment[7]

*Bridge* experiment [7] is conducted on a simply supported beam exposed to eccentric axial force (Figure 11). Axial compressive force  $N$  is monotonically increasing, while the mid-span deflection is being monitored. In this group of tests, the effects of loading eccentricity, column slenderness and biaxial bending on nonlinear behaviour of CFT columns are studied.



Slika 11. Testovi Bridge-a – dispozicija  
Figure 11. Bridge experiment disposition



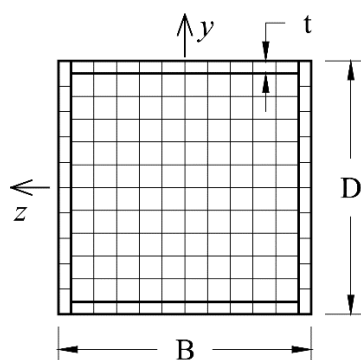
Slika 12. Orijentacija poprečnog preseka uzoraka a) SHC-1, SHC-7, SHC-8; b) SHC-3, SHC-5 i c) SHC-4, SHC-6  
Figure 12. Orientation of cross-section for samples a) SHC-1, SHC-7, SHC-8; b) SHC-3, SHC-5 and c) SHC-4, SHC-6

Tabela 5. Podaci o dispoziciji eksperimenta i dimenzijama uzoraka  
Table 5. Geometrical properties for Bridge experiment

Test	Čelična cev (mm) $D \times B \times t$ Steel tube (mm) $D \times B \times t$	D/t	L [m]	$\alpha$ (°)	e (mm)
SHC-1	203.7 x 203.9 x 9.96	20	2.13	0	38
SHC-3	203.3 x 202.8 x 10.03	20	2.13	30	38
SHC-4	202.8 x 203.4 x 9.88	20	2.13	45	38
SHC-5	202.6 x 203.2 x 10.01	20	3.05	30	38
SHC-6	203.2 x 202.1 x 9.78	20	3.05	45	64
SHC-7	152.5 x 152.3 x 6.48	23.5	3.05	0	38
SHC-8	152.5 x 152.3 x 6.48	23.5	3.05	0	64

Geometrijski podaci o uzorcima su prikazani u Tabeli 5, sa oznakama veličina prikazanim na slici 12.

Geometrical properties of samples are given in Table 5, while Figure 12 depicts the meaning of parameters  $e$  (eccentricity) and  $\alpha$  (angle).



Slika 13. Diskretizacija poprečnog preseka u testovima Bridge-a  
Figure 13. Cross-section discretization for Bridge experiments

Greda je modelirana sa dva konačna elementa s tri tačke integracije duž ose elementa. Budući da je u nekim testovima poprečni presek izložen savijanju oko obe ose, izvršena je diskretizacija poprečnog preseka i u y i u z pravcu sa ukupno 140 tačaka diskretizacije (za betonski deo preseka 100 tačaka integracije, za čelični deo preseka 40 tačaka diskretizacije [Slika 13]).

S obzirom na to što je u svim ispitivanim uzorcima ove grupe testova odnos  $e/D$  bio veći od 0.125, kao što je objašnjeno u delu 3, u numeričkom modelu zanemareno je povećanje čvrstoće betona pri pritisku, tj. usvojeno je da je  $f'_c = f_c$ . Ostali parametri modela materijala sračunati su prema izrazima (3)-(8) i njihove vrednosti date su u Tabeli 6. Parametar  $b$  ima vrednost 0.025.

Beam was modelled with two fiber elements with three integration points along the element axis. Considering that beam is exposed to biaxial bending, the discretization was done along both y and z directions. Total number of cross section integration points (fibers) is 140 (100 fibers for concrete core and 40 fibers for steel tube) (Figure 13).

The increase of compressive strength due to confinement for these samples was neglected, because  $e/D$  ratio was higher than 0.125, as explained in section 3. Remaining material model parameters are calculated using expressions (3-8) and their values are shown in Table 6. Parameter  $b$  has a value of 0.025

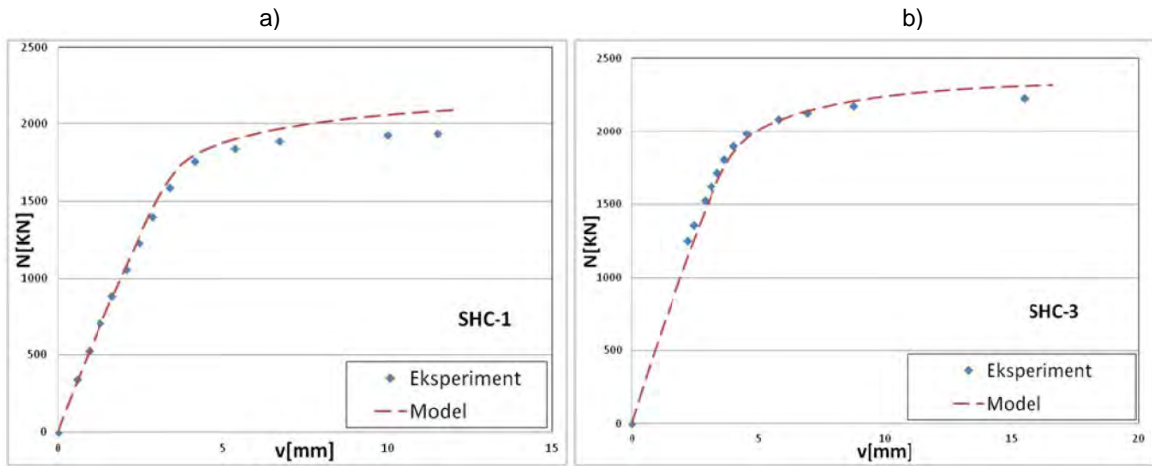
Tabela 6. Parametri konstitutivnih modela materijala  
Table 6. Parameters for models of materials

Test	$f'_c$ [MPa]	$\epsilon'_{c0}$	$E_c$ [MPa]	$r$	$f_y$ [MPa]	$E_y$ [GPa]
SHC-1	30.2	0.00641	23300	1.39	291	205
SHC-3	34.5	0.00585	26410	1.42	313	205
SHC-4	33.1	0.00603	27850	1.36	317	205
SHC-5	37.8	0.00548	28330	1.45	319	205
SHC-6	32.1	0.00615	27090	1.35	317	205
SHC-7	35.0	0.00573	24060	1.50	254	205
SHC-8	35.0	0.00573	24060	1.50	254	205

Slike 14-17 prikazuju poređenje numerički i eksperimentalno dobijenih relacija aksijalna sila ( $N$ ) - vertikalno pomeranje na sredini raspona ( $v$ ). Granične vrednosti aksijalne sile su date u Tabeli 7. Kao i u prethodnoj grupi testova, srednja vrednost i standardna devijacija potvrđuju visoku tačnost predloženog numeričkog modela. Najveće odstupanje numerički i eksperimentalno dobijenih rezultata je kod uzoraka SHC-7 i SHC-8 koji imaju najveću vitkost. Odnos  $L/D$  kod ovih uzoraka jeste 20, odnosno, veći je od 15. Kao što je objašnjeno u uvodnom delu, u ovim slučajevima stabilnost nosača dominantno određuje njegovo ponašanje, što dati numerički model ne uzima u obzir i što će biti predmet naknadne analize. Kod ostalih uzoraka, granična sila određena je s greškom manjom od 7%.

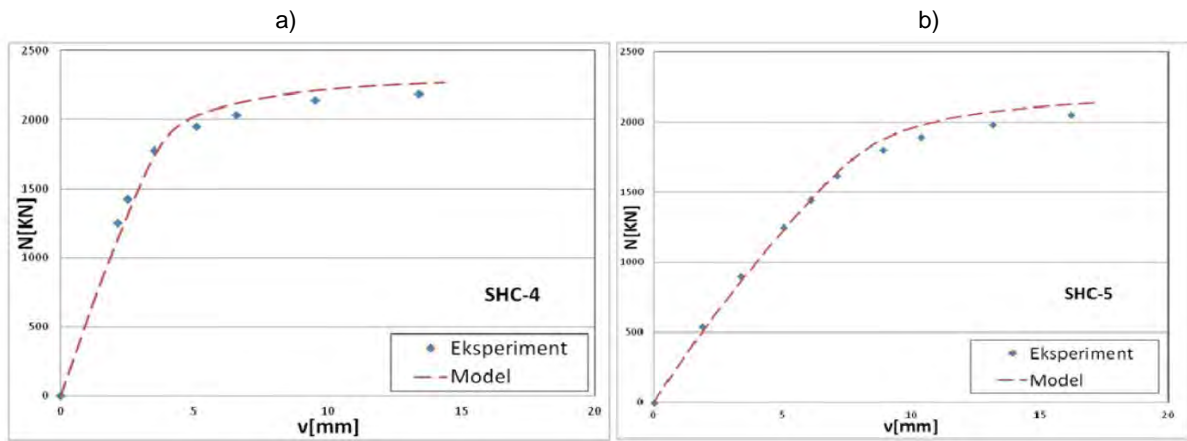
Figures 14-17 show numerically and experimentally obtained relations between axial force ( $N$ ) - vertical mid-section displacement ( $v$ ). Numerically and experimentally obtained values of ultimate axial forces are shown in Table 7. As with previous group of test, average value and standard deviation confirm the ability of the proposed numerical model to approximate well the nonlinear behaviour of CFT columns exposed to eccentric axial force. Larger differences between numerical and experimental results are observed in samples with higher slenderness such as SHC-7 and SHC-8. Ratio  $L/D$  for these samples is 20, i.e. higher than 15. As was explained in introduction, in these cases stability of CFT column dominantly governs the behaviour of the sample. In proposed numerical model, this is not considered and will be a subject of future study. In other samples,

ultimate axial force obtained numerically differs from experimental results by 7% at most.



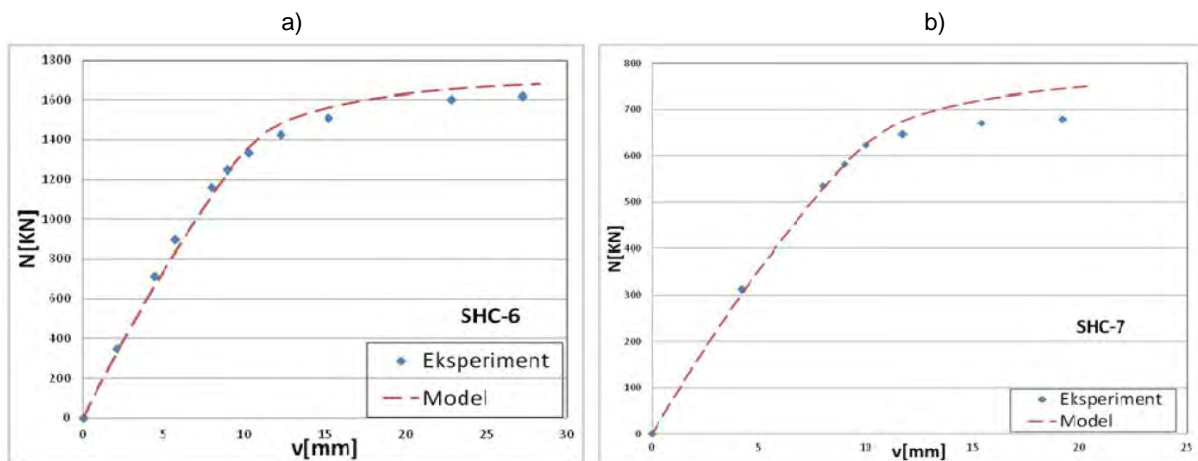
Slika 14. Aksijalna sila  $N$  – pomeranje  $v$  dijagram a) test SHC-1 i b) test SHC-3

Figure 14. Axial force  $N$  - mid-section displacement  $v$  diagram a) test SHC-1 and b) test SHC-3



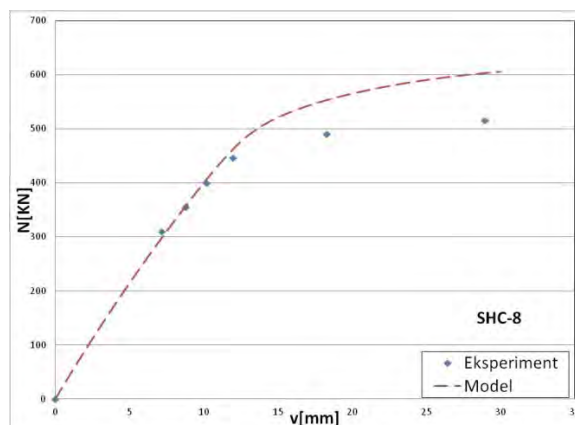
Slika 15. Aksijalna sila  $N$  – pomeranje  $v$  dijagram a) test SHC-4 i b) test SHC-5

Figure 15. Axial force  $N$  - mid-section displacement  $v$  diagram a) test SHC-4 and b) test SHC-5



Slika 16. Aksijalna sila  $N$  – pomeranje  $v$  dijagram a) test SHC-6 i b) test SHC-7

Figure 16. Axial force  $N$  - mid-section displacement  $v$  diagram a) test SHC-6 and b) test SHC-7



Slika 17. Aksijalna sila  $N$  – pomeranje  $v$  dijagram test SHC-8

Figure 17. Axial force  $N$  - mid-section displacement  $v$  diagram test SHC-8

Tabela 7. Poređenje numeričkih i eksperimentalnih rezultata za graničnu vrednost sile  $N$

Table 7. Numerical and experimental results

Test	$N_{u,exp}$ [kN]	$N_{u,num}$ [kN]	$N_{u,exp}/N_{u,num}$
SHC-1	1939.7	2081.2	0.93
SHC-3	2229.8	2314.1	0.96
SHC-4	2188.5	2263.7	0.97
SHC-5	2050.3	2147.3	0.95
SHC-6	1619.9	1671.6	0.97
SHC-7	678.8	744.2	0.91
SHC-8	515.9	603.5	0.85
Srednja vrednost Average			0.94
St. dev.			0.04

## 6 ZAKLJUČAK

U radu je predložen numerički model za nelinearnu analizu kvadratnih CFT stubova, primenom fiber elementa raspodeljene plastičnosti. Modelom su prikazani sledeći uticaji: nelinearno ponašanje betona i čelika primenom odgovarajućih konstitutivnih relacija, efekat utezanja betona i geometrijske nelinearnosti. Model zanemaruje lokalno izbočavanje čeličnog dela preseka, rezidualne napone prisutne u čeličnom delu preseka i viskozne deformacije betona.

Za modeliranje čeličnog i betonskog dela preseka CFT stuba analizirani su različiti konstitutivni modeli materijala. Parametarskom analizom utvrđeno je da se najmanja odstupanja od eksperimentalno dobijenih rezultata dobijaju primenom *Giuffré-Menegotto-Pinto* modela za čelični deo preseka i *Chang-Mander* modela za betonski deo preseka. Zatim, poređenjem sa eksperimentalnim rezultatima testova *Tomii* i *Sakina* i *Bridge-a*, izvršena je evaluacija tako definisanog numeričkog modela. U ovim testovima stubovi su bili izloženi

## 6 CONCLUSION

In this work, a numerical model for nonlinear analysis of square CFT columns using distributed plasticity fiber finite element is proposed. Numerical model presented in this work takes into consideration: nonlinear behaviour of concrete and steel using nonlinear material models, confinement effect and geometrical nonlinearities. Local buckling of steel tube, residual stresses and viscous characteristics of concrete are not considered.

For modelling of steel and concrete parts of the cross section various material models are analysed. Parametric study is conducted and it is concluded that the best agreement between numerical and experimental results is achieved using *Giuffré-Menegotto-Pinto* model for steel and *Chang-Mander* model for concrete. Afterwards, validation of such numerical model is done by comparing numerical and experimental results of CFT columns exposed to proportional and non-proportional monotonically increasing loading. Based on numerical results it can be concluded

statičkom, monotono rastućem opterećenju. Na osnovu dobijenih rezultata, može se zaključiti da predloženi numerički model karakteriše visok stepen tačnosti u nelinearnoj analizi kvadratnih CFT stubova pri statičkom opterećenju.

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## REZIME

### FIBER KONAČNI ELEMENT U NELINEARNOJ ANALIZI KVADRATNIH SPREGNUTIH CFT STUBOVA

Nikola BLAGOJEVIĆ  
Svetlana M. KOSTIĆ  
Saša STOŠIĆ

U radu je prikazana nelinearna analiza kvadratnih CFT stubova, pomoću fiber konačnog elemenata raspodeljene plastičnosti. CFT stubove karakteriše nelinearno ponašanje, što je potrebno imati u vidu u numeričkom modelu. Model predložen u ovom radu uzima u obzir sledeće nelinearne uticaje: nelinearno ponašanje betona i čelika primenom odgovarajućih konstitutivnih relacija, efekat utezanja betona i geometrijske nelinearnosti. Analizirani su uzorci izloženi delovanju statičkog opterećenja koji imaju različitu vitkost, odnos  $D/t$  (odnos ukupne dimenzije čeličnog profila  $[D]$  i debljine čeličnog profila  $[t]$ ), kao i uzorci izloženi različitim tipovima naprezanja. Na osnovu parametarske analize, određeni su modeli materijala za čelik i beton, kao i njihovi parametri koji najbolje aproksimiraju ponašanje CFT stubova. Tačnost modela proverena je zatim i upoređivanjem sa eksperimentalnim podacima dostupnim u literaturi.

**Ključne reči:** fiber konačni element, CFT stub, nelinearna analiza

## SUMMARY

### FIBER FINITE ELEMENT IN NONLINEAR ANALYSIS OF SQUARE CFT COLUMNS

Nikola BLAGOJEVIC  
Svetlana M. KOSTIC  
Sasa STOSIC

The paper presents nonlinear analysis of square CFT columns using distributed plasticity fiber elements. Behaviour of CFT columns is nonlinear and it is necessary to include different nonlinear effects into the numerical model in order to simulate their behaviour properly. Model proposed in this work considers: nonlinear behaviour of concrete and steel using nonlinear stress-strain models, confinement effect and geometrical nonlinearities. Tests exposed to static loading with different slenderness, different  $D/t$  ratio (where  $D$  is the total dimension of a cross section and  $t$  is the thickness of steel tube) and different loading conditions are analyzed. Stress-strain models that best approximate the behaviour of CFT columns are determined from a detailed parametric study. The proposed numerical model is validated by comparing numerical with experimental results available in the literature.

**Key words:** fiber beam/column element, CFT column, nonlinear analysis