Comparison of ARX- and AR- Models and of the Assumed Form of the Transfer Function when Examining Settlement of the Building

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Abstract

Experts in geodesy assess (measure) only the output signals of the dynamic system during the monitoring of buildings. Input signals that act on the structure behaviour are often unknown in practice. The aim of this paper is to show the possibilities to simulate and predict the subsidence of the structure when the input signals are not known (kinematic model). Forms of transfer function models (ARX- AutoRegressive with eXternal input and AR- AutoRegressive model) are used for the modelling of deformation processes. It also reveals a high level of applicability for defining the parameters of the model when the wide class of the processes is described with the assumed form of the transfer functions.
This paper describes the complex form of the input and output signals when examining subsidence of building. Subsidence is a process which is described by a differential equation of the first order. The whole system (structures, input signal and subsidence - output signal) is the first order system. Graphic layout of the first order system response and the form of transfer function for subsidence is shown in the paper. The difference in the percentage of simulations and predictions of these models is shown in a simulated example.

Key words: system identification, modelling, transfer function, subsidence

1 INTRODUCTION

Physical interpretation of the deformation process gives the mathematical relationship between input and output signals. Mathematical connections can be made using the natural laws of continuum mechanics or using the system identification. The application of the natural law leads to hypothetical assumptions and complex models. System identification involves modelling based on measurements.
The system identification is applied to models of shaped transfer function (Milovanovic 2011, Milovanovic 2012, Kaloop, Li 2014) or to models of state space in the case of a time-

invariant linear system (Eichhorn,2006; Kuhlmann, 2003; Mastelić-Ivić, Kahmen, 2001, Chatzi, Smyth 2009).
In most cases the nonlinear processes which occur in nature can successfully be described with this model. However, when the strong nonlinearity occurs then the artificial neural network with the back propagation algorithm which has two layers (one hidden layer and output layer) is generally applied (Heinen 1999, Kaloop, Li 2014).
Unfortunately, during the monitoring of structures the input signals are unknown to surveyors in most cases. Surveyors only measure the output signals - displacement vectors. Also, calculations of experts in civil engineering are based on hypothetical assumptions, which do not have to correspond to reality. The information regarding soil proves to be the most unreliable factor.
This paper analyzes the accuracy of the modelling in the time domain when the input signals are unknown in case of construction subsidence. The simulated data is used in the example. Modelling is done using the ARX- AutoRegressive with eXternal input model (known values of input and output signals at discrete moments), the AR- AutoRegressive model (known only to the value of output - shift) transfer function, and also the model when the form of transfer functions are assumed. Validation and comparison of these models is performed by applying criteria for the case of one step ahead prediction and simulation.

2 METHODS

2.1 THE COMPLEX FORM OF THE EXPONENTIAL FUNCTION

The exponential function $f(t) = Ae^{-at}h(t)$, as shown in Figure 1, is used to model the deformation process due to its own load (subsidence of the building). The complex form in the continuous system is

$$F(s) = \mathcal{L}[Ae^{-at}h(t)] = A\int_0^\infty e^{-(a+s)t}dt = \frac{A}{s+a}, \quad (1)$$

and in the discrete system is

$$G(z) = \mathcal{Z}[Ae^{-at}h(t)] = A\sum_{t=0}^{\infty} e^{-at}z^{-t} = \frac{Az}{z-e^{-aT}}. \quad (2)$$

where: $h(t)$- unit step (Heaviside) function, $\mathcal{L}$- Laplace operator, $\mathcal{Z}$- Z operator.

2.2 THE ASSUMED FORM OF THE TRANSFER FUNCTION FOR THE FIRST ORDER SYSTEM

Subsidence of construction, displacement caused by changes in temperature or by changes in the reservoir level can be described by the first order differential equations.

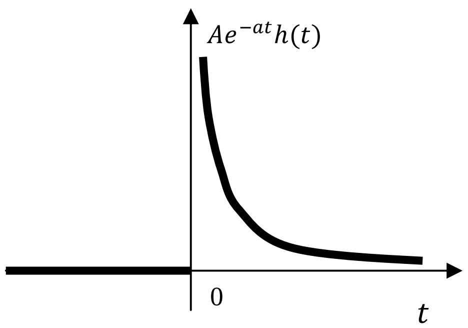

Fig. 1 Exponential function (Stojic, 1999)

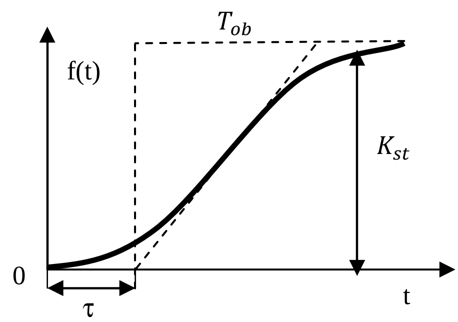

Fig.2 Response of the first order system (Stojic, 1999)

Let the input signal (construction's load on the soil or test load), which is constant, affect on the system currently (in the short term). The input signal is called Heaviside signal.
In Figure 2 it can be seen that the growth of functions is slower in the beginning, and then takes the form of an inverse exponential function, which indicates that it is the first-order system when by the step input signal. The transfer function is

$$F(s) = \frac{K_{st}}{T_{ob}s+1} e^{-\tau s}, \tag{3}$$

The static gain is obtained in the following manner

$$K_{st} = \frac{y_{st}-y_{nov}}{A}, \tag{4}$$

where y_{st} is the old stationary value of the output signal at the moment of excitation, and y_{nov} is the new stationary value of the output signal that is established enough long/some time after excitation with step signal of the size A. The time constant of the object T_{ob} is inversely proportional to the slope of the steepest tangent step response of the object. The system response is a function of $y(t) = \frac{K_{st} e^{-\frac{t}{T_{ob}}}}{T_{ob}} h(t-\tau)$, and the differential equation is

$$T_{ob}\dot{y} + y = K_{st} h(t-\tau). \tag{5}$$

It is well known that geodetic measurements of height difference by terrestrial methods are low-frequency measurements for the long periodic changes. The measurements are not performed continuously, for example they are performed at the moment of the load application, and the load is gradually applied during construction of the building. Because of the foregoing reasons it is not possible to determine the time delay with geodetic methods, so that the transfer functions (3) do not contain $e^{-\tau s}$ (τ - time delay).

3 RESULTS

Analysis of application possibilities of the AR- model and of the assumed form of the transfer function was performed on simulated data in the example, which is taken from Maksimovic, Santrac 2001. Simulation of ground subsidence is done by formulas for the surface subsidence due to consolidation of the clay layers every 30 days during the two years - 26 epochs of measurement. The standard deviation of the measurements of height differences on the station 0.3 mm were adopted. In this paper we describe the modelling of the height difference from the benchmark on the stable ground to the benchmark on the object. Modelling of the entire network is not shown due to the limitation of space in this paper.

SETTING OF THE PROBLEM

The load of $0.1\ \text{MN/m}^2$ was applied over a wide area in a short period of time. Profile of the field is a medium plastic clay of 5.5 m thickness. It is intersected by the horizontal layer of sand of 0,5 m thickness. Below the clay is an incompressible and waterproof surface. The module of compressibility of clay is $5.0\ \text{MN/m}^2$, the coefficient of consolidation is $5*10^{-4}\ \text{cm}^2/\text{s}$.

Figure 3 shows the calculated subsidence of the construction and the input signal - the average degree of consolidation. The values of the input signal are obtained from the geomechanical calculation of the field consolidation.

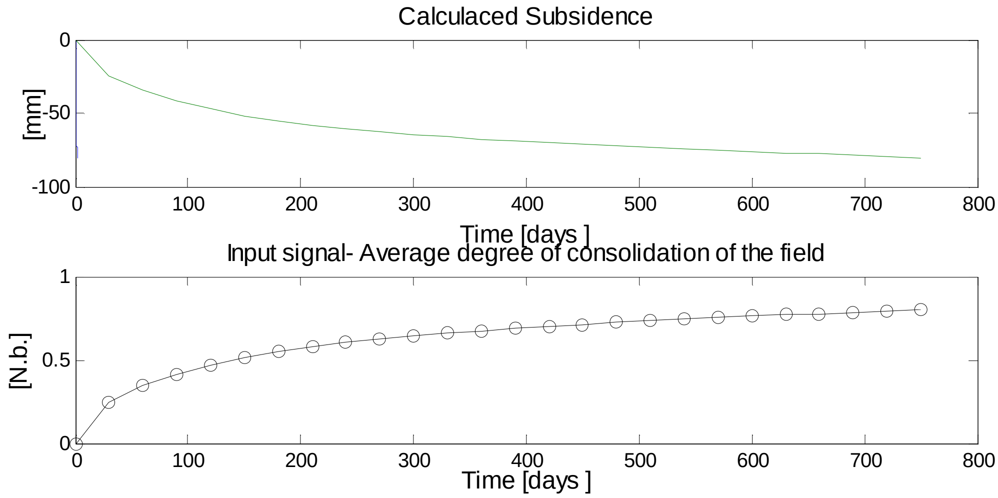

Fig.3 Calculated subsidence and the average degree of consolidation

The measured data are detrending by differences of the first order. This way of detrending did not lead to the stationary of the process. Modelling by stationary data was shown in Milovanovic 2012, and it was shown that there was no significant difference in the model with stationary and no stationary data. Also, the physical effect of the signal is preserved when the difference of the first order is used and the estimated parameters of the model can be physically explained. The structure of the model is determined on the analysis of time series in the time domain and the test of parameters significance. For the assessment of model parameters were used 70% of data, specifically the first 18 epochs of measurement. Parameter estimation was performed using the least squares. The parameters for the assumed form of transfer function (TF) are defined as follows:
The time constant of the object is obtained as the radio of the second and the first first-order difference

$$T_{ob} = \frac{w^1(2)}{w^1(1)},$$

and the static gain is determined from the expression

$$K_{st} = T_{ob}\frac{w^1(2)}{e^{-\frac{2T}{T_{ob}}}},$$

$T = 0.0822$ – sampling, w^1- the first-order difference.

Figure 4 shows a graph of fitting simulated data and predict one step ahead for ARX-, AR- and TF- models. Parameter estimation with a standard deviation of estimation and with percentages of fitting for simulation and prediction are given in the table below.

Model	Form of the model	Estimation of parameters	$\hat{\sigma}$	Simulation Fit [%]	Prediction Fit [%]
ARX	$y(t) = bu(t) + e(t)$	-100.39 mm	4.70 mm	96.8	95.6
AR	$y(t) + ay(t-1) = e(t)$	-0.506	0.209	47.4	92.6
TF	$y(t) = \frac{K_{st}}{T_{ob}} e^{-\frac{t}{T_{ob}}}$	$T_{ob} = 0.426$ [month] $K_{st} = -0.0156[\frac{\text{mm}}{\text{month}}$		87.1	94.7

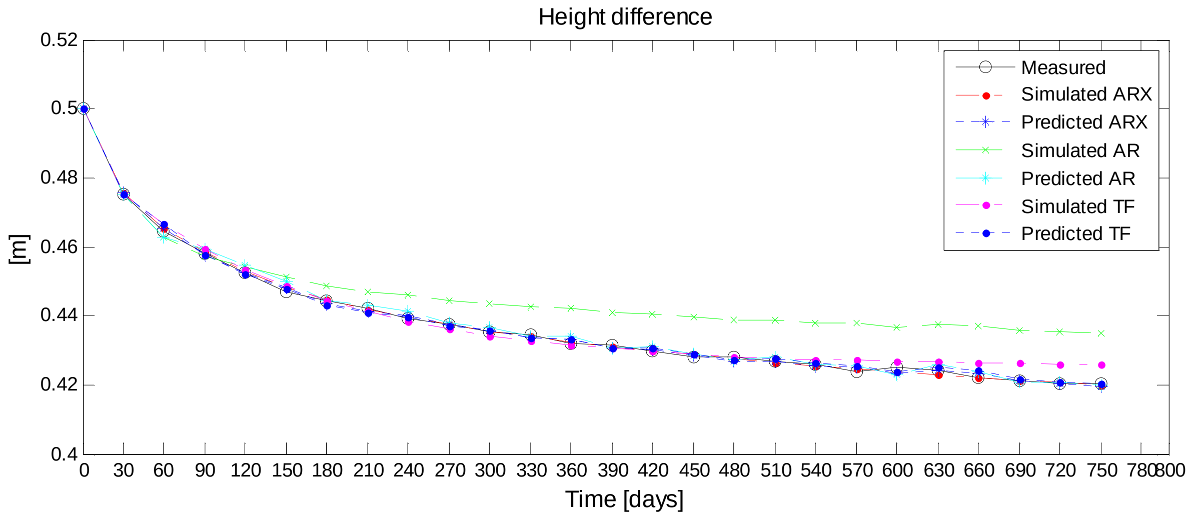

Fig.4 Simulated and predicted measurement

4 CONCLUSION

System identification based on the measurements are applied to predict the behaviour of the structure as a function of the input signal over time and also to check the static calculations of the structure. Reliability of construction calculation is between 70 – 80%. The reliability of the models obtained by system identification, as shown in the example, is high.
In this paper, it is shown that the AR- model is applicable only for prediction . Percentage of fitting for the prediction of one step ahead is very good: 92.6%. The TF- model can be successfully used for the simulation as well as for the prediction of subsidence. The advantage of this model is that the system parameters can be evaluated from the first three epochs of measurement. The verification of the building project, based on the model which is obtained at the beginning of the construction, is carried out early, so that it can be corrected at an early stage of construction if necessary. Timelines of observations must be carefully planned in order to monitor the course construction of the building adequatly and to get a reliable model of the process too. Timelines need to be defined on the basis of sampling theorem. It states that the sampling is $\frac{1}{4}$ of the time constant of the system, which corresponds to the time required when the output signal reaches 63% of the total value.

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