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SIMPLE MODEL FOR PREDICTION OF TRAFFIC INDUCED VIBRATION IN BUILDINGS

SUMMARY

Simple numerical model for prediction of traffic induced vertical vibration in buildings using wall, flexible floor and soil is presented in this paper. The wall is modeled using the axial spectral element, while the floor is modeled by spectral plate element considering only the first mode of vibration. The effect of soil is taken into account using the dynamic stiffness of rigid foundation on viscoelastic half space. The proposed numerical model is verified comparing calculated and measured vibrations for three different buildings. Although the predicted responses closely mimicked the measured ones, the maximum weighted vibration severities for predicted signals are higher than for measured ones. It means that calibration of the model is required.

Keywords: traffic induced vibration, numerical prediction, spectral elements

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ЕДНОСТАВЕН МОДЕЛ ЗА ПРЕДВИДУВАЊЕ НА ВИБРАЦИИ ВО ОБЈЕКТИ ПРЕДИЗВИКАНИ ОД СООБРАЌАЈ

РЕЗИМЕ

Во овој труд презентиран е едноставен нумерички модел за предвидување на вертикални вибрации предизвикани од сообраќај во објекти користејќи ѕид, флексибилен под и почва. Ѕидот е моделиран користејќи линиски спектрални елементи, додека подот со спектрални плочести елементи земајќи ја во предвид само прв мод на вибрации. Ефектот од почвата е земен во предвид преку динамичката крутост на крут темел врз вискоеластичен полупростор. Предложениот нумерички модел е потврден споредувајќо пресметани и мерени вибрации за три различни згради. И покрај тоа што прогнозираните одговори се блиску до измерените, максималната вредност на предвидените сигнали се поголеми од измерените. Тоа значи дека е потребно калибрирање на моделот.

Клучни зборови: вибрации предизвикани од сообраќај, нумеричка прогноза, спектрален елемент

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1. NUMERICAL MODELS

Traffic induced vibrations can cause nuisance among residents, malfunctioning to sensitive equipment and in the extreme cases some minor damages to historical buildings. Hence, the prediction of magnitude of the expected vibrations in buildings is of great interest. Many unknowns and uncertainties make the reliable predictions of traffic-relate vibration in a building practically impossible. The prediction can be reasonable only for low frequency range, (Petronijević and Nefovska-Danilović 2012).

Numerical models for prediction of vibrations in buildings caused by traffic are based on: (1) the numerical simulation of moving vehicle, (2) the transmission of vibrations trough the ground and (3) the analysis of building vibrations due to ground motions. They can be simple and detailed ones. The detailed models are based on complex numerical analysis and include all three above mentioned components of vibrations. Simple numerical models serve to determine whether the prescribed vibration limit is exceeded in the buildings or not.

The objective of the research presented in this paper is to develop a simple prediction model for propagation of traffic-induced vibrations in buildings using wave propagation theory. The previously developed frame model (Nefovska-Danilović et al. 2013), (Petronijević et al. 2013) and simple wall model (Petronijević et al. 2014) are improved taking into account the dynamics stiffness of plates, (Kovačević 2013), (Gudžulić 2014). The model consists of only one wall and one floor for each story. The presented theory is implemented in computer program MATLAB. The model is verified using the results of vibration measurements performed on the three-, eight- and fourteen-story buildings during the passage of a truck crossing the rubber unevenness. In order to check the possibility of prediction model to assess the vibration levels according to DIN4150-2 (DIN4150-2 1999), the weighted vibration severities for all buildings are calculated and compared for predicted and measured velocity signals, as well.

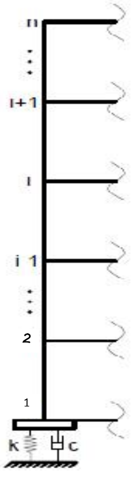

Figure 1. Numerical model

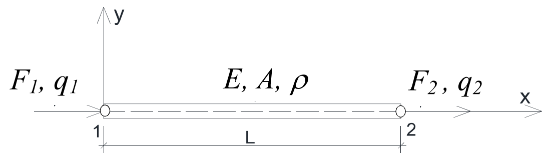

Figure 2. One-dimensional spectral element

1.1. Wall and column modeling

Only one wall in the facade parallel to the road is taken into account, Fig. 1. Wall is modeled as two-node axial spectral elements subjected to longitudinal waves, Fig. 2. The dynamic stiffness matrix **K**(ω) relates the amplitude vectors of nodal forces $\hat{\mathbf{F}} = \{F_1 \;\; F_2\}'$ and nodal displacements $\hat{\mathbf{q}} = \{q_1 \;\; q_2\}'$ at each frequency ω:

$$\hat{\mathbf{F}} = \mathbf{K}(\omega)\,\hat{\mathbf{q}} \tag{1}$$

The dynamic stiffness matrix **K**(ω) for an axial element is well known from a literature, and can be seen in (Kovačević 2013). It is obtained from the equation of motion in the following form:

$$\mathbf{K}(\omega) = \frac{EA}{e^{ikL} - e^{-ikL}} \begin{bmatrix} ik(e^{ikL} + e^{-ikL}) & -2ik \\ -2ik & ik(e^{ikL} + e^{-ikL}) \end{bmatrix}, \tag{2}$$

where $k = \omega / c_p$ is the wave number, $c_p = \sqrt{E / \rho}$ is the velocity of longitudinal waves, E is modulus of elasticity, ρ is mass density, A and L are area of cross section and length of element, respectively. In equation (2) the complex modulus of elasticity $E^* = E(1 + 2\xi i)$ can be used to represent damping within element, where ξ is damping ratio and $i = \sqrt{-1}$.

1.2. Floor modeling

The influence of floor is modeled by the dynamics stiffness coefficient K_f of plate, which represent vertical support force due to harmonic support displacement $\hat{q}_0 e^{i\omega t}$, Fig. 3.

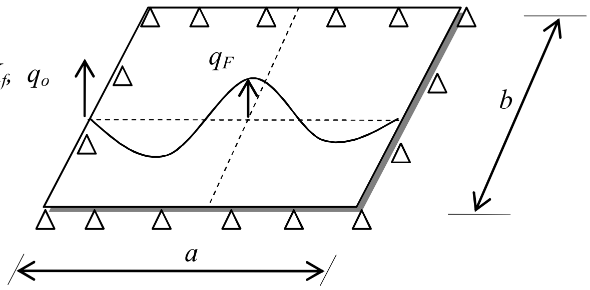

Figure 3. Levy-type plate spectral element

Auersh (Auersh 2010) developed the dynamic stiffness koefficient starting from the dynamic equation of motion due to harmonic support excitation of plate, $\hat{q}_0 e^{i\omega t}$:

$$K\ \hat{q}_r - \rho\omega^2 \hat{q}_r = \rho\omega^2 \hat{q}_o \tag{3}$$

where K is diferential stiffness operator, ρ is mass density, ω is the circular frequency, while $\hat{q}_r(x) = \hat{q}(x) - \hat{q}_o$ is the amplitude of displacement relative to $\hat{q}_o$. The relative displacement $\hat{q}_r$ was presented in the modal form:

$$\hat{q}_R = \sum_{i=1}^{\infty} y_i \Phi_i \tag{4}$$

where Φ_i is eigenvector and y_i is modal participation factor of mode i. The differential equation (3) was multiplied by eigenfunctions Φ_i and integrated to obtain the relation between y_i and $\hat{q}_o$. Taking into account the obtained relation, the support force was calculated by integrating the inertia force (Auersh 2010). Assuming that only first mode of vibration can be taken into account, the simply expression for the dynamic stiffness was obtained (Auersh 2010) in the following form:

$$K_f = (2\pi f)^2 \left(m_F + m' \frac{f^2}{(1 + 2\xi i) f_F^2 - f^2} \right) \tag{5}$$

where f is frequency of vibration, ξ is damping ratio, i is imaginary unit, m_F is floor mass, while m' and f_F are modal mass and first eigenfrequency of square plate, respectively (Kovačević 2013):

$$m' = \frac{64\rho hab}{\pi^4}, \qquad f_F = \frac{\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}}{2\pi} \sqrt{\frac{D}{\rho h}} \tag{6}$$

In Eq. (6) *h*, *a* and *b* are thickness and dimensions of square plate in *x* and *y* directions, respectively, while *D* is flexural stiffness:

$$D = \frac{Eh^3}{12\left(1-\nu^2\right)} \tag{7}$$

Finaly, the amplitude of displacement in the middle of the plate is calculated as:

$$\hat{q}_F = \left(1 + \alpha_{11}\frac{f^2}{\left(1+2i\xi\right)f_F^2 - f^2}\right)\hat{q}_o \tag{8}$$

where α_{11} is equal (Kovačević 2013):

$$\alpha_{11} = \frac{\int \Phi_1 dm}{\int \Phi_1^2 dm} \Phi_1(x,y) = \frac{16}{\pi^2}\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right) \tag{9}$$

1.3. Foundation modeling

The effect of soil-structure interaction (SSI) is taken into account applying the spring and dashpot, which represent the stiffness (*k*) and damping (*c*) of the soil, at the base of the model. The dynamic stiffness of the soil in vertical direction is complex number:

$$K_s = k + i\omega c\ . \tag{10}$$

The foundation parameters, stiffness *k* and damping *c*, for a mat foundation are set according to (Auersh 2010):

$$k = 3.4 G_S\sqrt{A_S} = 3.4\rho_S v_S^2\sqrt{A_S} \qquad c = 1.6\sqrt{G_S\rho_S}A_S = 1.6\rho_S v_s A_S \tag{11}$$

where $v_s = (G_s/\rho_s)^{0.5}$ is shear wave velocity, ρ_s is soil shear modulus and A_s is area of the foundation.

1.4. Dynamic equation in frequency domain

The dynamic equations for the soil-structure interaction analysis in the frequency domain is written as a function of storey displacements $\hat{q}_{r,i}$, *i=1,..,n*, relative to the base displacement $\hat{q}_0$ (free field displacement). The frequency-dependent system of equations, for the model with *n* kinematics DOF presented in Fig. 1, can be written in the matrix form

$$\begin{bmatrix} K_{n,n}+K_f & K_{n,n-1} & & & 0 \\ K_{n-1,n} & 2K_{n-1,n-1}+K_f & K_{n-1,n+1} & & \\ & \ddots & & & \\ & & K_{23} & 2K_{22}+K_f & K_{21} \\ 0 & & & K_{12} & K_{11}+K_f+K_s \end{bmatrix} \begin{Bmatrix} \hat{q}_{r,n} \\ \hat{q}_{r,n-1} \\ \vdots \\ \hat{q}_{r,2} \\ \hat{q}_{r,1} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -K_s\hat{q}_0 \end{Bmatrix} \tag{12}$$

The unknown story displacements $\hat{q}_{r,i}$ *i=1,..,n,* are obtained solving the system of linear algebraic equations (12) with complex coefficients for each frequency $\omega = 2\pi f$. Based on the presented methodology, the computer program in MATLAB is developed.

2. NUMERICAL ANALYSIS AND VERIFICATION OF RESULTS

The proposed numerical model was used to calculate the dynamic response of three typical low-, mid- and high-rise buildings subjected to ground vibrations induced by a 14t truck crossing a 3 cm thick rubber unevenness, at a speed of 50 km/h. The vibrations of the buildings were measured in New Belgrade (Petronijević and Nefovska-Danilović 2006). The measurements were carried out by the

Geophysical Institute, using I/O System One that consists of 5 three-component geophones. The velocities were measured simultaneously at five different points at the buildings site in three orthogonal directions, Fig. 4.

The measured velocities , $\dot{v}(t)$, in the vertical direction at the base of the structures (3) were used as input in the numerical models, while the measured velocities at the top of the structure near the wall (4) and in the middle of the floor (5) were used for validation of the numerical results. For the purpose of the analysis the measured velocity signal $\dot{v}(t)$ were transformed from time to the frequency domain using the Fast Fourier transform (Kovačević 2013):

$$\dot{v}(w) = \int_{-\infty}^{\infty} \dot{v}(t) e^{i\omega i} dt \tag{13}$$

The base displacements $q_o(\omega)$ were obtained in frequency domain dividing the velocities $\dot{v}(\omega)$ by $i\omega$:

$$q_o(\omega) = v(\omega) = \frac{\dot{v}(\omega)}{i\omega} \tag{14}$$

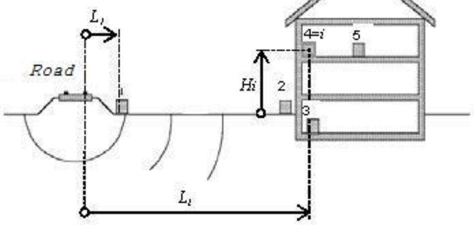

Figure 4. Measurement points

The main structural elements for all building are reinforced concrete (RC) walls. In all cases the same properties of the wall elements were used: $\rho_w = 2500$ kg/m^3, $E_w = 3 \cdot 10^{10}$ N/m^2, $\xi = 0.01$. For low and mid-rise building soil characteristics are $\rho_s = 1800$ kg/m^3, $v_s = 250$ m/s^2 , while for high-rise building are $\rho_s = 1750$ kg/m^3, $v_s = 300$ m/s^2.

Vertical displacements at the top of the wall (point 4 in Fig.4) and in the middle of the top floor (point 5 in Fig.4) are calculated using the proposed model (Gudžulić 2014).

Low-rise building

The low-rise building has ground floor and two floors above. The height of each floor is 3 m. Cross section of the RC wall parallel to the road is 20x0.25m. The wall is founded on a strip foundation 20x1.5m. Dimensions and thickness of the floor slabs are axb=5x5m and h=0.14cm, respectively.

The measured velocity time history of vertical vibration at the base of the building is presented in Fig. 5. Fig. 6 shows the amplitudes of predicted and measured vertical displacements plotted against the frequency. The velocity amplitudes are displayed in logarithmic form.

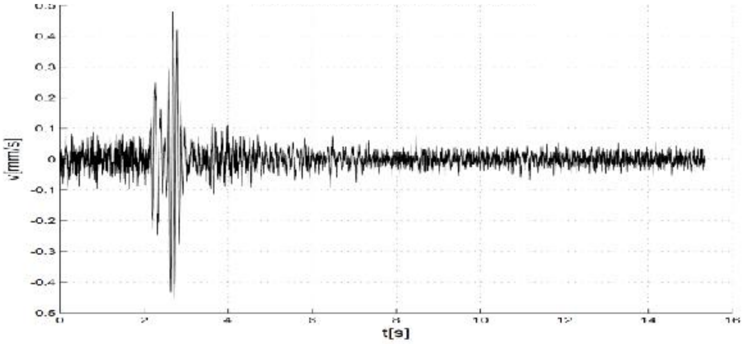

Figure 5. Ground velocity time history for low-rise building

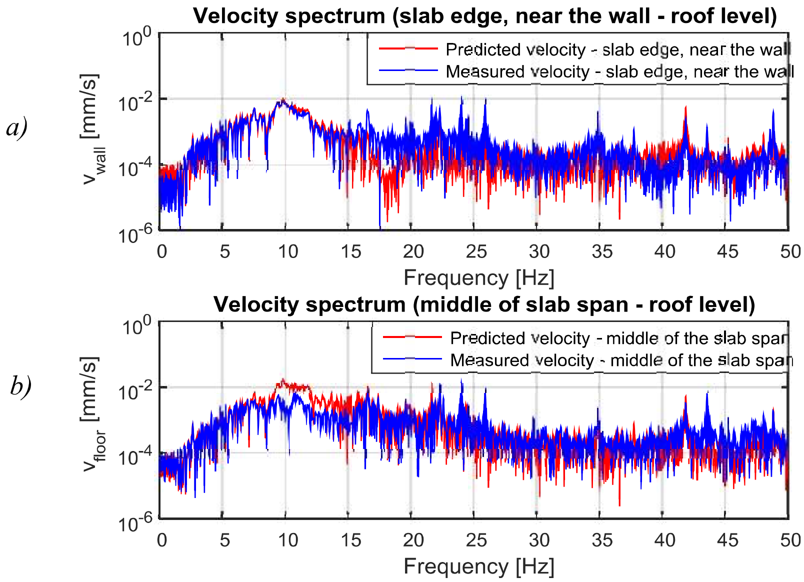

Figure 6. Predicted and measured velocities at the top of low-rise building:
a) near the wall, *b*) floor, (Gudžulić 2014)

The predicted and measured wall velocity at the top of building (point 4) are in good agreement in the low frequency range (0-15 Hz), Fig 6*a*. For higher frequencies, the predicted velocities are lower than the measured ones, generally. For the floor eigenfrequency, f_F=18 Hz, the predicted wall velocity is nearly zero [5], Fig. 6*a*. The measured velocity is nearly zero for a little bit higher frequency, f=16 Hz. This may be attributed to the insufficiently accurate assessment of floor characteristics and structural damping. Concerning the floor vibrations, Fig. 6*b,* the predicted velocities are higher than the measured ones in the frequency range between 9-16 Hz. For other frequencies, the predicted velocities are in good correlation with the measured ones.

Mid-rise building

The mid-rise building consists of a basement, ground floor and six floors above. The height of each floor is 3 m. The cross section of the RC wall is 20x0.25m while the dimensions of the foundation are 20x5m. The floor slabs are 5x5m wide and 0.14cm tick.

The measured ground velocity is presented in Fig. 7. The vertical displacements time history (measured and predicted) at the top of the wall and in the middle of the top floor are presented in Fig. 8 in the logarithmic form.

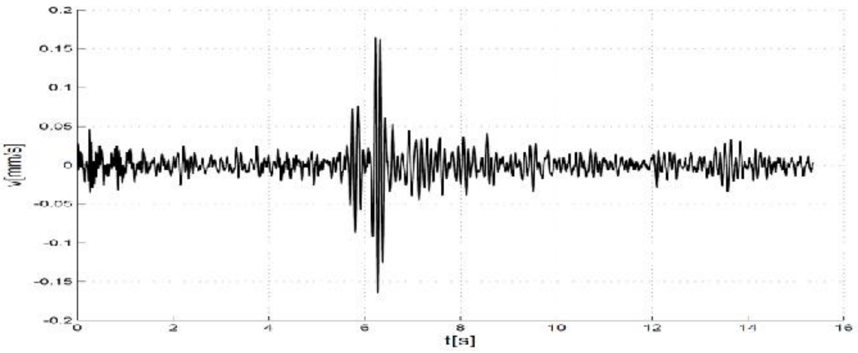

Figure 7. Ground velocity time history for mid-rise building

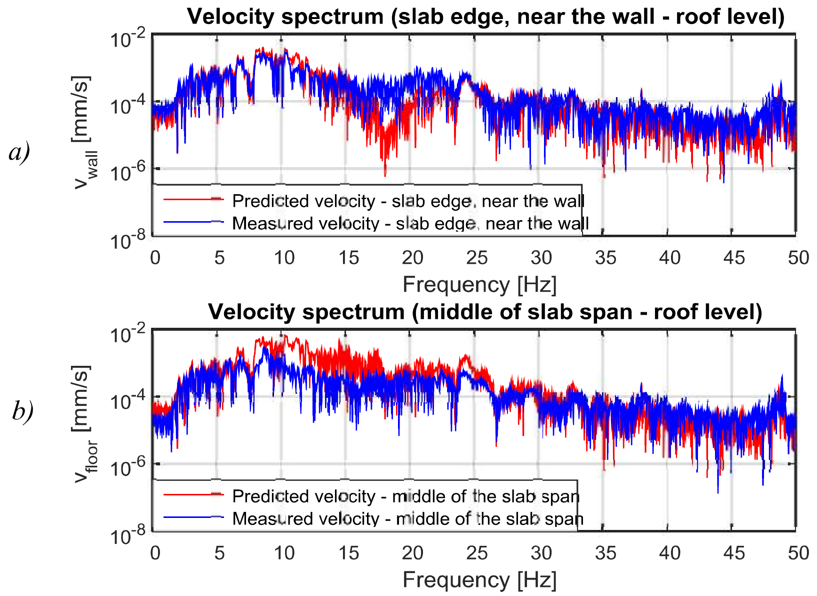

Figure 8. Predicted and measured velocities at the top of mid-rise building:
a) near the wall, *b*) floor (Gudžulić 2014)

Again, the predicted and measured wall velocities at the top of building (point 4) are in good agreement in the low frequency range (0-15 Hz) and the predicted wall velocities recede for the floor eigenfrequency, Fig 8*a*. The measured velocities do not exhibit such behavior. The predicted floor's vibrations are higher than the measured ones almost in the whole frequency range, except for frequencies between 2 and 8 Hz as well as for frequencies higher than 30 Hz, Fig 8*b*. This may be attributed to inaccurate assessment of floor characteristics and structural damping.

High-rise building

The high-rise building consists of a basement, ground floor and 12 floors. The height of each floor is 2.9 m. The reinforced concrete (RC) walls are founded on the reinforced grid foundation. The cross section of the RC wall parallel to the road is 20x0.25m. The dimensions of the foundation grid which belongs to the wall are 20x1m. The floor slabs are 5.5x4.9m wide and 0.12cm tick.

The velocity time history measured at the base of the building is presented in Fig. 9. The measured and predicted displacement time histories at the top floor near the wall and in the middle of the floor are presented in Fig 10. The velocity amplitudes are displayed in logarithmic form.

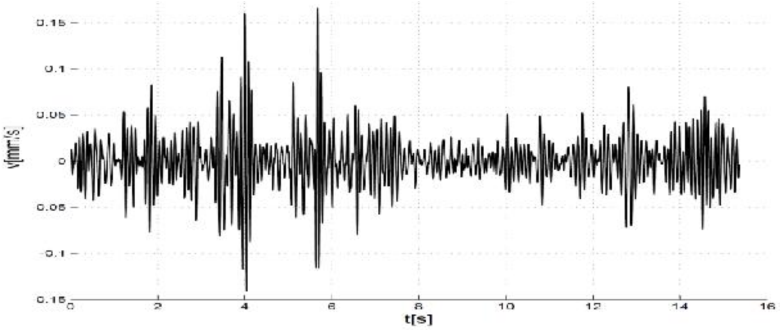

Figure 9. High-rise building - Ground velocity time history

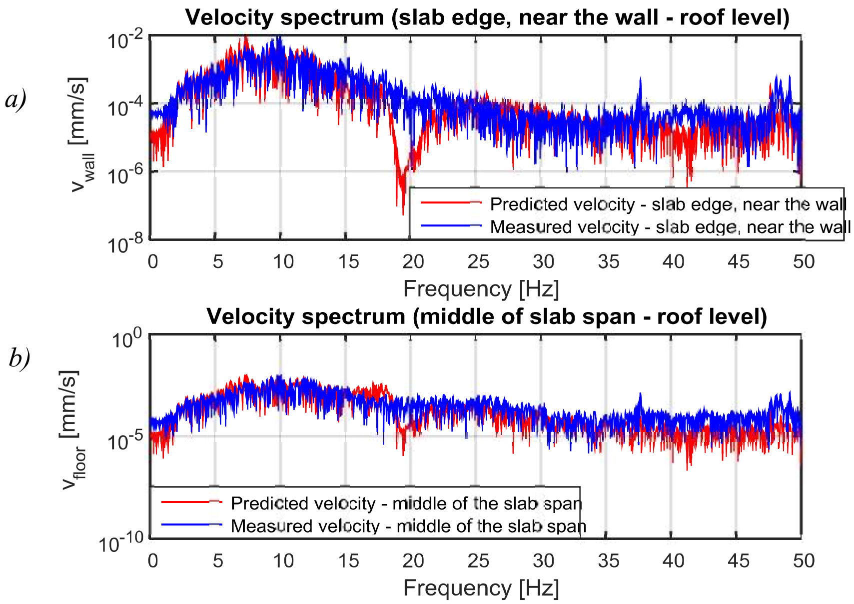

Figure 10. Predicted and measured velocities at the top of high-rise building:
a) near the wall, *b*) floor (Gudžulić 2014)

The predicted and measured wall velocities at the top of the building (point 4) are in good agreement in the entire frequency range, except in the zone of the floor eigenfrequency (19 Hz), Fig 10*a*, where predicted velocities recede. The measured velocities exhibit such behavior for frequency of 20 Hz. The predicted floor's vibrations are in good agreement with the measured values for frequencies between 7 and 16 Hz, and are less than the measured ones for higher frequencies. The predicted floor's vibrations show slight decrement nearby floor eigenfrequency, which might happened due to the highly damped wall amplitudes.

3. ASSESMENT OF TRAFFIC INDUCED VIBRATION ON HUMANS IN BUILDINGS

The effect of vibrations on humans in buildings is accessed according to DIN 4150-2 (DIN 4150-2 1999), by using maximum weighted vibration severity KB_{Fmax} and Nefovska-Danilović 2012). For determining the weighted vibration severity $KB_F(t)$, the velocity signal $v(t)$ is weighted and normalized as specified in DIN 45669-1 (DIN 45669-1 1995). The signal weighting is performed in MATLAB using *high pass* and *low pass* 2-poles Butterworth filter (*high pass* frekvency is 0.8 Hz, *low pass* frekvency is 80 Hz). The frequency weighting function is:

$$H_{Bnom}(if) = \frac{1}{1 - i\,\dfrac{5.6\,Hz}{f}} \tag{15}$$

The weighted vibration severity $KB_F(t)$ is calculated as r.m.s. (running mean square) of the weighted velocity signal $KB(t)$:

$$KB_F(t) = \sqrt{\frac{1}{\tau}\int_{\xi=0}^{t} e^{-\frac{t-\xi}{\tau}} KB^2(\xi)\,d\xi} \quad , \tag{16}$$

where τ is the time constant of 0,125 s and ξ is time.

In order to check the applicability of the proposed simple prediction model for the assessment of traffic induced vibration, the weighted vibration severity $KB_F(t)$, is calculated using the measured and predicted velocity signals near the wall and at the floor (Gudžulić 2014). The obtained results are presented in Figs. 11*a* and 11*b* for the mid-rise building.

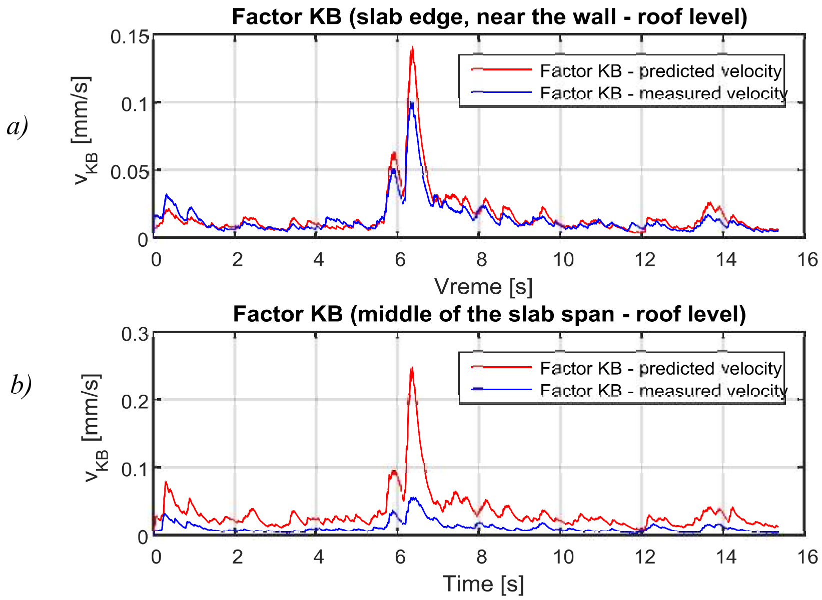

Figure 11. Predicted and measured weighted vibration severity $KB_F\,(t)$ for mid-rise building: *a*) near the wall, *b*) floor (Gudžulić 2014)

It is evident that vibration severity $KB_F(t)$ obtained from the predicted and measured velocity signals near the wall are in a good agreement, Fig. 11*a*. However, $KB_F(t)$ calculated from the predicted floor's velocity differs significantly from the $KB_F(t)$ obtained from the measured values. The reason why the computational model does not fit to the measured one lies in a fact that many structural and material parameters cannot be exactly determined in advance. In this case it happened with the floor characteristics which, obviously, were not adequately presented in the model.

Using the same approach the $KB_F(t)$ is calculated for two other types of buildings, and the maximum weighted vibration severities KB_{Fmax} are determined. The obtained values of KB_{Fmax} are presented in Table 1.

KB_{Fmax}			
Signal	Building		
	Low-rise	Mid-rise	High-rise
Predicted	0.6598	0.2477	0.1281
Measured	0.1893	0.0567	0.0965

Table 1. Maximum weighted vibration severity KB_{Fmax}

Apparently, the predicted velocities give higher KB_{Fmax}, in all cases, for the reasons listed above. Getting the model to be fitted to the measured ones, by means of the $KB_F(t)$ and KB_{Fmax}, the model update process is necessary. It will be the scope of our future work.

4. CONCLUSION

In this paper the simple numerical model for prediction of traffic induced vibration is presented. The model is based on the wave propagation theory and spectral elements. It consists of one wall, one floor for each storey and a soil spring and damper. The model was checked using vertical vibration measurements of three buildings with different heights. The buildings behavior is found in agreement with numerical modeling, although the amplitudes of the measured and predicted velocities show certain differences, especially at higher frequencies.

With the overall objective to make a simple numerical model for prediction of traffic induced vibration in buildings, the assessment of the vibration on humans in building was also analyzed. Despite the fact that the predicted maximum weighted vibration severity did not match the one obtained using the measured signal, in all cases, we must pointed out that the proposed model gives satisfactory results concerning the frequency content and velocity amplitudes. Obviously, the calibration of the obtained results and measurement is necessary.

5. ACKNOWLEDGEMENT

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