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MODELS RELATED TO THE MOVING LOAD PROBLEMS

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Abstract

In this paper different mathematical models of the vehicle load are presented, depending on the one of the moving load types. In general, vehicle load models can be classified into two types: static and dynamic moving load models. Static moving load model is model with nonvarying load value and shape in time. Dynamic moving load model is defined by differential equations of motion where the road roughness function affects the solution. Due to this influence some of the differential equations cannot be solved analytically and the numerical approach has to be performed. Therefore, in this work Matlab Simulink models have been developed to solve this problem. Validation of Simulink models is carried out and presented for all analysed dynamic moving load models.

Key words: moving vehicle load, dynamic, MATLAB Simulink

1. Introduction

Moving load problems are very common in engineering. Such problems are related to the soils subjected to load which moves in space and excite the soils into vibration. These vibrations affect the surrounding buildings which might have negative effects not only to buildings, but also to humans or sensitive equipment. In order to obtain numerical model for traffic induced vibration prediction, the appropriate vehicle load mathematical models are required. In addition, these models can be used in vehicle design, since the ride quality depends on vehicle vertical displacement, velocity and acceleration. Therefore, different types of vehicle load mathematical models for static, as well as for dynamic moving load are developed and presented in this paper.

2. Different moving load model

A mathematical models used to describe the behavior of the moving load, in our case vehicle load, can be classified into two groups, static and dynamic. Which mathematical model will be applied depends on the type of the requested output data. In the following, different static as well as the dynamic moving load models will be presented.

2.1 Static moving load model

First mathematical model, called the static mathematical model is analysed. The main assumption in this model is that only static component with constant load shape and constant velocity exists. Load shape in plane can be described as function [1]:

$$S = p_0 \left(\frac{\sin \frac{2\pi x}{L}}{\frac{2\pi x}{L}} \right)^2 \tag{1}$$

where p_0 is amplitude of vehicle wheel force and L is the length of wheel load transfer to road. In this model the dynamic and inertial part of loading are neglected.

Spatial representation of static load can be presented in the similar way:

$$S = p_0 \left(\frac{\sin \frac{2\pi x}{L_x} \cdot \sin \frac{2\pi y}{L_y}}{\frac{2\pi x}{L_x} \cdot \frac{2\pi y}{L_y}} \right)^2 \tag{2}$$

where p_0 also presents an amplitude of vehicle wheel force, L_x and L_y are the length of wheel load transfer to road respectively toward and perpendicular to the vehicle motion.

2.2 Dynamic moving load model

Due to the road roughness the dynamic and inertial parts of loading occur which have to be taken into account. Therefore as a second type, dynamic mathematical model which incorporates the dynamic behavior of the vehicle is considered. In Figure 1 several well known dynamic models of vehicle load are presented: 1-DOF model, the Quarter car model (QCM) with 2-DOF, as well as the Half car model with 2-DOF (HCM 2DOF) and 4-DOF (HCM 4DOF).

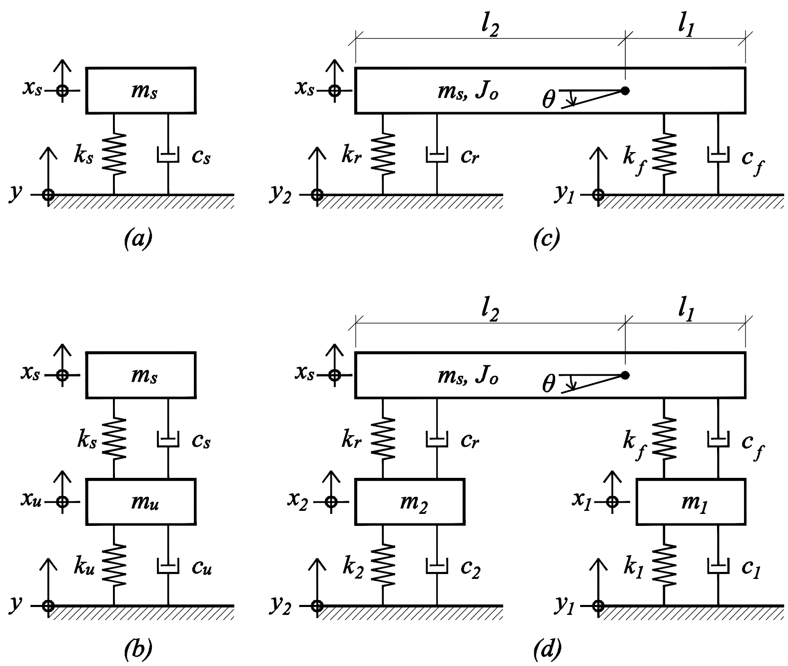

Fig. 1. Dynamic mathematical models of vehicle load: a-1DOF, b-2DOF (1/4 car model, QCM), c-2DOF (HCM 2DOF), d-4DOF (HCM 4DOF)

A mathematical model of a vehicle dynamic system is defined as a set of equations that represents the dynamics of the system. The equations of motion are the system of *n* coupled second-order ordinary differential equations, where *n* denotes number of degrees of freedom (DOF) of the system. In the following, the differential equations of motion of above mentioned dynamic moving load models are presented.

2.2.1 Dynamic 1DOF moving load model

The simplest dynamic moving load model represents 1DOF model (Figure 1a) [2,9], where vertical position of the vehicle body (m_s) is defined with one parameter, x_s. The differential equation of motion is given as:

$$m_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s = c_s\dot{y} + k_s y \tag{3}$$

where c_s is vehicle damping, k_s vehicle stiffness and y road roughness.

2.2.2 Dynamic 2DOF moving load model (Quarter Car Model (QCM))

Quarter Car Model (QCM) [3,7,9] analyses the behavior of the single wheel (m_u) and vehicle body (m_s) connected with main suspension (Figure 1b). Main suspension consists of vehicle damping (c_s) and stiffness (k_s). Single wheel is in turn connected to the ground via tyre damping (c_u) and stiffness (k_u). The differential equations of motion that describe the behavior of this model are given as:

$$\begin{aligned} &m_s\ddot{x}_s + c_s\dot{x}_s - c_s\dot{x}_u + k_s x_s - k_s x_u = 0 \\ &m_u\ddot{x}_u - c_s\dot{x}_s + \left(c_s + c_u\right)\dot{x}_u - k_s x_u + \left(k_s + k_u\right)x_u = k_u y + c_u\dot{y} \end{aligned} \tag{4}$$

which can be presented in the following matrix form:

$$\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix}\begin{Bmatrix} \ddot{x}_s \\ \ddot{x}_u \end{Bmatrix} + \begin{bmatrix} c_s & -c_s \\ -c_s & c_s + c_u \end{bmatrix}\begin{Bmatrix} \dot{x}_s \\ \dot{x}_u \end{Bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_u \end{bmatrix}\begin{Bmatrix} x_s \\ x_u \end{Bmatrix} = \begin{Bmatrix} 0 \\ k_u y + c_u\dot{y} \end{Bmatrix} \tag{5}$$

2.2.3 Dynamic 2DOF moving load model (Half car model with 2-DOF (HCM 2DOF))

Half car model with 2-DOF [4], vertical displacement x_s and angular movement θ of the vehicle body at the center of gravity, is presented in Figure 1c. The model is defined by vehicle body mass (m_s) and inertia moment (J_o), front and rear vehicle damping, c_f and c_r, and front and rear vehicle stiffness k_f and k_r. The differential equations of motion of this model are given as:

$$\begin{aligned} &m_s\ddot{x}_s + \left(c_f + c_r\right)\dot{x}_s + \left(c_f l_1 - c_r l_2\right)\dot{\theta} + \left(k_f + k_r\right)x_s + \left(k_f l_1 - k_r l_2\right)\theta = \\ &= c_f\dot{y}_f + c_r\dot{y}_r + k_f y_f + k_r y_r \\ &J\ddot{\theta} + \left(c_f l_1 - c_r l_2\right)\dot{x}_s + \left(c_f l_1^2 + c_r l_2^2\right)\dot{\theta} + \left(k_f l_1 - k_r l_2\right)x_s + \left(k_f l_1^2 + k_r l_2^2\right)\theta = \\ &= c_f\dot{y}_f l_1 - c_r\dot{y}_r l_2 + k_f y_f l_1 - k_r y_r l_2 \end{aligned} \tag{6}$$

The matrix form of the Equation (6) is:

$$\begin{bmatrix} m_s & 0 \\ 0 & J \end{bmatrix}\begin{Bmatrix} \ddot{x}_s \\ \ddot{\theta} \end{Bmatrix}+\begin{bmatrix} c_f+c_r & c_f l_1-c_r l_2 \\ c_f l_1-c_r l_2 & c_f l_1^2+c_r l_2^2 \end{bmatrix}\begin{Bmatrix} \dot{x}_s \\ \dot{\theta} \end{Bmatrix}+\begin{bmatrix} k_f+k_r & k_f l_1-k_r l_2 \\ k_f l_1-k_r l_2 & k_f l_1^2+k_r l_2^2 \end{bmatrix}\begin{Bmatrix} x_s \\ \theta \end{Bmatrix}= \tag{7}$$
$$=\begin{Bmatrix} c_f\dot{y}_f+c_r\dot{y}_r+k_f y_f+k_r y_r \\ c_f\dot{y}_f l_1-c_r\dot{y}_r l_2+k_f y_f l_1-k_r y_r l_2 \end{Bmatrix}$$

where l_1 and l_2 are the lengths between the front and rear axles and the vehicle center of gravity, respectively.

2.5 Dynamic 4DOF model (Half car model with 4-DOF (HCM 4DOF))

Half car model with 4-DOF [5, 6], which is also called pitch-bounce model, typically consists of either the left or right half of the vehicle. Also, either the front or rear half of the vehicle can be taken into account. In this paper the left (right) half of the vehicle, presented in Figure 1d, is considered. This vehicle model is composed of three rigid bodies: vehicle body, which is defined by vehicle body mass (m_s) and inertia moment (J_o), and two wheels, which masses are m_1 and m_2. This model has four degrees of freedom: the heave and pitch motion of vehicle body, x_s and θ, and heave motion of wheels, x_1 and x_2. The differential equations of motion are:

$$\begin{aligned} &m_1\ddot{x}_1+\left(c_f+c_1\right)\dot{x}_1-c_f\dot{x}_s-l_1c_f\dot{\theta}+\left(k_f+k_1\right)x_1-k_f x_s-l_1k_f\theta=c_1\dot{y}_1+k_1y_1 \\ &m_2\ddot{x}_2+\left(c_r+c_2\right)\dot{x}_2-c_r\dot{x}_s+l_2c_r\dot{\theta}+\left(k_r+k_2\right)x_2-k_r x_s+l_2k_r\theta=c_2\dot{y}_2+k_2y_2 \\ &m_s\ddot{x}_s-c_f\dot{x}_1-c_r\dot{x}_2+\left(c_f+c_r\right)\dot{x}_s+\left(l_1c_f-l_2c_r\right)\dot{\theta}-k_f x_1-k_r x_2+\left(k_f+k_r\right)x_s+ \\ &+\left(l_1k_f-l_2k_r\right)\theta=0 \\ &J_o\ddot{\theta}-l_1c_f\dot{x}_1+l_2c_r\dot{x}_2+\left(l_1c_f-l_2c_r\right)\dot{x}_s+\left(l_1^2c_f+l_2^2c_r\right)\dot{\theta}-l_1k_f x_1+l_2k_r x_2+\left(l_1k_f-l_2k_r\right)x_s+ \\ &+\left(l_1^2k_f+l_2^2k_r\right)\theta=0 \end{aligned} \tag{8}$$

The matrix presentation of Equation (8) can be written in the following form:

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_s & 0 \\ 0 & 0 & 0 & J_o \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{\theta} \end{Bmatrix}+\begin{bmatrix} c_f+c_1 & 0 & -c_f & -l_1c_f \\ 0 & c_r+c_2 & -c_r & l_2c_r \\ -c_f & -c_r & c_f+c_r & l_1c_f-l_2c_r \\ -l_1c_f & l_2c_r & l_1c_f-l_2c_r & l_1^2c_f+l_2^2c_r \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_s \\ \dot{\theta} \end{Bmatrix}+ \tag{9}$$
$$+\begin{bmatrix} k_f+k_1 & 0 & -k_f & -l_1k_f \\ 0 & k_r+k_2 & -k_r & l_2k_r \\ -k_f & -k_r & k_f+k_r & l_1k_f-l_2k_r \\ -l_1k_f & l_2k_r & l_1k_f-l_2k_r & l_1^2k_f+l_2^2k_r \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_s \\ \theta \end{Bmatrix}=\begin{Bmatrix} c_1\dot{y}_1+k_1y_1 \\ c_2\dot{y}_2+k_2y_2 \\ 0 \\ 0 \end{Bmatrix}$$

Where c_f and c_r are front and rear vehicle damping, k_f and k_r front and rear vehicle stiffness, c_1 and c_2 front and rear tyre damping, k_1 and k_2 front and rear tyre stiffness. Lengths between the front and rear axles and the vehicle center of gravity are given with l_1 and l_2, respectively.

3. Simulink dynamic moving load models

Since the equations of motion, with arbitrary road roughness function (y), cannot be solved analytically the Matlab Simulink model is developed to solve this problem. The Simulink [9] is a software package that enables user to model, simulate, and analyse systems whose outputs change over time, i.e. the dynamic systems. User creates a block diagram, using the Simulink model editor that graphically presents time-dependent mathematical relationships among the system's inputs and outputs. The block diagram consists of the basic building blocks from the Simulink library. In the following, Simulink models for previously specified dynamic moving load models are presented.

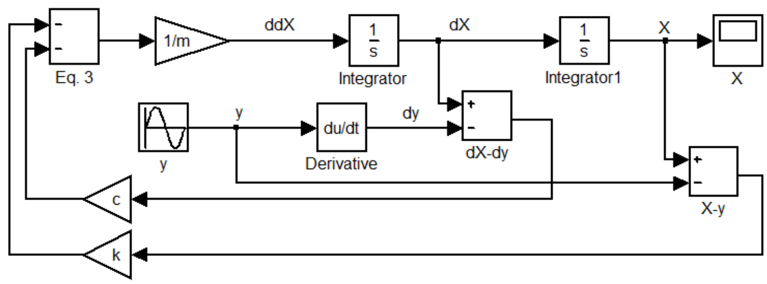

Fig. 1. Matlab Simulink model for 1DOF dynamic system (Equation (3))

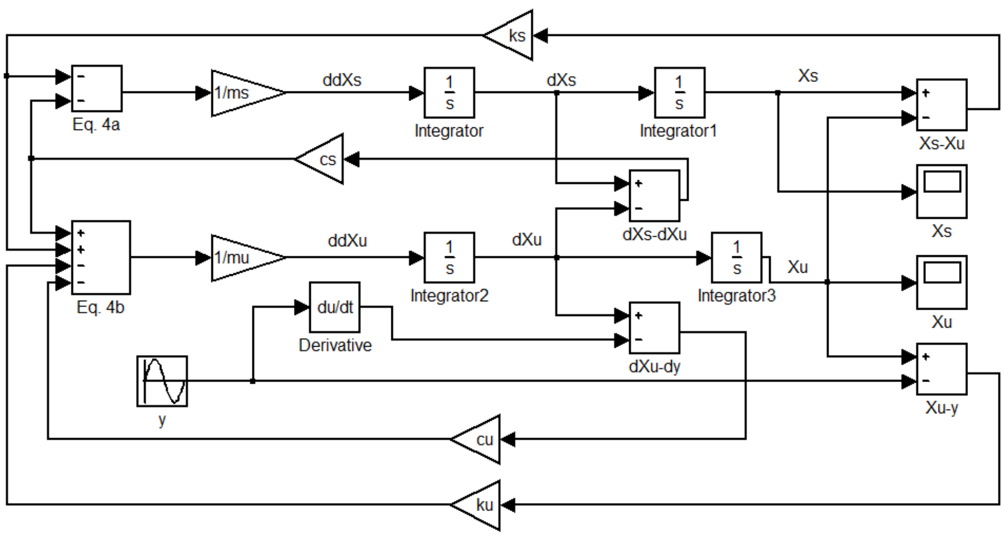

Fig. 2. Matlab Simulink model for QCM (Equation (4))

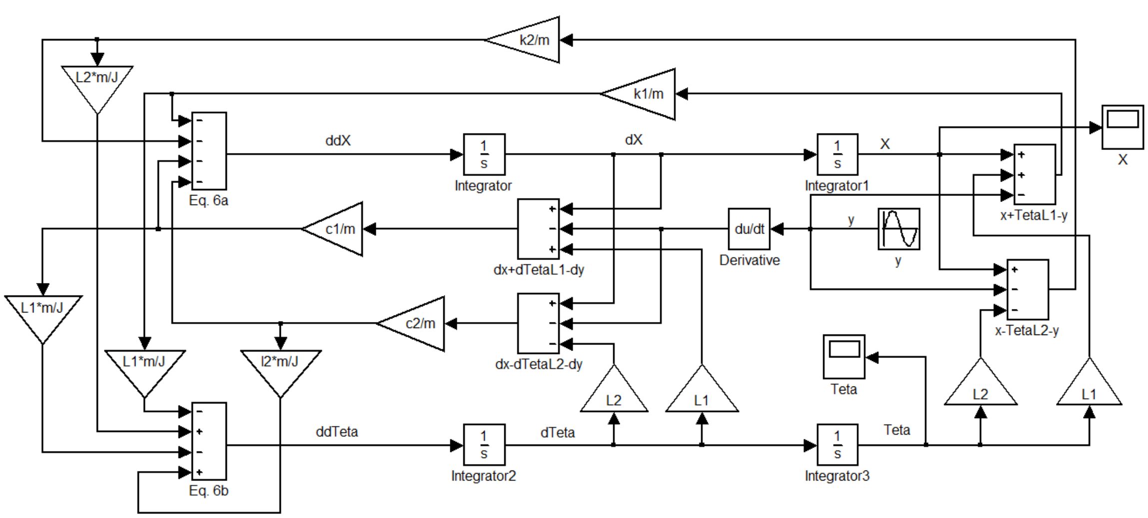

Fig. 3. Matlab Simulink model for HCM 2DOF (Equation (6))

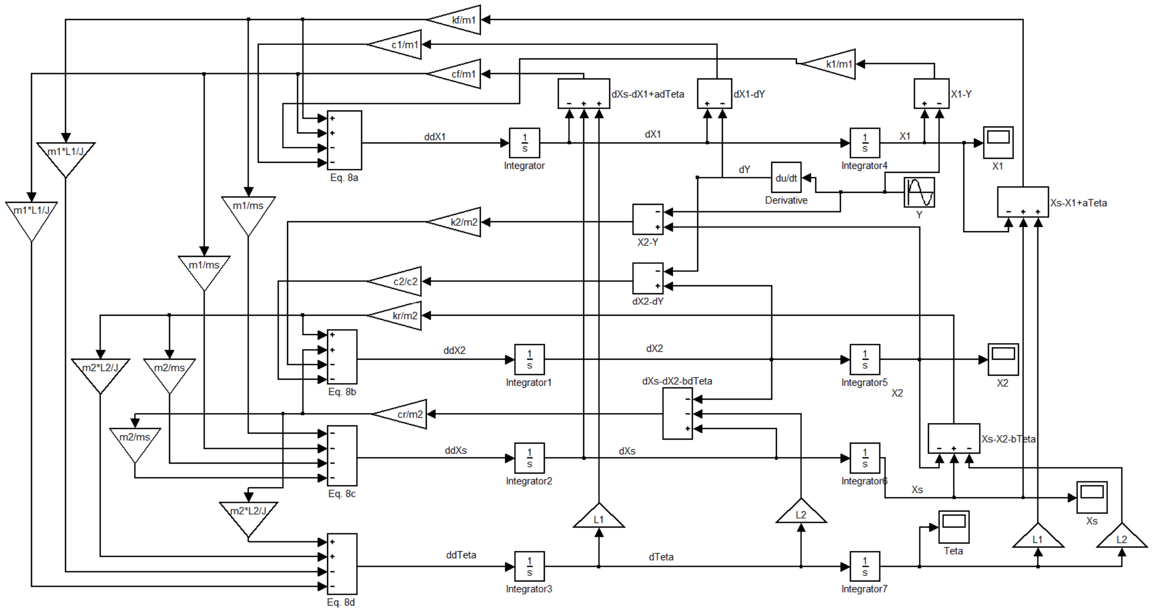

Fig. 4. Matlab Simulink model for HCM 2DOF (Equation (8))

4. Validation of Simulink models

In order to validate Simulink models the solutions of differential equations of motion, for all cases presented in Figure 1, obtained by Simulink models are compared with analytical solutions. The dynamic input is a steady state harmonic excitation $y = 0.002 \cdot e^{i350t}$, presented in Figure 5. The vehicle properties used for simulations are shown in Table 1.

	1DOF [2]	2DOF (QCM) [3]	HCM 2DOF [5]	HCM 4DOF [5]
m_s [kg]	236.124	241.5	1200	1200
c_s [Ns/m]	1385.4	300	-	-
k_s [N/m]	12394	6000	-	-
J_o [kgm^2]	-	-	2100	2100
l_1 [m]	-	-	0.847	0.847
l_2 [m]	-	-	1.513	1.513
c_f [Ns/m]	-	-	2500	2500
k_f [N/m]	-	-	28000	28000
c_r [Ns/m]	-	-	2000	2000
k_r [N/m]	-	-	21000	21000
m_u [kg]	-	41.5	-	-
c_u [Ns/m]	-	1500	-	-
k_u [N/m]	-	140000	-	-
m_1 [kg]	-	-	-	62.2
c_1 [Ns/m]	-	-	-	700
k_1 [N/m]	-	-	-	134000
m_2 [kg]	-	-	-	60
c_1 [Ns/m]	-	-	-	700
k_1 [N/s]	-	-	-	134000

Table 1. Dynamic moving load model properties

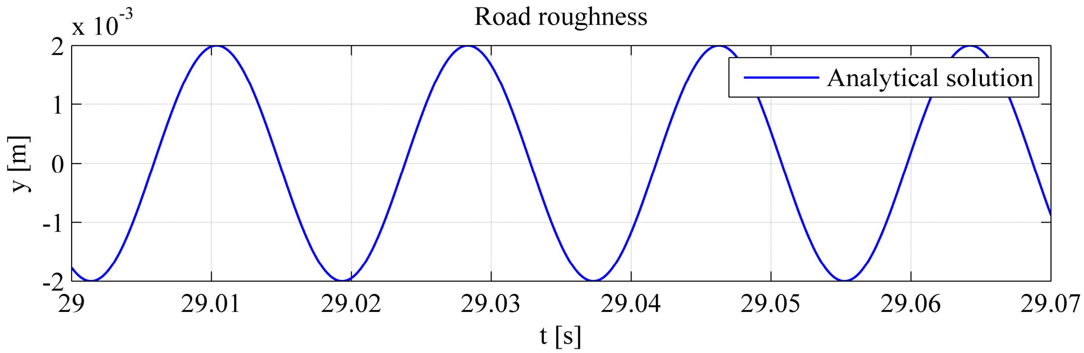

Fig. 5. Road roughness function $y = 0.002 \cdot e^{i350t}$

In Figure 6 the vehicle body displacement obtained for 1DOF dynamic model using the Simulink as well as the analytical solution are presented.

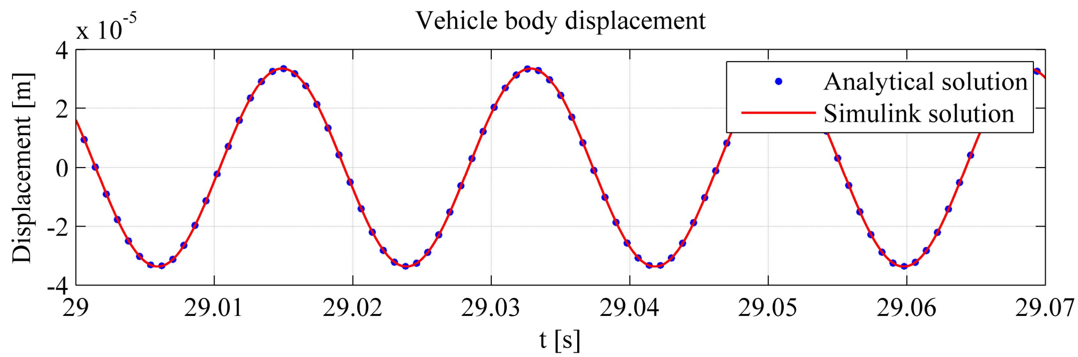

Fig. 6. Vehicle body displacement x_s, 1DOF model

The results obtained using both methods are the same, which means that the Simulink model for 1DOF system is well-set and can be used for any type of road roughness functions.

The same procedure is repeated for QCM, using the same road roughness function presented in Figure 5. Analytical solution as well as numerical solution obtained using the Simulink model for QCM is presented in Figures 7 and 8 for vehicle body displacement and wheel displacement, respectively. As shown in figures, displacements obtained with both methods are identical.

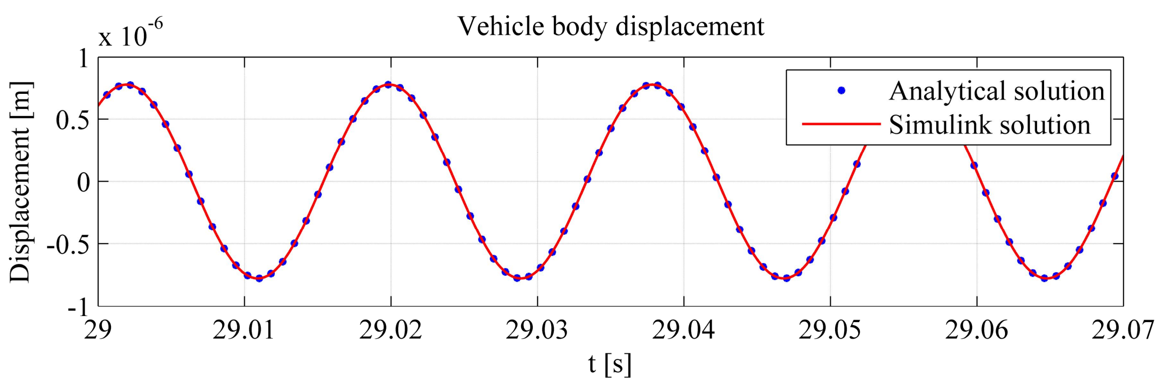

Fig. 7. Vehicle body displacement x_s, QCM

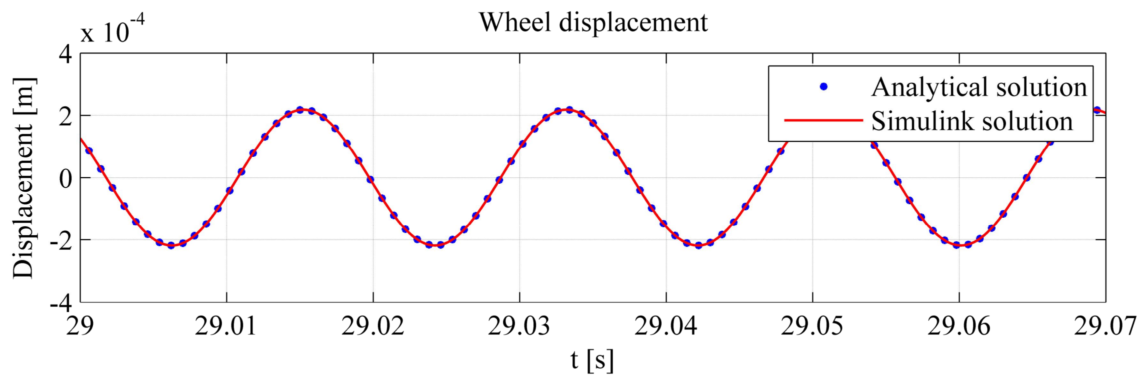

Fig. 8. Wheel displacement x_u, QCM

In order to validate HCM 2DOF, analytical and numerical solutions using the Simulink are compared. The model is excited by the same road roughness function with a time delay, which induces the diferences between front and rear suspension movements, y_1 and y_2. Vehicle body displacements x_s and vehicle body rotation θ of this model is presented in Figure 9 and Figure 10, respectively. Obtained results show that developed Simulink model is adequate.

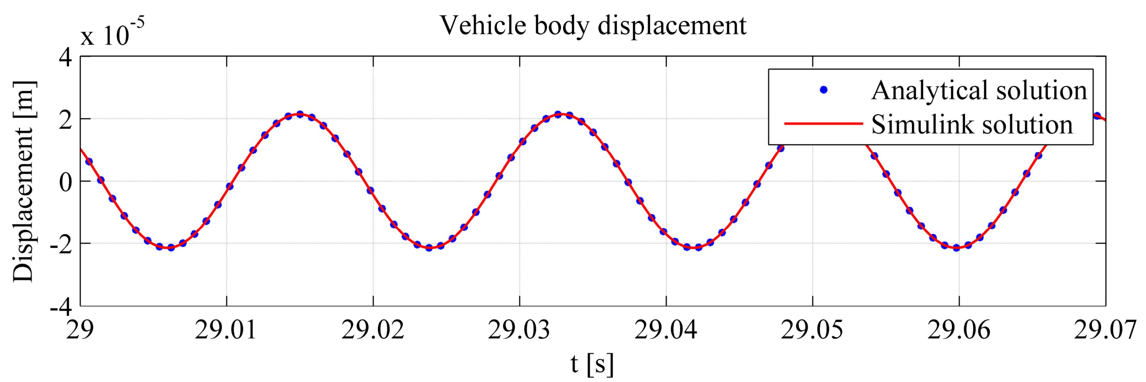

Fig. 9. Vehicle body displacement x_s, HCM 2DOF

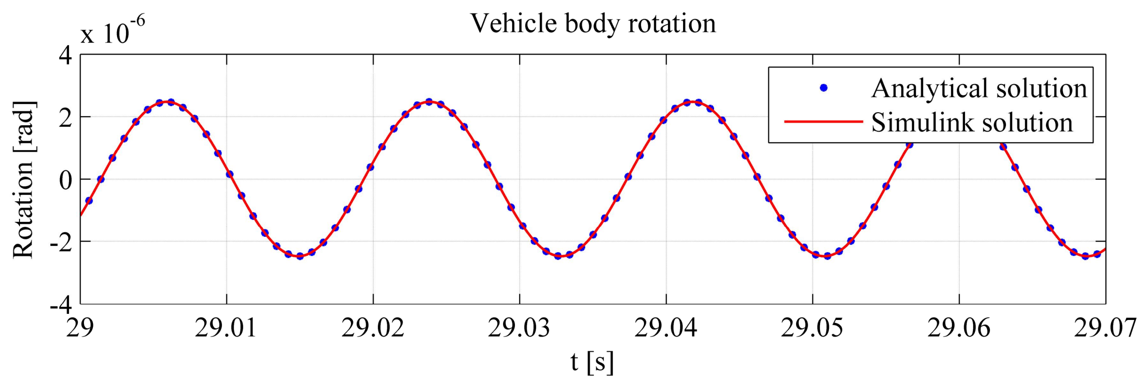

Fig. 10. Vehicle body rotation θ , HCM 2DOF

At the end, HCM 4DOF is analysed. Analytical and numerical solutions obtained using Simulink for all four displacements of half car model subjected to harmonic excitation are presented in Figures 11 to 14. The same solutions indicate that the Simulink model can be used to solve differential equations for arbitrarily road roughness function.

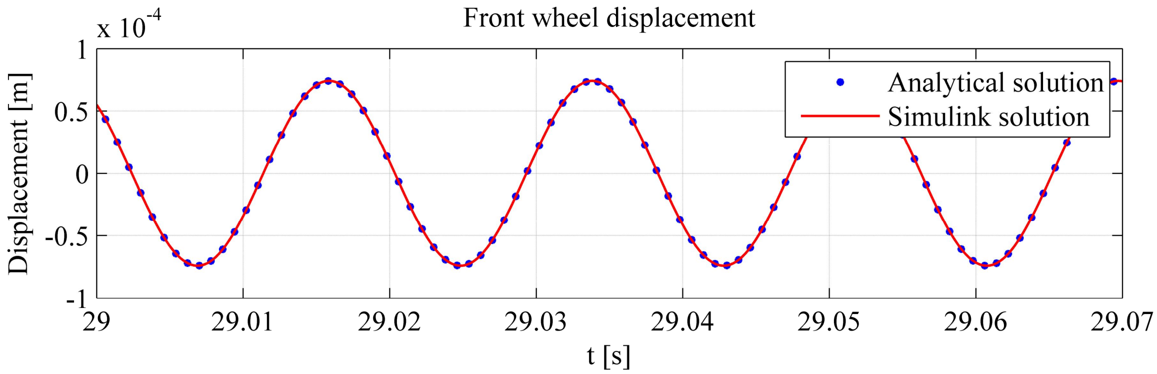

Fig. 11. Front wheel displacement x_1, HCM 4DOF

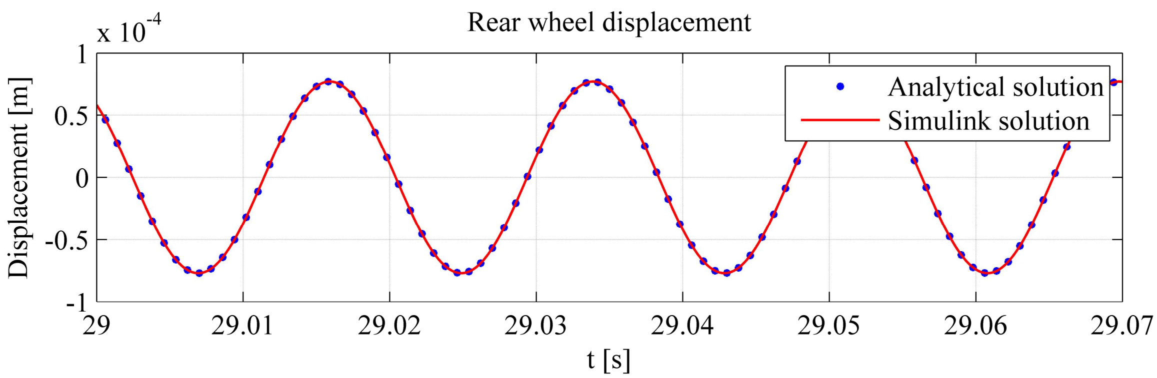

Fig. 12. Rear wheel displacement x_2, HCM 4DOF

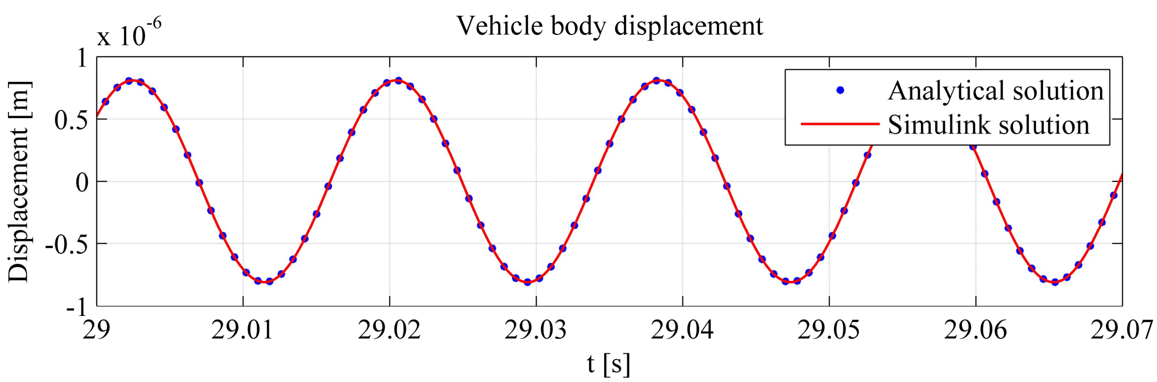

Fig. 13. Vehicle body displacement x_s, HCM 4DOF

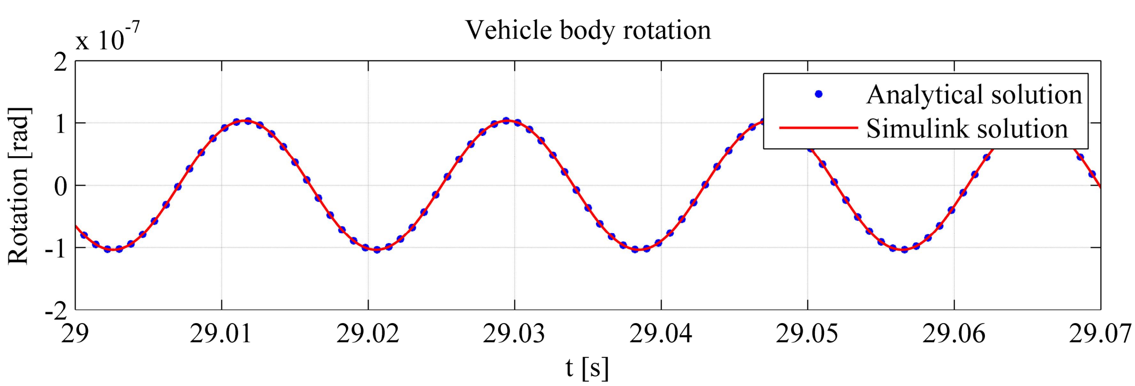

Fig. 14. Vehicle body rotation θ , HCM 4DOF

5. Conclusion

The main objective of this paper is to find the solution for different moving load models, which can be used to describe behavior of vehicle moving on the road with arbitrary roughness. Static moving load model is based on assumption that dynamic and inertial part of loading can be neglected, while these components of loading in dynamic moving load models have to be taken into account due to the effects of road roughness. Four dynamic vehicle models are analysed in this paper (1DOF, QCM, HCM 2DOF, HCM 4DOF). Dynamic moving load models cannot be solved analytically for all cases of road roughness functions. Due to this, the Simulink models are developed in order to define the behavior of vehicle subjected to arbitrarily road roughness function. Validation of Simulink models via harmonic excitation follows presented dynamic vehicle models.

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