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INFLUENCE OF SOIL-STRUCTURE-INTERACTION ON NONLINEAR TIME HISTORY SEISMIC RESPONSE OF RC FRAMES

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ABSTRACT

Modern regulations require from engineers to take into account a non-linear behavior and the appearance of plastic hinges in the selected structural elements during the earthquake. Also, regulations more insist on considering the soil-structure interaction (SSI) in the computation of dynamic response of buildings. In this paper, the nonlinear time history analysis of multi-storey RC frame structure is carried out taking into account SSI according to the Eurocode 8 and FEMA 450 recommendations. In order to illustrate the impact of the SSI, the response of the structure is calculated with and without the influence of the soil. Nine accelerograms are selected to correspond to the frequency spectra defined in EC8. Two standard types of soil, (ie. type B and C) according to the EC8, are adopted. The appropriate conclusions and recommendations for further work in this area are carried out.

KEY WORDS: soil-structure interaction, nonlinear time history seismic analysis

UTICAJ INTERAKCIJE TLA I KONSTRUKCIJE NA NELINEARAN VREMENSKI SEIZMIČKI ODGOVOR AB RAMOVA

REZIME

Savremeni propisi pred inženjere postavljaju zahtev da se u analizi zgrada na uticaj zemljotresa dozvoli nelinearno ponašanje i pojava zglobova u izabranim konstruktivnim elementima. Takođe, u propisima se sve više insistira na uzimanju u obzir sadejstva tla i konstrukcije pri sračunavanju dinamičkog odgovora. U ovom radu prikazana je nelinearna vremenska (Time History) analiza višespratne AB ramovske konstrukcije u sadejstvu sa tlom, primenom preporuka iz Evrokoda 8 i FEMA 450. Da bi se ilustrovao uticaj interakcije tla i objekta, sračunat je odgovor konstrukcije sa i bez uticaja okolnog tla. Izabrano je 9 akcelerograma čiji frekventni sastav odgovara spektrima definisanim u EC8. Usvojena su dva standardna tipa tla, (tj. tlo B i C) prema EC8. Dati su odgovarajući zaključci i preporuke za dalji rad u ovoj oblasti.

KLJUČNE REČI: interakcija tla i konstrukcije, nelinearna time history seizmička analiza

INTRODUCTION

Influence of the soil-structure interaction on the seismic behavior of buildings is important in the case when a stiff structure is founded on the relatively soft ground. In those cases, SSI might cause negative effects, which means that displacements and forces in the structures might be increased in comparison with a case where SSI is not taken into account. In all other cases SSI could have detrimental effects, reducing the forces and displacements in the structures (Petronijevic & Rašeta, 2010).

Effect of SSI has to be taken into analysis according to the recommendations of Building Seismic Safety Council (BBSC) (1996), or Applied Technology Council Committee (ATC) (1998). Recommendations of the NEHRP (National Earthquake Hazards Reduction Program) Federal Emergency Management Agency - FEMA450 (2004), contains a procedure for SSI analysis developed in 1977, from the ATC3 Committee 2C (Applied Technology Council Committee on Soil-Structure Interaction). The procedure has, meanwhile, been verified in practice and supplemented by (Stewart et al., 2003). It is related to the simplified SSI linear analysis of frame structures.

Detailed study of the effects of SSI on the performance of 3-, 5-, and 10-storeys buildings was presented by (Petronijevic at all, 2014). The time history analysis was carried out to obtain linear response of 3D-structures due to several earthquake excitations.

The question is whether the SSI has an influence on non-linear seismic response of regular frame structures? In order to analyze its effects on the non-linear behavior of structures, simple plane frame is analyzed with and without the SSI.

The SSI can be calculated using the substructure method in time or frequency domain, or by modeling the whole structure and a soil in a single step by using finite element approach. The solution of the SSI problem in time domain using the substructuring method consists of two main steps: (i) calculation of the dynamic stiffness of the foundation, and (ii) calculation of the nonlinear dynamic response of the structure founded on springs and dashpots which characteristics are equal to the dynamic stiffness of the soil obtained for the first natural frequency of the structure. In this paper the substructure method in time domain is adopted and some conclusions are carried out.

MODELING OF SOIL INFLUENCE

In the dynamic analysis of soil-structure interaction, the coupling of two different media occurs. During the earthquakes, shear deformation in the soil changes. Increase of the shear deformation leads to the decrease of shear modulus and increase of damping in the soil.
Dynamic stiffness of the foundation is the complex function which depends on the frequency of vibrations, geometry of the soil-foundation contact surface and the soil properties. Its real part k_j represents the stiffness of the foundation in the considered direction, while its imaginary part c_j represents the damping. The dynamic stiffness of the foundation can be represented as a function of nondimenzionalized frequency a_0:

$$\overline{k}_j = \overline{k}_j(a_0), \quad a_0 = \omega \frac{r}{v_s} \tag{1}$$

where r is the characteristic dimension of the foundation, $v_s = \sqrt{G/\rho}$ is the shear wave velocity, G is the shear modulus of the soil, while ρ is the mass density. The components of the dynamic stiffness of the foundation can be represented in the following form:

$$k_y = \alpha_y K_y \ , \quad k_z = \alpha_z K_z \, , \quad k_\Theta = \alpha_\theta K_\Theta \tag{2}$$

where K_y, K_z and K_Θ are horizontal, vertical and rotational static stiffness of the foundation, while α_y, α_z and α_Θ are frequency-dependent functions which relate static and dynamic stiffness of the foundation. These functions decrease with the increase of the frequency, thus the dynamic stiffness of the foundation decreases under the high frequency excitation.

Having in mind that the earthquake loading is generally of a low frequency, for the practical purpose, without the loss of accuracy, we can adopt α_y=α_z=α_Θ=1 (dynamic stiffness is equal to the static stiffness) (Petronijević & Rašeta, 2010). The static stiffnesses for the circular footing on the homogeneous and isotropic layer of the soil (of constant thickness H), resting on the rigid substratum, can be calculated according to the following relations (Sieffert et al., 1992):

$$K_y = \frac{8Gr_y}{2-\nu}\left(1+\frac{r_y}{2H}\right), \quad K_z = \frac{4Gr_z}{1-\nu}\left(1+1.28\frac{r_z}{H}\right), \quad K_\Theta = \frac{8Gr^3}{3(1-\nu)}\left(1+\frac{r_\Theta}{6H}\right) \tag{3}$$

where ν is the Poisson ratio of the soil, H is the thickness of the soil layer, while r_y, r_z and r_Θ are the equivalent radii of the circular footing:

$$r_y = r_z = \sqrt{\frac{4L_xL_y}{\pi}}, \quad r_\Theta = \sqrt[4]{\frac{16L_xL_y{}^3}{3\pi}} \tag{4}$$

where L_x and L_y are the dimensions of a rectangular footing. Note that the damping of the footing for very low frequencies can be neglected.

The equations (3) are valid for $H/r_y > 2$, $H/r_z > 1$ and $4 \geq H/r_\Theta > 1$. If some of the conditions are not satisfied, the corresponding stiffness for the foundation on the soil layer is reduced to the stiffness for the foundation on the homogeneous elastic halfspace ($H \to \infty$).
The foundation stiffnesses depend on the sher modulus of the soil that changes during earthquake due to the changes of shear deformations. It can be calculated according to FEMA450, taking into account the soil shear strain levels associated with the design earthquake motion, or by using 1D-Equivalent linear analysis (1D-ELA).

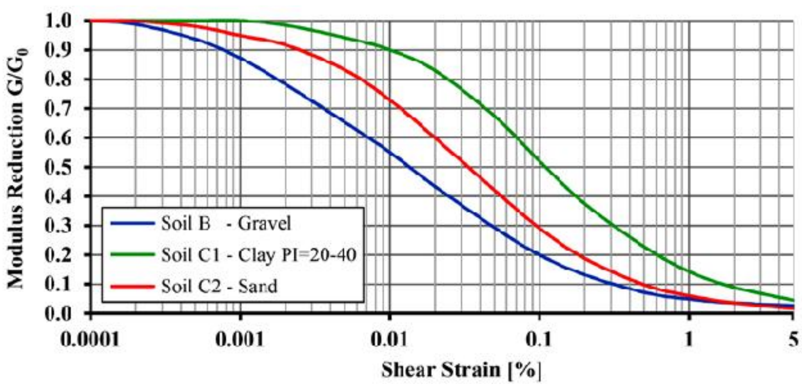
In this paper, the modulus reduction is calculated in both ways and results are used to estimate the influence of the soil shear modulus reduction on the dynamic response of RC frame. The ratio of effective shear modulus G and shear modulus G_0, associated with small strains as a function of S_{DS} (FEMA450, 2004), is given in Table 1:

Tabela 1. Vrednosti G/G_0 prema (FEMA450, 2004)
Table 1. Values of G/G_0 according to (FEMA450, 2004)

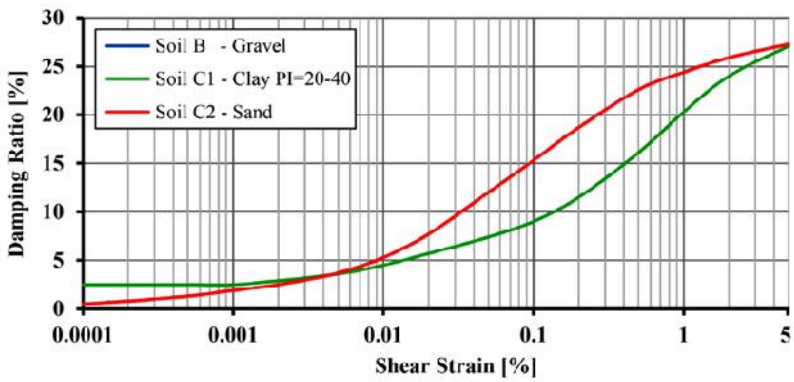
	$S_{DS} / 2.5$			
	≤ 0.10	**0.15**	**0.20**	**≥ 0.30**
G/G_0	0.81	0.64	0.49	0.42

1D-Equivalent linear analysis of wave propagation in the soil layer (Marjanović, 2013) was used to calculate the reduction of shear modulus G and enhacement of damping ratio ξ due to earthquake excitation. The iterative procedure based on the Kelvin's material model, the modulus reduction curve, as well as the damping curve for soil, have been used in the performed numerical analysis.

For the purpose of this work, three different soils, corresponding to soil types B and C according to EN1998-1-1, have been analyzed. The soil characteristics are taken from the Seismic micro-zoning of the location of Ada Bridge in Belgrade (Radovanović, 2008), and presented in Table 2. The modulus reduction curves and damping curves, plotted in Figures 1-2, are taken from the following references: for gravel - (Seed et al., 1986), for marl clay - (Sun et al., 1988) and for sand - (Seed et al., 1986). In all calculations, the height of soil is H=6 m.



Slika 1. Krive redukcije modula za tri razmatrana tla
Figure 1. Modulus reduction curves for three considered soils



Slika 2. Krive prigušenja za tri razmatrana tla
Figure 2. Damping ratio curves for three considered soils

Tabela 2. Karakteristike tla
Table 2. Soil properties

N^0	Soil	Soil mark	H	$V_{s,30}$	γ	ξ	G_0	ν
			[m]	[m/s]	[kN/m^3]	[%]	[MPa]	[-]
1	Alluvial gravel	B	6	450	20	1	405	0.25
2	Marl Clay	C1	6	325	20	1	210	0.30
3	Sand	C2	6	220	20	1	168	0.25

For all soil types and chosen accelerograms the ratios G/G_0 are calculated using 1D-Equivalent linear analysis. The following average reductions of the shear modulus are obtained: $\boldsymbol{G/G_0}$**=0.560** for Soil B, $\boldsymbol{G/G_0}$**=0.966** for Soil C1 and $\boldsymbol{G/G_0}$**=0.802** for Soil C2. The obtained values are kept constant during the analysis and used to calculate the foundation stiffness in the numerical examples.

Using FEMA's recommendations (FEMA450, 2004), the spectral response accelerations $S_{DS} = S_e(T) = 2.5a_g(g)S\eta$ are calculated for the considered soil types using a_g=0.23g. Then the G/G_0 ratios are obtained. The results are presented in Table 3.

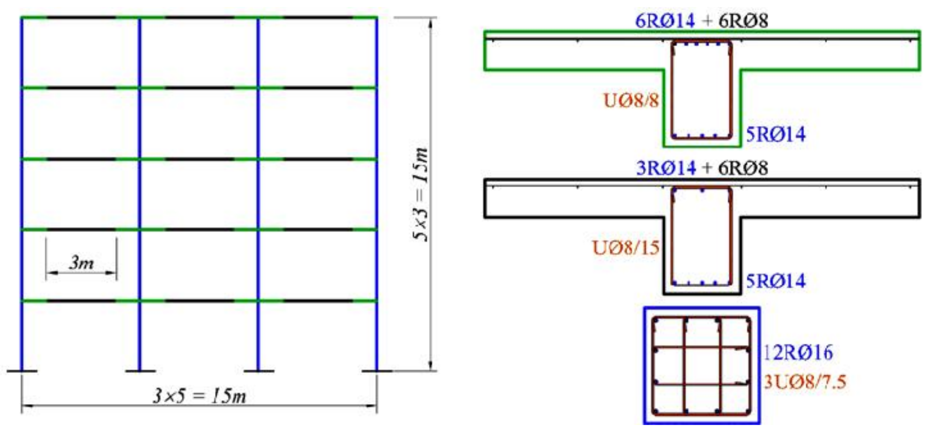
Tabela 3. Odnosi G/G_0 za različite tipove tla, prema (FEMA450, 2004)
Table 3. G/G_0 ratios for different soil types, according to (FEMA450, 2004)

Soil	Soil mark	η	S	S_{DS}	G/G_0 (see Table 1)
Alluvial gravel	B	1	1.20	0.276g	0.437
Marl Clay	C1	1	1.15	0.265g	0.455
Sand	C2	1	1.15	0.265g	0.455

As shown in Table 3, the modulus reductions according to FEMA are higher in comparison with G/G_0 ratios obtained from the 1D-Equivalent linear analysis, which means that FEMA450 regulations are conservative regarding the estimation of the soil properties in the dynamic loading environment.

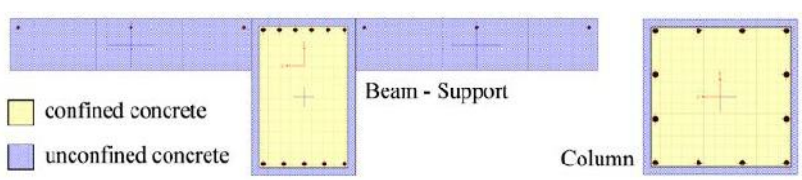
NUMERICAL EXAMPLE

The influence of a soil deposit on the nonlinear seismic time history response is illustrated in this example. A five-story reinforced-concrete frame structure (see Figure 3), which geometry and reinforcement are adopted from Refs. (Stančev-Ćurčin et al., 2014), is considered. The beams have rectangular cross section 30/45 cm, while the columns are of 45/45 cm. Slab thickness is 15 cm, while the effective width of the slab is 170 cm (therefore, the beams are modeled as T-section members). In order to consider the influence of cracks, flexion and shear, properties of the structural elements are reduced to 50%. All members are made of concrete C35/45 (f_c=43MPa, ε_c=0.2%, ε_u=0.4%), with the Poisson ratio ν=0. The reinforcement is adopted as steel class B500c (f_y=500MPa, ε_y=0.25%, f_u=600MPa, ε_u=7.5%).

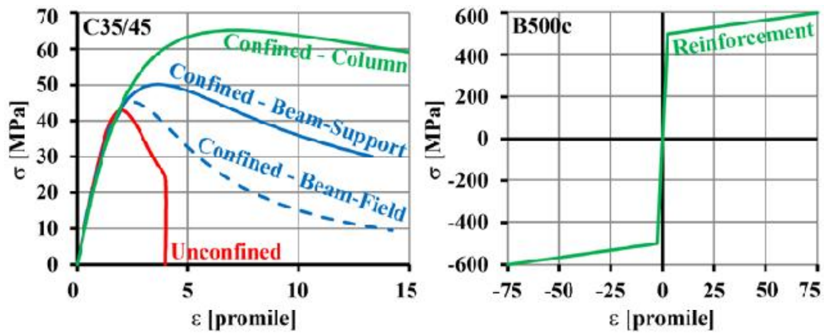


Slika 3. Geometrija i usvojena armatura razmatranog AB rama
Figure 3. Geometry and adopted reinforcement of the considered RC frame

The nonlinear analysis has been performed using the software package SAP2000 v14.2. The cross-sections are modeled using the Section Designer Tool in SAP2000, introducing the Mander's (Mander et al., 1984) models for the confined (the core) and the unconfined part of the concrete section (a protective layer of concrete to reinforcement) – see Figure 4, as well as the bilinear material model for the B500c reinforcement. The corresponding material models of concrete and reinforcement are given in Figure 5.



Slika 4. Poprečni preseci T-grede (nad osloncima) i stuba u SAP2000 Section Designer-u
Figure 4. T-beam (at the supports) and column cross section in SAP2000 Section Designer



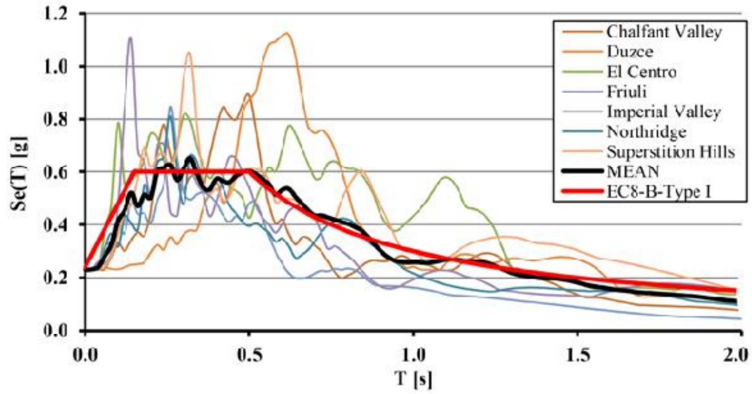
Slika 5. Materijalni modeli za beton C35/45 i armaturu B500c
Figure 5. Material models of concrete C35/45 and reinforcement B500c

The structural members have been previously designed according to (EN1992-1-1, 2006) and (EN1998-1, 2004), for the ductility class DCH, under the combined action of the following vertical loads: self-weight, added dead permanent load ΔG = 2.5 kN/m^2 and the imposed load Q = 2.5 kN/m^2 for the building category "B" according to (EN1991-1-1, 2009). The introduced seismic action is based on the design response spectrum S_d(g), which is reduced in comparison with the elastic response spectrum (soil category C, a_g=0.20g), using the behavior factor q=5.85. The building is classified as a building of significant class II and the importance factor γ=1. As shown in (Stančev-Ćurčin et al., 2014), the total mass of the designed frame structure is 271.65t. The dominant periods are T_1=0.791s and T_2=0.254s.

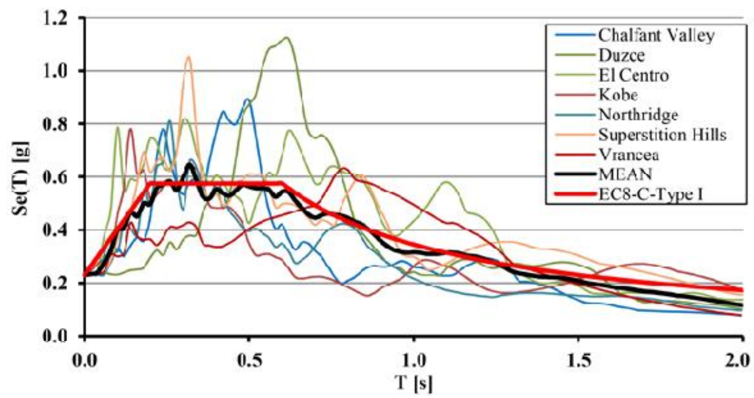
For the seismic analysis, 9 recorded (real) accelerograms have been selected and scaled to a_{max}~0.23g. The characteristics of applied ground motions are presented in Table 4. The elastic response spectra of all accelerograms have been calculated. Finally, two combinations (each composed of 7 accelerograms) corresponding to two different soil types (B and C according to EN1998-1, 2004), have been made. As shown in Figures 6-7, the mean response spectra of selected accelerograms, for both combinations, well match the horizontal elastic response spectra $S_e(g)$ Type I (a_g=0.20g), for soils B and C, according to EC8.

Tabela 4. Karakteristike stvarnih zemljotresa i usvojene kombinacije za tla B i C
Table 4. Properties of real accelerograms and adopted combinations for soils B and C

N°	Earthquake	Date	M	Soil B	Soil C
1	Chalfant Valley	July 21, 1986	6.2	+	+
2	Duzce	November 12, 1999	7.2	+	+
3	El Centro	May 18, 1940	6.9	+	+
4	Friuli	May 6, 1976	6.5	+	-
5	Imperial Valley	October 15, 1979	6.4	+	-
6	Kobe	January 17, 1995	6.9	-	+
7	Northridge	January 17, 1994	6.7	+	+
8	Superstition Hills	November 24, 1987	6.6	+	+
9	Vrancea	August 30, 1986	7.1	-	+



Slika 6. Spektri odgovora (stvarni, prosečni i prema EN1998-1 Tip I) za tlo tipa B
Figure 6. Response spectra (real, average and EN1998-1 Type I) for soil type B



Slika 7. Spektri odgovora (stvarni, prosečni i prema EN1998-1 Tip I) za tlo tipa C
Figure 7. Response spectra (real, average and EN1998-1 Type I) for soil type C

The formation of plastic hinges is allowed at the ends of the beams, as well as at the foundation level in columns. The hinge properties are calculated automatically using the default options in SAP2000 (reccomendations of FEMA 450) and previously imposed nonlinear material models of structural members.

The nonlinear seismic time history analysis was carried out using SAP2000 for: (i) clamped model and (ii) spring model, (i.e. plane frame on elastic supports). For the spring model, three different soils have been considered: (1) gravel, corresponding to the soil type B, (2) marl clay and (3) sand, corresponding to the soil type C, according to EC8.

For the clamped models, boundary conditions are assigned by restraining all generalized displacements in columns at the foundation level. On the other hand, springs have been assigned in horizontal, vertical and rotation direction, in all columns at the foundation level. The foundation dimensions, necessary for the calculation of spring stiffnesses, are L_x=L_y=2.5m. The spring stifnesses, calculated according to the procedure given in the previous section and taking into account two different values for G/G_0 ratio, are presented in Table 5:

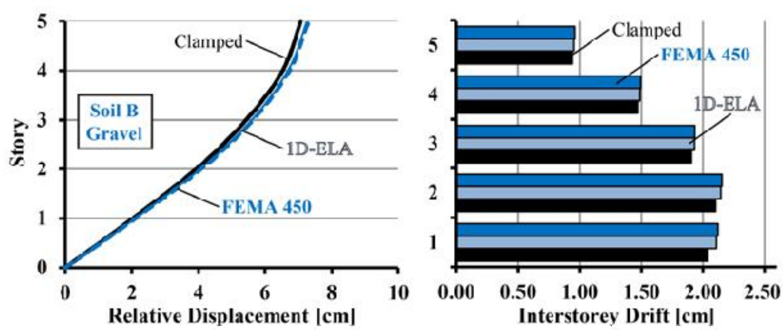
Tabela 5. Krutosti opruga korišćene u numeričkim modelima
Table 5. Spring stiffnesses used in numerical models

Model	1D-Equivalent Linear Analysis			FEMA 450		
Soil	B	C1	C2	B	C1	C2
G/G_0	0.560	0.966	0.802	0.437	0.455	0.455
K_y [kN/m]	1634267	1504754	970875	1275312	708761	550808
K_z [kN/m]	2219479	2127001	1318535	1731987	1001848	748047
K_Θ [kNm]	2342433	2244832	1391579	1827934	1057348	789487

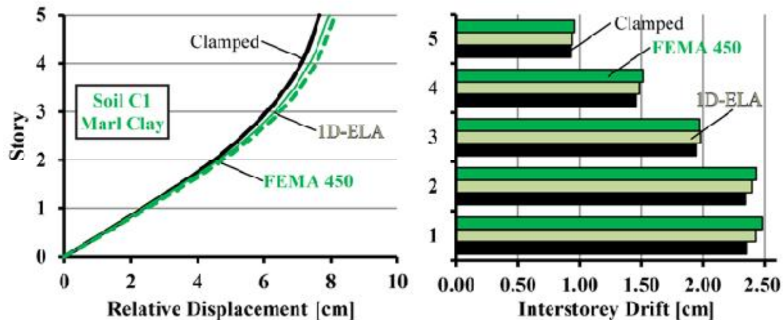
After performing the set of nonlinear seismic time history analyses, the maximum values of story displacement and interstorey drifts have been calculated for the following scenarios: (i) clamped model (soils B and C), (ii) model with springs according to one-dimensional elastic analysis of wave propagation in soil layer accounting for shear modulus reduction of the soil (Marjanović, 2013) (soils B, C1 and C2), and (iii) model with springs with reduced soil shear modulus according to the recommendations from (FEMA450, 2004) (soils B, C1 and C2).

All calculations have been performed using previously shown ground motion records. The average values of maximum relative story displacement and interstorey drift for each combination of accelerograms are plotted in Figures 8-10.

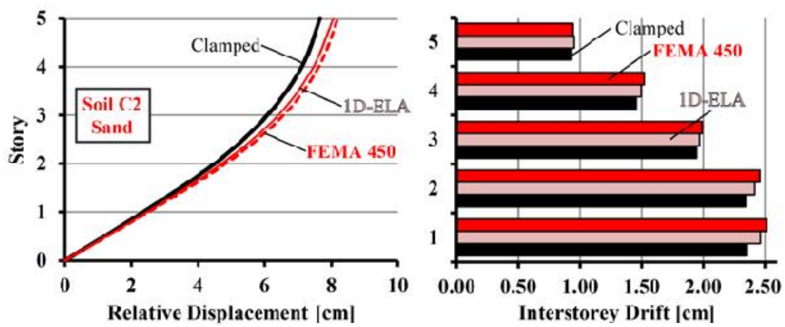
Relative displacements for elastically supported frames are greater than for clamped ones. It can be seen from Figs. 8-10 that relative displacemens of all floors, for the soil type C2, are the largest. The relative displacement at the top of frame founded on the soil type C2 is 7.0% higher than for the clamped frame. The difference between relative displacement at the top of the frame founded on the soil type B and C2 is 12.2%, while this difference between the cases of the soil type C1 and C2 is only 1.0% .



Slika 8. Prosečna relativna pomeranja spratova i međuspratni "driftovi" za tlo tipa B (šljunak)
Figure 8. Average relative story displacements and interstorey drifts for soil type B (gravel)



Slika 9. Prosečna relativna pomeranja spratova i međuspratni "driftovi" za tlo tipa C (laporovita glina)
Figure 9. Average relative story displacements and interstorey drifts for soil type C (marl clay)



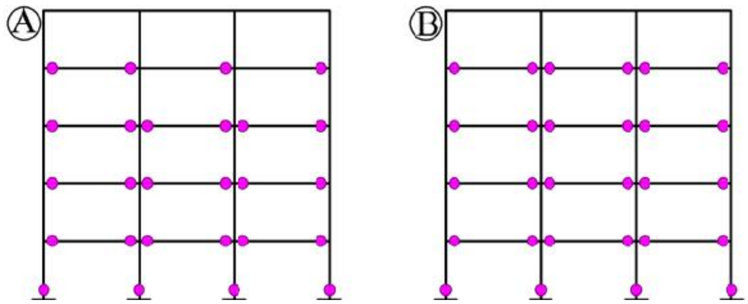
Slika 10. Prosečna relativna pomeranja spratova i međuspratni "driftovi" za tlo tipa C (pesak)
Figure 10. Average relative story displacements and interstorey drifts for soil type C (sand)

Interstory drift for the frame founded on the soil type B has the largest value between the 1st and the 2nd floor. For elastically support frame, it is approximately 2.2 cm. Influence of SSI on the interstory drift is very small, increasing the drift by 0.1 cm, only.

For the soil types C1 and C2, the interstory drift is the largest between the basement and the 1st floor. Maximum value is 2.5 cm for the elastically supported frame on the soil type C2. The difference between interstory drifts for clamped and elastically support structures is max 0.2 cm for the soil type C2, or less, for the other cases.

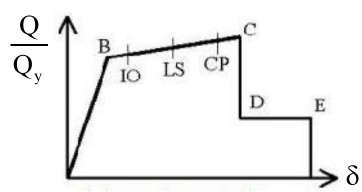
The main conclusion is that the effect of SSI on the relative displacements and interstory drifts is not so important, for soil types B and C. The higher influence is expected for the soils of a relatively low quality (types D and E according to EC8). The FEMA 450 recommandations gives higher value for modulus reduction, but it doesn't have significant influence on the nonlinear dynamic response.

The final arrangement of the plastic hinges is shown in Figure 11, for two "average" ground motions which action caused the displacements of the structure that are in the best agreement with the average displacements shown in Figures 8-10. The "average" earthquake records are Northridge, for soil B, and El Centro, for soils C1 and C2.



Slika 11. Konačan raspored plastičnih zglobova za (A) Northridge zemljotres – tlo B, (B) El Centro zemljotres – tla C1 i C2
Figure 11. Final fomation of plastic hinges for (A) Northridge EQ – soil B, (B) El Centro EQ – soils C1 and C2

From Figure 11 it is obvious that hinges type B (FEMA450 – see Figure 12) are formed at the ends of the beams at levels 1-4, as well as in columns at the foundation level. The structure is in the IO (Immediate occupancy) region, which means that visible damage occurred, but less than 67% of deformation limit for the Life Safety, see Figure 12.



Slika 12. Tipična veza opterećenje-deformacija i ciljni nivoi ponašanja (FEMA 450)
Figure 12. Typical load – deformation relation and target performance levels (FEMA 450)

CONCLUSIONS

The nonlinear time history analysis of 5 story frame has been analyzed taking into account SSI or not. Three types of soil have been used – one soil corresponding to the type B and two soils corresponding to the type C, according to EC8. The obtained results are presented and discussed. The main conclusion is that for soil type B and C, the effect of SSI on the non-linear response of frame structure is not so important, leading to the marginal increase of the interstorey drifts and relative story displacements in comparison with the clamped model. The influence is higher for lower-quality soils.

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