ESTIMATION OF DOWNSTREAM SEQUENT DEPTH IN B-F HYDRAULIC JUMPS FOR STILLING BASINS OF STEPPED SPILLWAYS

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Summary: *Study of the stability of the B-F type hydraulic jump was performed using results obtained from a physical hydraulic model. B-F type jump occurs in reverse slopped stilling basins, if the beginning of the jump is located in the spillway chute. Results were analyzed and compared to the existing literature which deals with both B-F and F type jumps., A modified procedure for estimation of the downstream sequent depth was presented for stilling basins of stepped spillways.*

Keywords: *hydraulics, stilling basin, hydraulic jump, stepped spillway*

1. INTRODUCTION

Hydraulic jump in the stilling basins presents an everlasting subject due to its effectiveness as an energy dissipator for dam spillways, and a long list of theoretical and experimental studies has been conducted over the course of the last century. Categorization of hydraulic jumps based on their location in reference to the channel geometry is first presented by Kindsvater (1944) [1]. According to this classification a B type jump originates in the upstream channel of a positive slope (usually a spillway), and extends into the horizontal downstream channel (usually a stilling basin). An F type jump occurs in a channel of adverse (negative) slope, and as described by Rouse (1938) [2], is inherently unstable, with an agitated free surface profile. However, Ohashi et al. (1973) [3] showed that, although an F type jump is indeed unstable for all slopes (except for slopes close to zero),

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it can be stabilized if the beginning of the roller is located along the positively sloped spillway chute. This type of jump is defined as a B-F type. In the Figure 7 are presented B, F and B-F types of hydraulic jump .

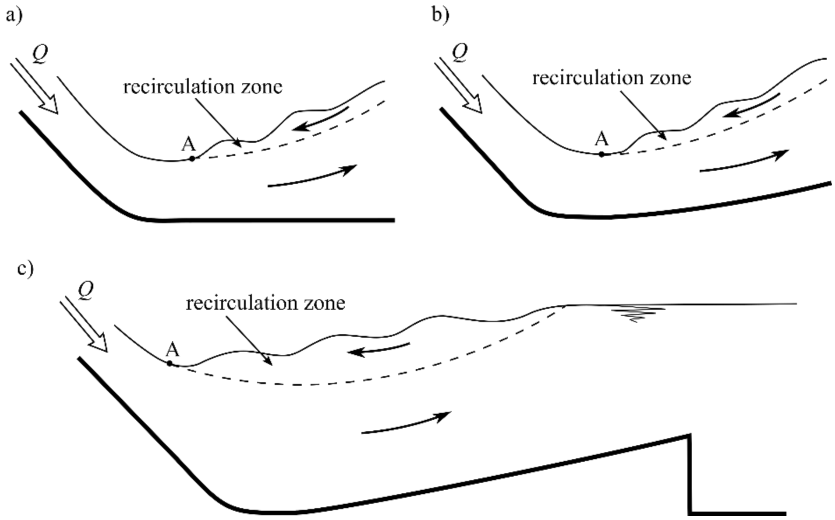

Figure 7. Certain types of hydraulic jumps: (a) B type in a horizontal basin, (b) F type and (c) B-F type in a basin with negatively sloped apron. Point A indicates the beginning of the jump

The attractiveness of B-F type jumps lies with the opportunity of using the gravitational force for the stabilization of the jump (Figure 7). The contribution of the gravitational forces can help reduce both the roller length and the sequent depth, when compared to the jumps in horizontal basins. This would lead to the reduction of the stilling basin length and side wall height. However, hydraulic benefits of the B-F jumps should be considered alongside the economic cost of the construction of the sloped stilling basin when choosing the optimal design.

In the last decades, stepped spillways have become regular overflow structures, particularly for roller-compacted concrete dams. The steps significantly affect the energy dissipation along the chute and reduce the size of the stilling basin. Bateni and Yazdandoost (2009) [4] and Beirami and Chamani (2010) [5] studied B-F jumps for standard ogee weirs with smooth chutes, and provided expressions for the estimation of the roller length, sequent depth and energy loss.

However, before these results can be safelly applied to the stilling basins of the stepped spillway chutes, they need to be verified experimentally. With that in mind, an experimental research of the B-F jump was conducted in the in the hydraulic laboratory of the University of Belgrade, Civil Engineering Faculty. The goal of the study was to determine whether the conclusion of the studies performed for standard chutes can be equally applied to stepped chutes.

2. EXPERIMENTAL SETUP

Experiments were performed on a rectangular channel with a stepped spillway chute, presented in the Figure 8. Slopped apron was constructed in the stilling basin at the angle of –5° relative to the horizontal. Apron was fixed to the chute toe using hinges so that the angle could be varied throughout the experiment. The apron was also secured to the horizontal basin bottom to avoid any movement and/or damage due to lift forces acting on the apron.

Side walls and bottom of the spillway chute were constructed from wood. Horizontal channel in which the apron was constructed was 2.50 m in length, from the toe of the chute to the downstream gate. Channel, chute and stilling basin widths were 0.46 m. Basin walls and apron were constructed using acrylic glass. Total height of the model, measured from the weir crest to the lowest point in the basin, was 0.94 m. Spillway chute length in the direction of the *x* axis was 1.02 m. Height and width of the steps were 4.50 cm, except for the first 6 steps which were half the size. Length of the apron was 1.50 m, which proved to be enough to form and stabilize the hydraulic jump at the maximum discharge for the experiment. Discharge was measured using two identical side-by-side V notch (triangular) weirs, using point gauge with measurement precision of up to ±0.1 mm. Weir rating curve was previously determined using ultrasonic flow meters on a downstream outlet pipe. Water level in the stilling basin was controlled using vertical rising flat sluice gate at the downstream end on the horizontal channel. Gate opening height was also measured, with the precision of up to ±0.5 mm, to ensure the repeatability of the experiment at any given discharge.

Discharge was varied within the range from 25 to 50 L/s. Flow depths were measured in the chute immediately before the beginning of the jump, h_1, and at the end of the roller, h_2. Incoming depth h_1 proved to be very difficult to measure due to the intense aeration of the flow. Depth h_2 at the end of the roller, was determined by direct measurement using measuring tape. However, owing to the highly unsteady nature of the hydraulic jump and the region downstream of the jump, measurement was done with the accuracy of up to ±0.5 cm. Length of the roller was determined by observing light tracer particles on the surface of the jump. Accuracy of the roller length measurement was up to ±1.0 cm.

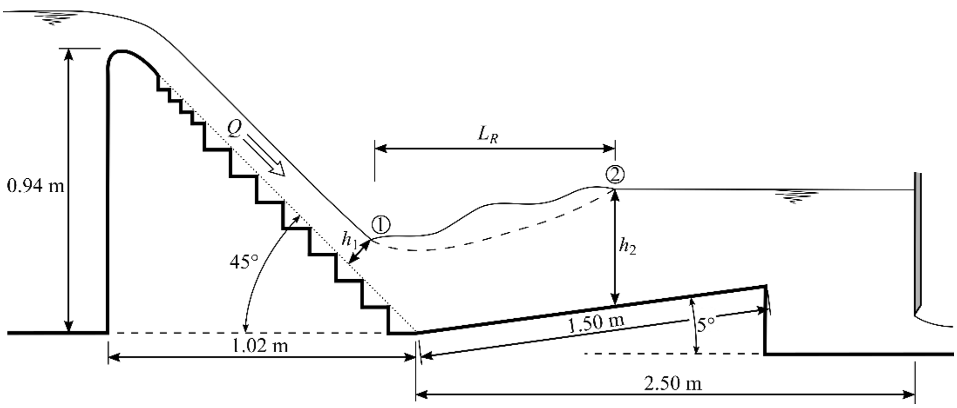

Figure 8. Experimental setup

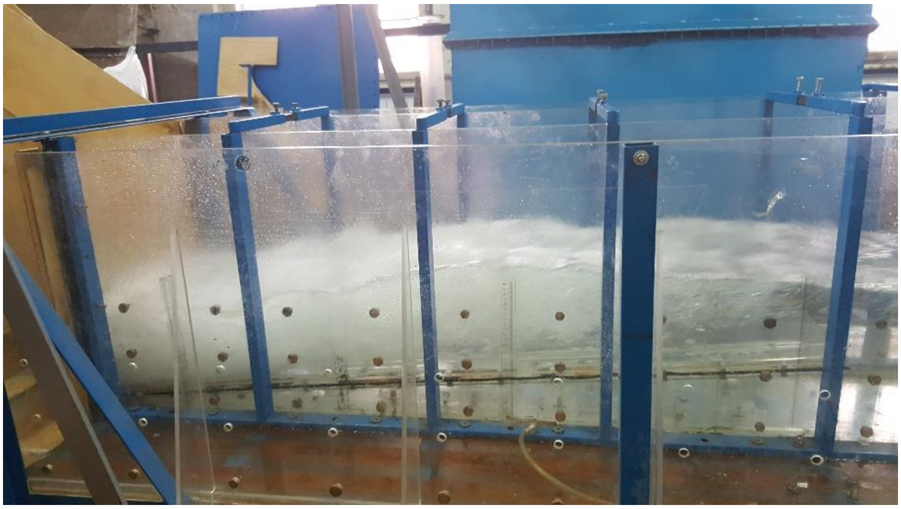

Figure 9. Water level and aeration in the hydraulic jump at maximum discharge of 50 L/s

3. METHODOLOGY

Methodology used was initially developed by Bateni and Yazdandoost (2009) [4]. Several distinct methods and assumptions were used in their study:

1. **The incoming (upstream) velocity and depth are known**. This can be problematic, considering the number of variables involved. It can easily be shown that the value of the downstream depth is very sensitive to the change of the upstream depth. Furthermore, Bateni and Yazdandoost do not mention the occurrence of the self-aeration of the flow. Since aeration of the flow causes bulking of the water jet, flow depth increases upstream of the jump.
2. **Position of the beginning of the jump is known**. Position of the jump cannot be determined analytically or numerically before conducting the experiment.
3. Instead of calculating the force of gravity acting on the control volume between sections 1 and 2, authors **assumed that hydrostatic pressure distribution law holds for sections 1 and 2** (Figure 10). Gravitational force has not been considered, and instead the reaction pressure force was used (R_1 and R_2 in Figure 10) with the assumption of **linearly distributed bed pressure along segments *CD* and *DE*** (Figure 10). This assumption is reasonable considering the difficulties of determining the volume of water in the hydraulic jump and the resulting gravitational force. Consequently, the increase of pressure on the sloped apron caused by the impact of water jet from the chute was neglected, if it exists. Centrifugal force acting on the curved part of the weir has also been neglected, to allow for easier calculation.

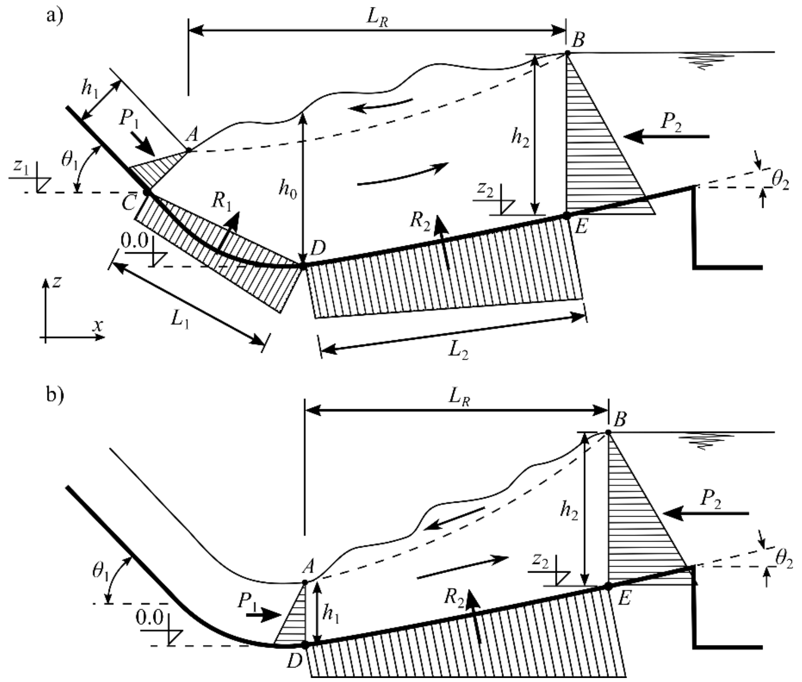

Figure 10. Pressure distribution on the control volume for (a) B-F type jump and (b) F type jump, according to [4]

Momentum conservation law for the unit-length control volume of the B-F jump in the horizontal direction (Figure 10a) can be written as:

$$\frac{\rho g}{2}\left(h_1^2\cos^2\theta_1 - h_2^2\right) + R_x = \rho q^2\left[\frac{1}{h_2} - \frac{\cos\theta_1}{h_1}\right], \quad (1)$$

where ρ is the water density (1000 kg/m^3), g is the gravitational constant (9.81 m/s^2), q is the unit discharge ($q = Q/B$, where $B = 0.46$ m is the channel width), and h_0, h_1, h_2, θ_1 are geometric characteristics presented in the Figure 10. The right side of the expression contains the pressure forces acting on the control volume, while the right side presents the change of the momentum between sections 1 and 2. The pressure force R_x is the horizontal component of the bed pressure forces:

$$R_x = R_{1x} - R_{2x}. \quad (2)$$

In their paper, Bateni and Yazdandoost [4] provided expressions for the components R_{1x} and R_{2x}, assuming the channel bed pressure distribution presented in Figure 10:

$$R_{1x} = \frac{\rho g L_1}{2}\left(h_0 + h_1\cos\theta_1\right)\sin\overline{\theta_1}, \quad (3)$$

$$R_{2x} = \frac{\rho g L_2}{2}\left(h_2 + h_0\right)\sin\theta_2. \quad (4)$$

where L_1, L_2, $\overline{\theta_1}$ and θ_2 are geometric characteristics presented in Figure 10 ($\overline{\theta_1}$ is defined by the slope of the segment *CD* in Figure 10a). However, expressions (3) and (4) require prior knowledge of position, as well as the depths and lengths of the jump, which can be difficult to determine. Non-dimensional form of the momentum conservation law for the B-F type jump was derived from (1):

$$D^3 + A_2 D^2 + \left(-A_1 A_0 + A_2 A_0 - A_1 \cos\theta_1 - 2Fr_1 \cos\theta_1 - \cos^2\theta_1\right) D + 2Fr_1 = 0, \quad (5)$$

where $D = h_2/h_1$, $A_0 = h_0/h_1$, $A_1 = z_1/h_1$, $A_2 = z_2/h_1$, and $Fr_1 = v_1^2/(gh_1)$ is the inflow Froude number (v_1 is the flow velocity at section 1).

In the case of the F type jump (Figure 10b), only one bed pressure force exists (it was named R_2 due to the similarity with the force R_2 in the B-F type jump):

$$R_{2x} = \frac{\rho g L_R}{2\cos\theta_2}\left(h_1 + h_2\right)\sin\theta_2. \quad (6)$$

Non-dimensional form of the momentum conservation law for the F type jump can be written as:

$$D^3 + A_2 D^2 + \left(-1 + A_2 - 2Fr_1\right) D + 2Fr_1 = 0. \quad (7)$$

Assuming that the inflow depth and velocity can be obtained using some empirical or analytical method, expression (7) contains 2 independent unknown variables: downstream depth h_2 and roller length L_R, and cannot be solved without using additional data.

4. EXPERIMENTAL RESULTS AND DISCUSSION

Flow characteristics along the chute can be obtained by solving the energy equation for free surface flow [6] (adapted form):

$$\frac{dE}{dL} = \frac{Q^2 n^2}{A^2 R^{4/3}}, \quad (8)$$

where E is the total energy of the flow, L is the coordinate axis along the spillway chute, Q is the discharge, n is the Manning friction coefficient, A is the flow area, and R is the hydraulic radius ($R = A/O$, where O is the wetted perimeter of the flow). It is important to notice that for stepped chutes, the Manning friction coefficient can be assessed using the equivalent channel bed roughness [6]:

$$k_e = s\cos\theta_1, \quad (9)$$

where s is the step height/width. Manning coefficient can be calculated by [7]:

$$n = \frac{k^{1/6}}{26}. \tag{10}$$

Previous experimental research on the same physical model showed that the combination of eq. (8) – (10) gives fairly accurate predictions in terms of the total forces of the flow at the section 1 (toe of the chute in Figure 10) [8]. For stepped chutes, Boes and Hager (2003a, 2003b) [9, 10] have presented an alternative method for the estimation of flow characteristics along the chute. Both methods yield very similar results, so the inflow data presented in Table 2 was determined using eq. (8) – (10).

Table 2. Measured inflow and geometric data

Q [L/s]	h_1 [cm]	h_2 [cm]	v_1 [m/s]	Fr_1 [-]	L_R [cm]	L_1 [cm]	L_2 [cm]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
25	1.724	15.0	3.152	58.73	75	12.7	66.2
30	2.002	15.5	3.257	54.01	85	10.6	78.3
35	2.330	16.0	3.265	46.63	93	6.4	88.3
40	2.683	17.0	3.241	39.92	101	6.4	96.4
45	3.033	18.5	3.225	34.96	115	12.7	106.4
50	3.335	19.0	3.239	31.88	123	11.3	116.4

Table 3. Sequent depths predicted by eq. (5) (indicated by B-F) and (7) (indicated by F), with relative errors to the observed depths

Q [L/s]	h_2 [cm]	h_2^{B-F} [cm]	h_2^{F} [cm]	ε^{B-F} [%]	ε^{F} [%]
(1)	(2)	(3)	(4)	(5)	(6)
25	15.0	8.07	14.72	46.2	1.9
30	15.5	9.22	16.13	40.5	-4.1
35	16.0	9.62	17.14	39.9	-7.1
40	17.0	10.85	17.97	36.2	-5.7
45	18.5	13.3	18.64	28.1	-0.8
50	19.0	13.7	19.36	27.9	-1.9

Experimental data acquired during this research (Table 2, Table 3) showed a significant deviation from the results obtained from the eq. (5) as proposed for B-F type jump by Bateni and Yazdandoost. In fact, the B-F jump on stepped spillways appeared to share more features with the F type jump for classic ogee spillway. This can be explained by considering all the differences between the two experimental setups:

a) Bateni and Yazdandoost conducted experiments on a classic ogee spillway, while in this study research was conducted on a stepped chute. On stepped chute recirculation zones develops between the steps and the chute tangent line (presented by a dotted line in Figure 8). The pressure distribution on the horizontal and vertical contour of the step is not defined.
b) Adding to the previous consideration is the fact that the pressure force on the horizontal contour of the step produces a vertical reaction force that does not

contribute to the momentum conservation law (1), since it was written for the horizontal direction.

c) Significant aeration of the flow is a characteristic feature of the stepped spillway chutes, and presents an additional difficulty when calculating the flow characteristics. Also, it has been observed that because of the intensity of the surface aeration, inflow water velocity remains practically unchanged from the start of the hydraulic jump until it reaches the chute toe. This indicates that the inflow forces $\sum F_1$ are easily transferred into the stilling basin.

Points a) and b) lead to a conclusion that the bed pressure force R_{1x} should be neglected. Point c) indicates that the B-F type hydraulic jump on stepped spillway chutes shares a lot of features with the F type jump for ogee spillways. To verify this hypothesis, gravitational force acting on the control volume was determined by approximating the water level in the basin with the line segment *AB* in Figure 10a and calculating the corresponding volume of the water in the jump. Apron reaction force R_x^V was then determined and compared with the reaction force R_x obtained from the eq. (1). The results are presented in the Table 4. The length of the roller is similar to the lengths of the jump in horizontal channel, and corresponds to the results provided by Peterka [11] (Figure 12).

Table 4. Comparison of the apron reaction force Rx obtained from eq. (1) with the reaction force RxV calculated from the measurement of the volume of water in the jump

Q [L/s]	A [cm^2]	V [cm^3]	R_x^V [N]	R_x [N]	$\sum F_1$ [N]	$\sum F_2+R_x^V$ [N]	$\varepsilon^{\Sigma F}$ [%]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
25	627.2	28852	24.67	19.65	79.47	84.49	6.3
30	752.0	34593	29.58	31.79	98.62	96.41	-2.2
35	829.2	38143	32.61	44.11	115.50	104.00	-9.9
40	993.5	45700	39.07	45.61	131.27	124.74	-5.0
45	1243.7	57211	48.92	46.19	147.21	149.93	1.8
50	1382.0	63572	54.35	54.46	164.51	164.41	-0.1

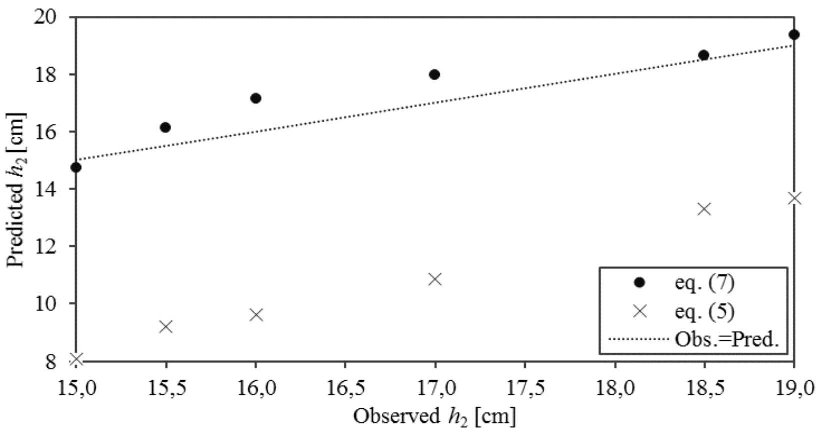

Figure 11. Observed sequent depth h2 against predictions by eq. (5) and (7)

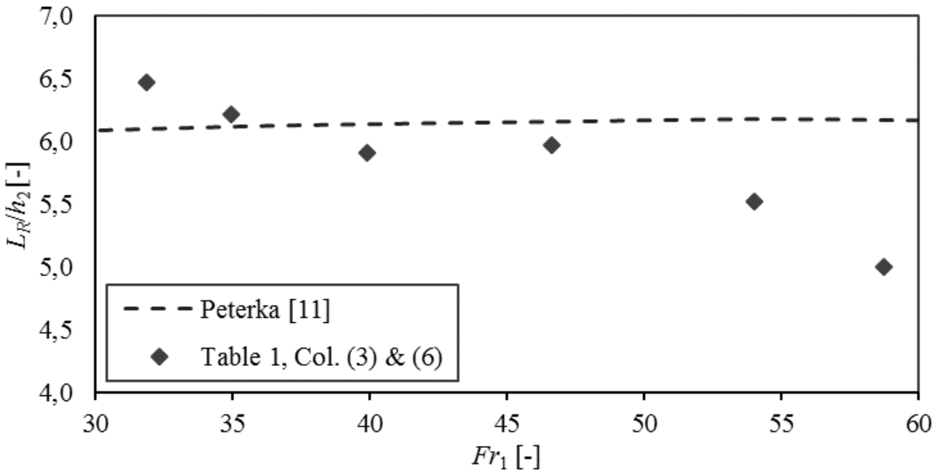

Figure 12. Relative length of the roller in terms of the inflow Froude number, compared to Peterka (1958) [11]

5. CONCLUSIONS

Experimental results showed that the behavior of the B-F type jump in stilling basins of stepped spillways is different from the jump in smooth ogee spillways. The main disagreement comes from the differences in bed pressure distribution and the effect of intensive aeration in stepped chutes. Methodology presented by Bateni and Yazdandoost can still provide good estimations for sequent depths, if the procedure for F type jumps in used instead. It has been shown that the volume of water in the jump can be approximated as a linear segment in order to estimate the contribution of the gravitational force of the stability of the B-F jump.

Further research should be conducted, preferably on a larger physical model, before more conclusions can be given. Additional measurements are due to include different apron slopes so that the effect of such basin on the reduction of the downstream sequent depth can be fully described for stepped spillway chutes.

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PRORAČUN SPREGNUTE DUBINE U HIDRAULIČKOM SKOKU B-F TIPA KOD UMIRUJUĆIH BAZENA STEPENASTIH BRZOTOKA

Rezime: *U cilju analize stabilnosti hidrauličkog skoka B-F tipa u umirujućim bazenima stepenastih brzotoka obavljena su odgovarajuća hidraulička modelska ispitivanja. Skok tipa B-F nastaje u umirujućem bazenu sa negativnim nagibom dna, pri čemu se početak skoka obrazuje u okviru brzotoka. Dobijeni rezultati su analizirani sa osvrtom na postojeću literaturu koja se bavi B-F i F tipom hidrauličkog skoka. Predložena je modifikacija postupka za proračun nizvodne spregnute dubine za ovaj tip umirujućih bazena, za slučaj kada se on nalazi u nastavku stepenastih brzotoka.*

Ključne reči: *hidraulika, umirujući bazen, hidraulički skok, stepenasti brzotok*