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Edited by:

Mihailo Lazarević
Srboljub Simić
Damir Madjarević
Ivana Atanasovska
Andjelka Hedrih
Bojan Jeremić

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FREE VIBRATION ANALYSIS OF MULTIPLE CRACKED FRAMES USING DYNAMIC STIFFNESS METHOD

Marija D. Milojević[1], Marija T. Nefovska-Danilović[2], Miroslav S. Marjanović[3]

Faculty of Civil Engineering,
University of Belgrade, Bulevar kralja Aleksandra 73, 11000 Belgrade, Serbia
[1] e-mail: mmilojevic@grf.bg.ac.rs
[2] e-mail: marija@grf.bg.ac.rs
[3] e-mail: mmarjanovic@grf.bg.ac.rs

Abstract

Cracks are the most common type of structural damage in many engineering structures, introducing the local change in stiffness of structural elements and changing the dynamic properties of the structure. Therefore, it is of great importance to detect and locate the existing structural damage in its early phase to prevent crack propagation.

In the paper, free vibration analysis of beams and frames with multiple cracks has been carried out. First, the dynamic stiffness matrix of beam element with multiple cracks based on Bernoulli-Euler theory has been derived. Using the developed model, extensive free vibration study of multiple-cracked frame structures has been conducted. Original Matlab code has been developed for the derivation of natural frequencies and plot of mode shapes.

After the validation of the model against the existing data from the literature, several benchmark examples are provided. As expected, increasing size and number of cracks lead to decrease of natural frequencies due to the local stiffness degradation.

Key words: dynamic stiffness method, crack, beam, frame, free vibration

1. Introduction

Cracks are the most common type of structural damage in many engineering structures. In order to ensure structural safety and integrity, it is of great importance to detect and locate the existing damage in its early phase, as well as to develop reliable experimental and numerical methods for damage detection. Generally, cracks introduce local change in stiffness of structural elements resulting in the change of dynamic characteristics of the structure in terms of natural frequencies and associated mode shapes. Consequently, most of the methods for damage detection are based on the evaluation and comparison of dynamic characteristics of damaged and undamaged structure [1-3].

Numerical modeling and computation of the free vibration characteristics of cracked structures is most frequently performed using the finite element method (FEM) [4]. Alternatively, vibration analysis of cracked beams and frames can be efficiently carried out using the dynamic stiffness method, [5, 6]. In this approach, beam parts between cracks are represented by the dynamic stiffness elements based on Bernoulli-Euler [7] or Timoshenko beam theory [8]. Shape functions of the dynamic stiffness elements represent exact analytical solutions of the corresponding equations of motion defined in the frequency domain. Consequently, the dynamic stiffness matrix is exact and frequency dependent. In addition, the number of dynamic stiffness elements necessary for structural discretization is frequency independent and influenced only by the change of the geometrical and/or material properties of the structure. For crack modeling, a crack element of zero length consisting of rotational and translational springs is used. Compliance properties of crack element are defined using fracture mechanics theory [9]. In that manner, by modeling beam as a complex structure consisting of multiple elements whose number depends on the number of cracks, the dynamic stiffness matrix of cracked beam element can be formed. Moreover, using the same assembly procedure as in the FEM, the global dynamic stiffness matrix of the structure can be formed.

In the paper, free vibration study of beams and frames with multiple cracks has been presented. The dynamic stiffness matrix of beam element with multiple cracks based on Bernoulli-Euler theory has been derived using condensation procedure. Afterwards, free vibration study of frame structures with multiple cracks have been performed in order to reveal the effect of crack size and location as well as number of cracks on the free vibration characteristics of the analyzed frames.

2. Multiple Cracked Beam Element

2.1 Crack modeling

Crack introduces local stiffness reduction of beam element. Furthermore, the change in stiffness causes the change of the free vibration characteristics of beam. Crack itself can be modelled using the equivalent rotational spring model [10-11], where the open crack is represented by rotational spring of stiffness k^*. This model accounts for change in the bending stiffness, while the axial and shear stiffness remain unchanged. Zheng and Kessissoglou [9] formulated a crack model taking into account bending, axial and shear stiffness reduction. Using the fracture mechanics theory, the elements of the compliance matrix of the crack have been derived explicitly for both rectangular and circular cross-sections. For the rectangular cross sections, expressions are presented in terms of width *b,* depth *h* and the crack length *a* through the beam depth, as follows:

$$\mathbf{C}=\frac{1-\nu^2}{Eb}\begin{bmatrix} F(1,1) & 0 & 0 \\ 0 & F(2,2) & 0 \\ 0 & 0 & \frac{1}{h^2}F(3,3) \end{bmatrix} \tag{1}$$

where ν is the Poisson's ratio and E is the Young's modulus. Expressions for dimensionless functions $F(1,1)$, $F(2,2)$ and $F(3,3)$ are given in the Appendix.

2.2 Dynamic stiffness matrix for multiple crack beam element

Beam element with a single crack represented with two Bernoulli-Euler dynamic stiffness elements *a* and *b* and a zero length crack element *c* is shown in Fig. 1. Lengths of the elements *a*, *b* and *c* are denoted as L_a, L_b and L_{c1}, respectively. Consequently, a single crack beam element has four nodes and twelve degrees of freedom (DOF). Dynamic stiffness matrices of elements *a* and *b* are given as:

$$\mathbf{K}_a(\omega)=\begin{bmatrix} \mathbf{k}_{ii}^a & \mathbf{k}_{ij}^a \\ \mathbf{k}_{ji}^a & \mathbf{k}_{jj}^a \end{bmatrix} \quad , \quad \mathbf{K}_b(\omega)=\begin{bmatrix} \mathbf{k}_{ii}^b & \mathbf{k}_{ij}^b \\ \mathbf{k}_{ji}^b & \mathbf{k}_{jj}^b \end{bmatrix} \tag{2}$$

where subscripts *i* and *j* denote end nodes of the dynamic stiffness elements *a* and *b*. Elements of the dynamic stiffness matrices $\mathbf{K}_a$ and $\mathbf{K}_b$ are given in [7]. Applying the assembly procedure, the global dynamic stiffness matrix of a single crack beam element is obtained in the following form:

$$\mathbf{K}(\boldsymbol{\omega}) = \begin{bmatrix} \mathbf{k}_{11}^{a} & \mathbf{k}_{13}^{a} & & \\ \mathbf{k}_{31}^{a} & \mathbf{k}_{33}^{a} + \mathbf{k}^{c} & -\mathbf{k}^{c} & \\ & -\mathbf{k}^{c} & \mathbf{k}_{44}^{b} + \mathbf{k}^{c} & \mathbf{k}_{42}^{b} \\ & & \mathbf{k}_{24}^{b} & \mathbf{k}_{22}^{b} \end{bmatrix} \tag{3}$$

where $\mathbf{k}^{c} = \mathbf{C}^{-1}$ is the nodal stiffness matrix of the crack element.

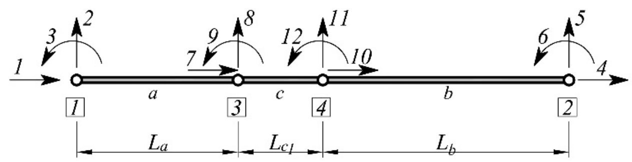

Fig. 1. Single crack dynamic stiffness beam element with adopted nodes and nodal displacements

In the same manner, dynamic stiffness matrix of beam element with n cracks can be formed as follows:

$$\begin{array}{ccccccc} 1 & 3 & \cdots & i & \cdots & 2n+2 & 2 \end{array}$$

$$\mathbf{K}(\boldsymbol{\omega}) = \begin{bmatrix} \mathbf{k}_{11} & \mathbf{k}_{13} & & & & & \\ \mathbf{k}_{31} & \mathbf{k}_{33} + \mathbf{k}^{c_1} & -\mathbf{k}^{c_1} & & & & \\ & -\mathbf{k}^{c_1} & \ddots & -\mathbf{k}^{c} & & & \\ & & -\mathbf{k}^{c} & \mathbf{k}_{i,i} + \mathbf{k}^{c} & -\mathbf{k}^{c} & & \\ & & & -\mathbf{k}^{c} & \ddots & -\mathbf{k}^{c_n} & \\ & & & & -\mathbf{k}^{c_n} & \mathbf{k}_{2n+2,2n+2} + \mathbf{k}^{c_n} & \mathbf{k}_{2n+2,2} \\ & & & & & \mathbf{k}_{2,2n+2} & \mathbf{k}_{22} \end{bmatrix} \tag{4}$$

Decomposing the dynamic stiffness matrix given by Eq. (4) to the *master* (m) and *slave* (s) DOFs and using the condensation procedure, the slave DOFs that correspond to the internal nodes 3, 4, …., ($2n$+2) of the multiple crack beam element can be eliminated and the condensed dynamic stiffness matrix is obtained in the following form:

$$\mathbf{K}_{c}^{e} = \mathbf{K}_{mm} - \mathbf{K}_{ms}\mathbf{K}_{ss}^{-1}\mathbf{K}_{sm} \tag{5}$$

3. Free Vibration of Cracked Frames

The global dynamic stiffness matrix of a frame structure is obtained using the well-known transformation and assembling techniques as in the FEM. Based on the theoretical considerations presented in previous sections, the Matlab program [12] has been extended enabling the computation of natural frequencies and mode shapes of 2D cracked frames. Natural frequencies of frame structure are obtained from the following equation:

$$\mathbf{K}_{ff}^{D}\mathbf{q}_{f} = 0 \tag{6}$$

where $\mathbf{K}_{ff}^{D}$ is the global dynamic stiffness matrix of frame structure corresponding to the unknown displacement components $\mathbf{q}_f$. When the natural frequencies have been determined, the mode shapes are computed from Eq. (6) by setting one of the nodal displacement components to an arbitrary value.

4. Numerical Study

4.1. Free vibration analysis of cracked cantilever beam

To illustrate and validate the method described in previous sections, cantilever cracked beam has been considered. Geometry and material properties are taken from [7] and [11]: Young's modulus E = 216 *GPa,* Poisson's ratio $\nu = 0.28$, density $\rho = 7850$ kg/m^3, beam length L = 20 c*m,* cross-sectional dimensions b = 2.5 c*m,* $h = 0.78$ *cm.* Single crack is located at x = 8 c*m,* as shown in Fig. 2. The first three natural frequencies for undamaged and damaged beam with crack to depth ratio $a/h = 0.6$ are computed and given in Table 1 along with results taken from [7] and [11].

As shown in Table 1, the agreement between results is very good. Associated mode shapes for the first 3 natural frequencies are plotted in Fig. 3.

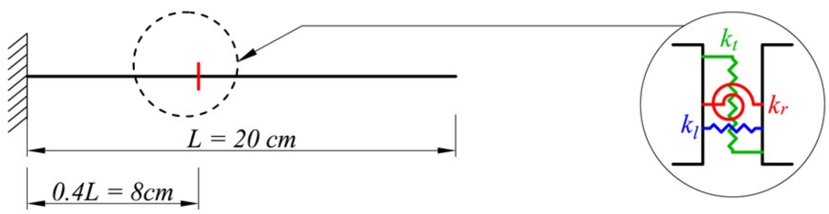

Fig. 2. Considered cantilever beam (left) and three springs to represent crack (right)

	Undamaged			Damaged		
Mode	Present model	[7]	[11]	Present model	[7]	[11]
1	1038.2	1038.2	1038.2	950.1	949.8	985.0
2	6506.4	6506.4	6506.3	5774.9	5768.5	6036.0
3	18218	18218	18218	17078	17015	17447

Table 1. Natural frequencies ω [*rad/s*] of the undamaged and damaged beam

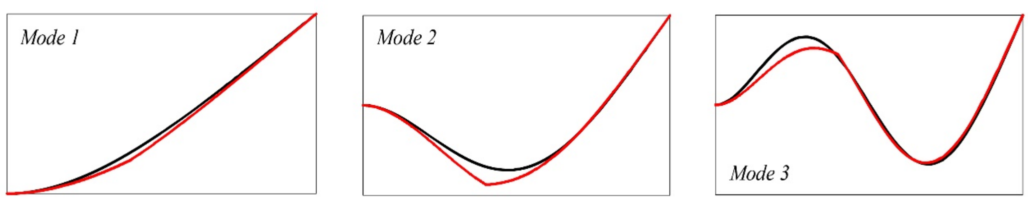

Fig. 3. Mode shapes for the first three natural frequencies of the cantilever beam, obtained using the presented model (black lines – undamaged beam, red lines – damaged beam)

4.2. Free vibration analysis of two-bay damaged frame

The second validation example deals with the free vibration analysis of 2-bay, single storey frame with five discrete cracks, as shown in Fig. 4. Geometrical and material properties of beams and columns are: E = 206 *GPa,* $\nu = 0.28$, $\rho = 7675$ kg/m^3, $b/h = 0.198/0.122$ *m*. Crack to depth ratio is $a/h = 0.9$. The results obtained using the presented theory are compared with the results given in [11] and are presented in Table 2. The first three mode shapes of the undamaged and damaged frame are shown in Fig. 5.

Note that in [11] only rotational spring crack model has been used, which introduces the differences in the results, especially for higher modes. As shown in Table 2, the agreement for the intact frame is excellent.

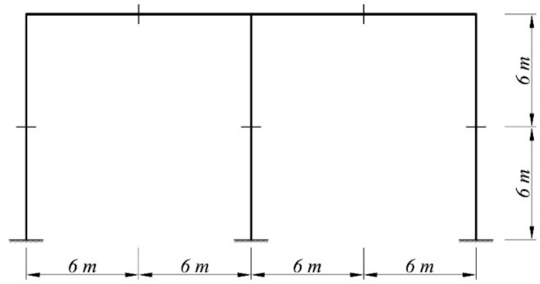

Fig. 4. Two-bay single-storey frame with discrete cracks

	Undamaged			Damaged		
Mode	Presented theory	[11]	[%]	Presented theory	[11]	[%]
1	0.599	0.5987	0.050	0.578	0.5919	2.348
2	2.466	2.4662	0.008	1.232	1.7167	28.234
3	3.108	3.1080	0.000	1.824	2.2836	20.126

Table 2. Natural frequencies ω [Hz] of the undamaged and damaged frame

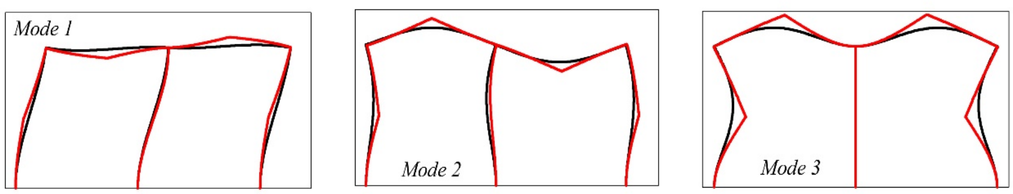

Fig. 5. Mode shapes for the first three natural frequencies of a two-bay single-storey frame, obtained using the presented model (black lines – intact frame, red lines – damaged frame)

4.3. Free vibration analysis of simple-supported beam with multiple cracks

In order to compare the effect of the crack size and number of cracks on the dynamic characteristics of the beam, free vibration of the simple supported beam has been analyzed. The beam is 3 *m* long, with rectangular cross section b/h = 0.2/0.3 *m* (Fig. 6). Material properties are E = 30 *GPa*, ν = 0.2, ρ = 2500 kg/m^3. Several different cases are investigated, starting from undamaged beam. Then, the number of cracks was gradually increasing as shown in Fig. 6. Three different crack-to-depth ratios were used: a/h = 0.1; 0.2; 0.4.

Fig. 7 shows decrease in the first three natural frequencies as the number of cracks increases, where ω_{i0} denotes the i^{th} natural frequency of the undamaged beam. It is noticeable that the crack size has greater impact on the natural frequency reduction than the number of cracks. However, even greater influence has the crack location. As it can be seen in Fig. 7, there is nearly no change in the second natural frequency when the 8th and 9th cracks are introduced. This is because the cracks occur near the middle of the beam span (Fig. 6), where the curvature of the second mode is zero. On the other hand, the highest effect on the frequency reduction has the crack that occurred near the location of the maximum curvature of the corresponding mode shape.

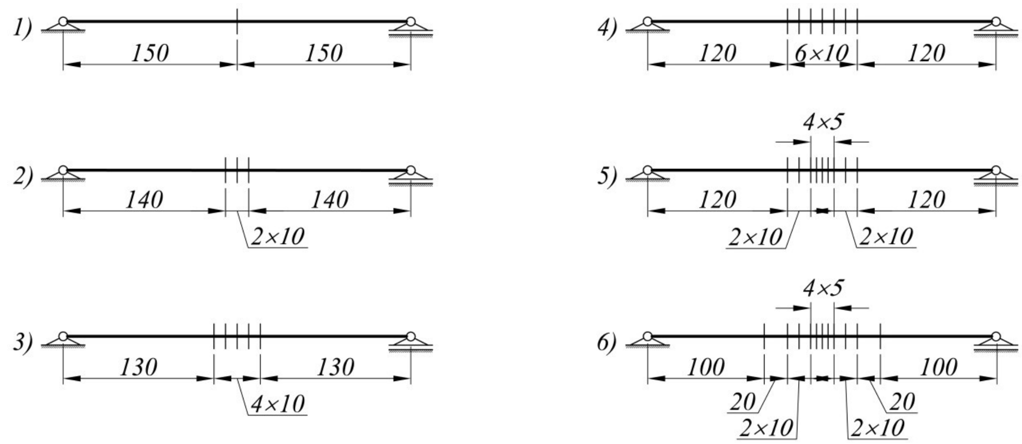

Fig. 6. Considered cases of cracked simple-supported beam

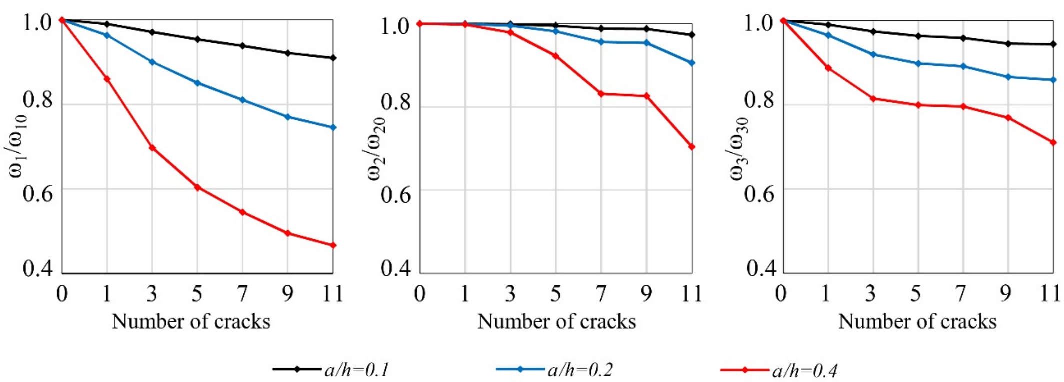

Fig. 7. Frequency reduction curves for the first three mode shapes of damaged simple-supported beam

First three mode shapes of the undamaged and damaged beams are shown in Fig. 8. As previously concluded, the free vibration characteristics of beam are more affected by one deep crack, than by many small cracks. Crack in the middle of the span induces the peak in the first and the third mode shape, but it has no influence on the second mode.

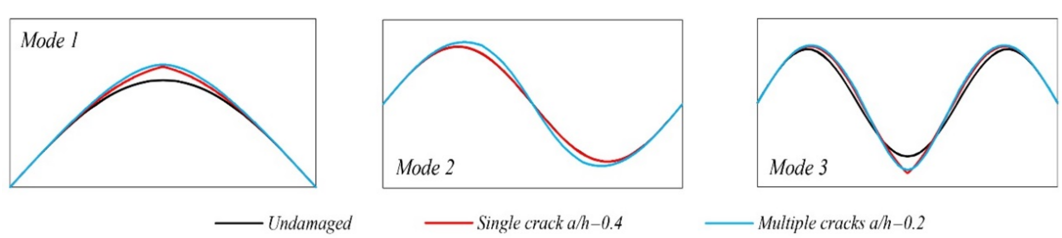

Fig. 8. First three mode shapes of both damaged and undamaged simple-supported beam (multiple cracks = case 6, see Fig. 6)

4.4. Free vibration analysis of frame with multiple cracks

In this example, a single storey frame shown in Fig. 9 has been examined. Material properties are the same as in the previous example, beam cross – section is rectangular with b/h = 0.2/0.3 *m* and columns are of the square cross – section with h = 0.2 *m*. Three different cases of damaged frame have been analyzed with four different crack-to-depth ratios: a/h = 0.1, 0.2, 0.4, and 0.6.

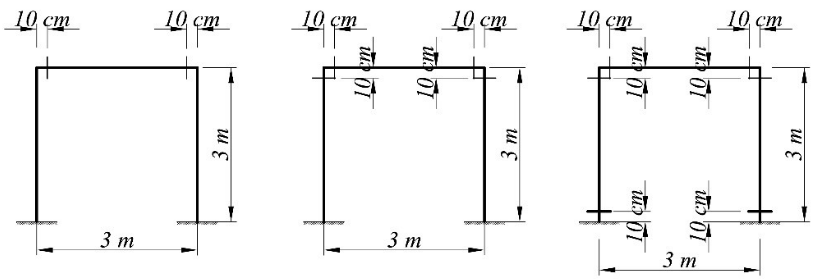

Fig. 9. Single storey frame with different layout of multiple cracks

Natural frequencies for all cases are elaborated in Table 3, while the normalized frequencies are graphically interpreted in Fig. 10 (ω_{i0} denotes the i^{th} natural frequency of the undamaged frame).

It is noticeable that the first four natural frequencies remain almost unchanged for crack to depth ratio 0.1. Slightly greater effect is shown for crack to depth ratio 0.2. Finally, crack to depth ratio 0.6 induces much greater frequency drop – about 45% in the first mode shape and more than 30% in the third and fourth mode shapes. The first four mode shapes for both undamaged and damaged frames (case 3, a/h = 0.6) are presented in Fig. 11. It can be seen that additional cross sectional rotation appeared at the crack location due to the decrease of the bending stiffness of the beam element.

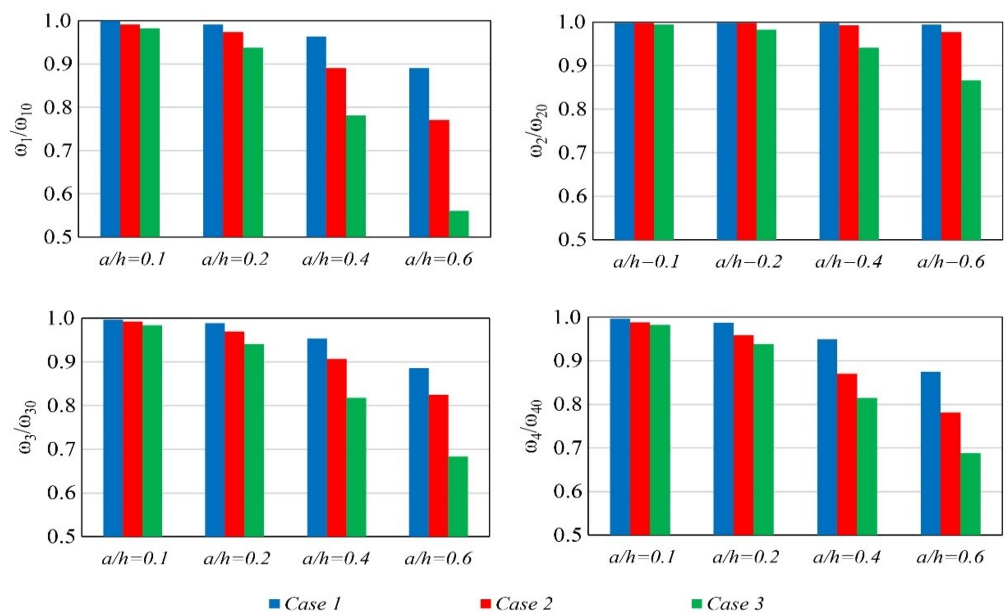

Fig. 10. Normalized natural frequencies of frames with different crack layouts and depths

Mode	Intact	a/h=0.1			a/h=0.2			a/h=0.4			a/h=0.6		
		Case 1	Case 2	Case 3	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
1	10.9	10.9	10.8	10.7	10.8	10.6	10.2	10.5	9.7	8.5	9.7	8.4	6.1
2	52.5	52.4	52.4	52.2	52.4	52.4	51.6	52.4	52.1	49.4	52.2	51.3	45.5
3	79.2	78.9	78.6	77.9	78.3	76.8	74.5	75.5	71.8	64.8	70.1	65.3	54.1
4	84.6	84.3	83.6	83.1	83.5	81.1	79.4	80.3	73.6	68.9	74.0	66.1	58.2

Table 3. Natural frequencies ω [Hz] of the undamaged and damaged single storey frame

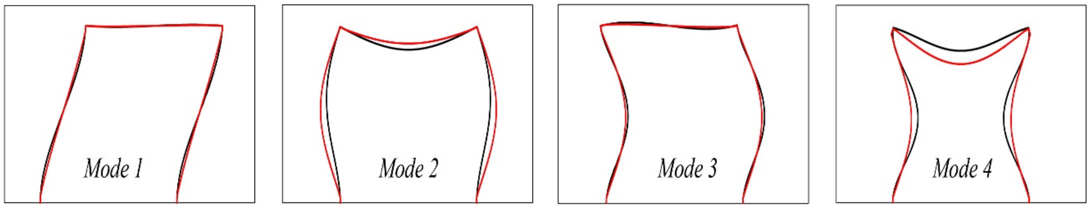

Fig. 11. First four mode shapes of damaged and undamaged frame for *a/h* = 0.6 (black lines – intact frame, red lines – damaged frame)

4.5. Free vibration analysis of two-storey frame with multiple cracks

Two storey frame having the same material and geometrical properties as in the previous example has been analyzed. Cracks are located in the columns as shown in Fig. 12. Four different crack-to-depth ratios have been analyzed: *a/h* = 0.1, 0.2, 0.4 and 0.6. As previously concluded, natural frequencies are not significantly affected by the crack size of 10% or 20% of the beam and column height (Table 4). However, when crack to depth ratio is increased to 0.4 and 0.6, differences between the natural frequencies of the undamaged and damaged frames are not negligible. First two mode shapes are dominantly affected by cracks located in the columns.

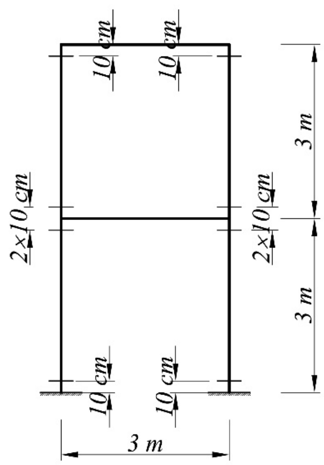

Fig. 12. Two storey frame with discrete cracks

First four mode shapes for damaged frame with cracks having rack to depth ratio *a/h* = 0.6 are graphically presented in Fig. 13. Once again, the difference between the cross – sectional rotations of the beam parts separated by the crack can be noticed.

ω	Undamaged	Damaged *a/h*=0.1		Damaged *a/h*=0.2		Damaged *a/h*=0.4		Damaged *a/h*=0.6	
Mode	[*Hz*]	[*Hz*]	[%]	[*Hz*]	[%]	[*Hz*]	[%]	[*Hz*]	[%]
1	5.710	5.610	1.750	5.410	3.566	4.710	17.513	3.510	38.529
2	16.810	16.510	1.785	15.910	3.633	13.610	19.036	10.110	39.857
3	46.210	46.110	0.216	45.910	0.434	45.010	2.597	43.510	5.843
4	57.910	57.610	0.518	56.710	1.562	53.294	7.971	47.410	18.132

Table 4. Natural frequencies of the undamaged and damaged two storey frame

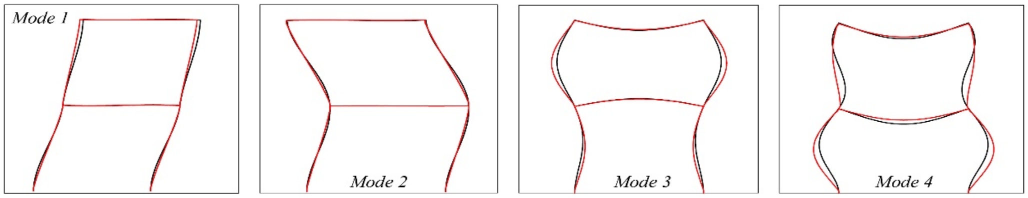

Fig. 13. First four mode shapes for damaged frame for *a/h* = 0.6 (black lines – intact frame, red lines – damaged frame)

5. Conclusions

In order to determine the effect of crack presence in beam and frame structures on the natural frequencies and mode shapes, free vibration of beams and frames with multiple cracks has been analysed. Crack model takes into account bending, axial and shear stiffness reduction of the beam element. Dynamic stiffness matrix of beam element with multiple cracks has been derived. Several numerical examples have been investigated. The results obtained in the first two examples have shown good agreement with the existing data [7, 11].

As expected, increasing the size and number of cracks leads to decrease of natural frequencies associated with the stiffness degradation. In addition, the decrease due to deeper single crack or two cracks is greater than due to multiple smaller ones. The influence of small cracks with length of 10% of cross-sectional height or less on the free vibration is almost imperceptible.

Cracks located at the zero curvature point show no influence on the corresponding natural frequency and mode shape of the beam. Finally, mode shapes can be used in detecting the crack location in frames. Due to bending stiffness decrease, the difference between cross-sectional rotation on the left and the right side of the crack is noticeable.

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Appendix

Dimensionless functions *F* (1,1), *F* (2,2) and *F* (3,3) are:

$$F(1,1) = e^{\frac{1}{1-\xi}}(-0.326584\times10^{-5}\xi + 1.455190\xi^2 - 0.984690\xi^3 + 4.895396\xi^4 - 6.501832\xi^5 + 12.792091\xi^6 - 26.723556\xi^7 + 35.073593\xi^8 - 34.954632\xi^9 + 9.054062\xi^{10})$$

$$F(2,2) = e^{\frac{1}{1-\xi}}(-0.326018\times10^{-6}\xi + 1.454954\xi^2 - 1.455784\xi^3 - 0.421981\xi^4 - 0.279522\xi^5 + 0.455399\xi^6 - 2.432830\xi^7 + 5.427219\xi^8 - 6.643057\xi^9 + 4.466758\xi^{10})$$

$$F(3,3) = e^{\frac{1}{1-\xi}}(-0.219628\times10^{-4}\xi + 52.379034\xi^2 - 130.248317\xi^3 + 308.442769\xi^4 - 602.445544\xi^5 + 939.044538\xi^6 - 1310.950293\xi^7 + 1406.523682\xi^8 - 1067.49982\xi^9 + 391.536356\xi^{10})$$

where $\xi = a/h$ is non – dimensional crack length.

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