UNIVERZITET CRNE GORE
GRAĐEVINSKI FAKULTET
U PODGORICI

PETI INTERNACIONALNI NAUČNO-STRUČNI SKUP

GRAĐEVINARSTVO - NAUKA I PRAKSA

ZBORNIK RADOVA

ŽABLJAK, 17-21. FEBRUARA 2014.

ISBN 978-86-82707-23-3

ZBORNIK RADOVA GNP 2014

Izdavač

UNIVERZITET CRNE GORE
GRAĐEVINSKI FAKULTET

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400 primjeraka

CIP - Каталогизација у публикацији
Национална библиотека Црне Горе, Цетиње

ISBN 978-86-82707-23-3
COBISS.CG-ID 24170256

5. INTERNACIONALNI NAUČNO-STRUČNI SKUP
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ŽABLJAK, 17-21. FEBRUARA 2014.

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TERMIČKA ANALIZA LAMINATNIH KOMPOZITNIH I SENDVIČ PLOČA PRIMENOM SLOJEVITOG KONAČNOG ELEMENTA

Rezime

U ovom radu analiziran je kvazi statički odgovor laminatnih kompozitnih i sendvič ploča ploča izloženih temperaturnom polju linearno promenlivom po debljini i sinusoiudalno promenlijvom u ravni. Matematički model, zasnovan na slojevitom polju pomeranja koje je predložio Reddy [15], formulisan je koristeći pretpostavke o geometrijskoj, materijalnoj i statičkoj linearnosti problema. Princip virtulanih pomeranja je primenjen za dobijanje slabe forme matematičkog modela. Slaba forma je diskretizovana koristeći izoparametarsku aproksimaciju konačnim elementom. Originalan MATLAB računski program je korišćen za analizu uticaja razlicitih debljina ploče na termo-elastičan odgovor laminatnih kompozitnih i sendvič ploča. Tačnost numeričkog modela je potvrđena poređenjem sa rešenjima iz literature.

Ključne reči

Termička analiza, kompozitna ploča, slojeviti model, konačni element

THERMAL ANALYSIS OF LAMINATED COMPOSITE AND SANDWICH PLATE USING LAYERWISE FINITE ELEMENT

Summary

In this paper the quasi static response of laminated composite and sandwich plates subjected to lineary varaying through the thickness and sinusoidall distributed in plane temperature field, is analyzed. Mathematical model, based on layer-wise displacement field of Reddy [15], is formulated using small deflection linear-elasticity theory. The principle of virtual displacements (PVD) is used to obtain the weak form of the mathematical model. The weak form is discretized utilizing isoparametric finite element approximation. The originally coded MATLAB program is used to investigate the influence of plate thickness on thermo-elastic response of laminate composite and sandwich plates. The accuracy of the numerical model is verified by comparison with the available results from the literature.

Key words

Thermal analysis, layer wise model, composite plates, finite element

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1. INTRODUCTION

Multi layered composite structures are widely used in high-temperature environments such as aerospace applications, nuclear reactors and chemical planets, which attracted considerable investigations of researchers in order to meet an optimal design parameters.

In the presently published studies, generally two major problems are have appeared, when studying thermal problem in composite materials. The first is concerned with definition of realistic temperature distribution taking into account the material in use and second is formulating an appropriate material structural model. In other words, homogenization, have to be performed on two scales, the temperature and the structural scale.

On the temperature scale, it is well known that temperature distribution is three-dimensional and time dependent. This demand analyzing all three heat transfer mechanisms, i.e. heat conduction, radiation and convection. The heat conduction problem, described by second Fourier’s law, results in solving nonlinear second order differential equation, in order to find actual nonlinear temperature distribution. Nonlinearity of temperature filed comes from temperature dependent thermal conductivities of multi-layered structure.

On the structural scale, nonlinear temperature field results in geometrically and materially nonlinear response of composite structure. Namely, high temperature regime, accompanied by large deflections and large deformations may cause deviation from initial structural configuration and the change of its geometry. On the other hand, the thermal stresses accompanying the unsteady heating may cause thermal fatigue and considerable plastic strains may lead to complete or progressive destruction of the composite structures.

In order to find the appropriate mathematical model which will meet the requirement of two above mentioned scales, many papers have been reported to date. They generally attend to adjust present Laminated Plate Theories, initially formulated for mechanical loading, to meet the complex nature of temperature field. The key factor in understanding laminate plate deformation due to thermal load is the coefficients of thermal expansion. Namely, the coefficient of thermal expansions for selected orthotropic lamina in transverse direction is much larger than those in the in-plane direction. This implies that standard assumption of constant transverse deflection is acceptable in case of mechanical loading, but may become invalid if the load is thermal. On the stresses level, different thermal expansion coefficients will produce high transverse and normal stresses at the interfaces between adjacent layers, thus a realistic prediction of transverse normal stress and transverse shear stresses become a key issue, in obtaining response of composite plate to thermal load. Depending on how the two mentioned displacement and/or stress field are assumed, mathematical models for thermal analysis of composite laminates have been formulated using three dimensional theory of elasticity, Equivalent Single Layer Theories, Layer Wise Theories or Zig-Zag theories.

Tungikar and Rao [1] obtained 3D elasticity solution for temperature distribution and thermal stresses in simply supported rectangular laminates. The actual temperature distribution across the thickness of the laminate is evaluated by solving the ordinary differential equations (ODEs) of heat conduction without internal heat generation. Savoia and Reddy [2] also solved transient heat conduction equation for exact temperature distribution across the thickness of laminates for 3D stress analysis of symmetric four-layered square laminate subjected to sudden uniform temperature change. Based on the three-dimensional elasticity solution described in the references of Pagano [3], Noor et al. [4] obtained the three-dimensional thermo elasticity solutions to compare with those of the two- and three-dimensional finite element models which considered only the effect of transverse shear stresses. In order to reduce computational cost of three-dimensional theories, and reach acceptable accuracy in the field of application, several solutions of thermal problems in composites have been proposed using Equivalent Single Layer, which are Classical Laminated Plate Theory(CLPT), First-order Shear Deformation Theory (FSDT) and Higher-order Shear Deformation Theory (HSDT).

Using the CLPT based on the Kirchhoff hypothesis, Wu and Tauchert [5] studied the thermal deformation and stress results in symmetric and antisymmetric laminates. However, this kind of approach is inaccurate for laminated plates with relatively soft transverse shear modulus. The inaccuracy is due to neglecting the transverse shear and normal strains in the composite laminates.

To take into account the effects of low ratio of transverse shear modulus to the in-plane modulus, the FSDT has been developed by Rolfes et al. [6]. Due to assumed displacement field, the transverse shear strains are constant through the plate thickness, obtained by the direct constitutive equation approach and the shear correction factors have to be adopted. However, the shear correction factors are not easy accurately to predict and a number of global higher-order shear deformation theories have been developed for thermal analysis.

Kant and Khare [7] developed a simple Co isoparametric finite element displacement model based on HSDT formulations for the analysis of symmetric and antisymmetric laminates subjected to thermal gradient. Rohwer et al. [8] removed the deficiencies in the FSDT by incorporating third and fifth order displacement approximations through the plate thickness. In order to study the effects of the higher-order displacement terms on thermal stresses Matsunaga [9] proposed a ninth-order theory for analysis of thermal behaviors of laminated composite and sandwich plates, in which the in-plane displacement field consists of a ninth-order polynomial in global thickness coordinate z whereas the transverse deflection is represented by an eighth-order polynomial of global coordinate z. Moreover, the ninth-order theory is extended to compute critical temperatures in cross-ply laminated composite plates and shells subjected to thermal loads.

Although results of the global ESL plate theories may be sufficiently accurate for predicting global responses, their results may exhibit considerable errors in predicting the local behavior. In such cases, layerwise plate theories have been proposed to over-come some of the shortcomings of the equivalent single layer theories [10]. The layer-wise theories assume separate field expansions within each layer, thus being able to more accurately predict plate behavior due to complex, generally nonlinear temperature filed. However, as the number of layers in the laminates increase, layerwise models become computationally expensive for global laminate analysis.

In order to retain the benefits of less computational times of the equivalent single-layer theories and the accuracy of the layerwise theories, zigzag theories with linear or high-order layerwise zigzag functions were proposed. By imposing the continuity conditions of the transverse shear stresses, the number of unknowns in these models are independent on the number of the layers. Kapuria and Achary [11] presented a third-order zig-zag theory for the sandwich plates. Ganapathi et al. [12] investigated the nonlinear dynamics behavior of the sandwich plates based on a zigzag theory. In addition to the contact conditions on the interlaminar shears, those on the transverse normal stress and stress gradient have been also fulfilled.

Since, there are so many papers on thermal analysis using layerwise plate theories, and limited 3D elasticity solutions are available, in this paper a layerwise finite element for thermo elastic analysis is formulated. The quasi static thermo elastic problem with linear through the thickness and sinusoidal in plane variation of the temperature field is studied. The temperature field is incorporated into the layerwise mechanical material model, which assumes layerwise variation of in-plane displacements and constant transverse displacement. With assumed displacement field, linear strain displacement relations and linear thermo mechanical materials properties, the governing equilibrium equations are formulated using principle if virtual displacements (PVD) and weak form is obtained. The weak form is discretized using isoparametric finite element approximation. The original MATLAB program is coded and used to investigate the influence of different side to thickness ratio on thermo elastic response of laminated composite and sandwich plates. The accuracy of the numerical model is verified by comparison with the available results from the literature.

2. THEORETICAL FORMULATION

2.1 DISPLACEMENT FIELD

In the LW theory of Reddy [15] in-plane displacements components (u,v) are interpolated through the thickness using 1D linear Lagrangian interpolation function $\Phi^I(z)$, while transverse displacement component w is assumed to be constant through the plate thickness.

$$
\begin{aligned}
u_1(x,y,z) &= u(x,y) + \sum_{I=1}^{N+1} U^I(x,y)\cdot\Phi^I(z) \\
u_2(x,y,z) &= v(x,y) + \sum_{I=1}^{N+1} V^I(x,y)\cdot\Phi^I(z), \\
u_3(x,y,z) &= w(x,y)
\end{aligned}
\tag{1}
$$

2.2 STRAIN-DISPLACEMENT RELATIONS

The Green Lagrange strain tensor associated with the displacement field Eq.(1) is computed using linear strain-displacement:

$$\varepsilon_{xx} = \frac{\partial u_1}{\partial x} = \frac{\partial u}{\partial x} + \sum_{I=1}^{N+1} \frac{\partial U^I}{\partial x}\Phi^I ,$$

$$\varepsilon_{yy} = \frac{\partial u_2}{\partial y} = \frac{\partial v}{\partial y} + \sum_{I=1}^{N+1} \frac{\partial V^I}{\partial y}\Phi^I ,$$

$$\gamma_{xy} = \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \sum_{I=1}^{N+1}\left(\frac{\partial U^I}{\partial y} + \frac{\partial V^I}{\partial x}\right)\Phi^I , \tag{2}$$

$$\gamma_{xz} = \frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x} = \sum_{I=1}^{N+1} U^I \frac{d\Phi^I}{dz} + \frac{\partial w}{\partial x} ,$$

$$\gamma_{yz} = \frac{\partial u_2}{\partial z} + \frac{\partial u_3}{\partial y} = \sum_{I=1}^{N+1} V^I \frac{d\Phi^I}{dz} + \frac{\partial w}{\partial y} .$$

With linearly assumed strain displacement kinematics, an orthotropic linear Hook's material is assumed for each lamina, to formulate constitutive equations as:

$$\{\boldsymbol{\sigma}\}^{(k)} = [\mathbf{Q}]^{(k)} \cdot \left(\{\boldsymbol{\varepsilon}\}^{(k)} - \{\alpha\}^{(k)} \Delta T\right). \tag{3}$$

where $\boldsymbol{\sigma}^{(k)} = \{\sigma_{xx}\ \sigma_{yy}\ \tau_{xy}\ \tau_{xz}\ \tau_{yz}\}^{(k)T}$ and $\boldsymbol{\varepsilon}^{(k)} = \{\varepsilon_{xx}\ \varepsilon_{yy}\ \gamma_{xy}\ \gamma_{xz}\ \gamma_{yz}\}^{(k)T}$ are stress and strain components respectively, $\mathbf{Q}_{ij}^{(k)}$ and $\alpha^{(k)} = \{\alpha_{xx}\ \alpha_{yy}\ \alpha_{xy}\ \ 0\ \ 0\}^{(k)T}$ are transformed reduced elastic stiffness [16] and coefficients of thermal expansion, of k-th lamina in global coordinates, while ΔT is prescribed temperature field.

2.3 EQUILIBRIUM EQUATIONS

Using assumed displacement field, linear strain displacement relations and Hook's linear material, the principle of virtual displacements is employed to derive the governing equilibrium equations:

$$\begin{aligned}\delta\pi = \int_{\Omega} \Bigg\{ & N_{xx}\frac{\partial \delta u}{\partial x} + \sum_{I=1}^{N} N^I{}_{xx}\frac{\partial \delta U^I}{\partial x} + N_{yy}\frac{\partial \delta v}{\partial y} + \sum_{I=1}^{N} N^I{}_{yy}\frac{\partial \delta V^I}{\partial y} + N_{xy}\left(\frac{\partial \delta u}{\partial x} + \frac{\partial \delta v}{\partial y}\right) + \\ & \sum_{I=1}^{N} N^I{}_{xy}\left(\frac{\partial \delta U^I}{\partial y} + \frac{\partial \delta V^I}{\partial x}\right) + Q_x\frac{\partial \delta w}{\partial x} + \sum_{I=1}^{N} Q^I{}_x\ \delta U^I + Q_y\frac{\partial \delta w}{\partial y} + \sum_{I=1}^{N} Q^I{}_y\ \delta V^I \Bigg\} dxdy = 0\end{aligned} \tag{4}$$

where internal force vectors are:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \sum_{k=1}^{n}\int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz , \quad \begin{Bmatrix} N^I{}_{xx} \\ N^I{}_{yy} \\ N^I{}_{xy} \\ Q^I{}_x \\ Q^I{}_y \end{Bmatrix} = \sum_{k=1}^{n}\int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} \left\{\Phi^I\ \ \Phi^I\ \ \Phi^I\ \ \frac{d\Phi^I}{dz}\ \ \frac{d\Phi^I}{dz}\right\} dz \tag{5}$$

3. FINITE ELEMENT MODEL

The governing equations (4) are discretized using GLPT finite element [16]. Over each element, the displacements are expressed as linear combination of shape functions and primary nodal variables as:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}^e = \begin{Bmatrix} \sum_{j=1}^{m} u_j \Psi_j \\ \sum_{j=1}^{m} v_j \Psi_j \\ \sum_{j=1}^{m} w_j \Psi_j \end{Bmatrix}^e = \sum_{j=1}^{m} \left[\boldsymbol{\Psi}_j\right]^e \left\{\mathbf{d}_j\right\}^e \quad \begin{Bmatrix} U^I \\ V^I \end{Bmatrix}^e = \begin{Bmatrix} \sum_{j=1}^{m} U_j^I \Psi_j \\ \sum_{j=1}^{m} V_j^I \Psi_j \end{Bmatrix}^e = \sum_{j=1}^{m} \left[\overline{\boldsymbol{\Psi}}_j\right]^e \left\{\mathbf{d}_j^I\right\}^e \tag{6}$$

where $\{\mathbf{d}_j\}^e = \{u_j^e \; v_j^e \; w_j^e\}^T, \{\mathbf{d}_j^I\}^e = \{U_j^I \; V_j^I\}^T$ are displacement vectors, in the middle plane and I-th plane, respectively, Ψ_j^e are interpolation functions, while $\lfloor \boldsymbol{\Psi}_j \rfloor^e, \lceil \overline{\boldsymbol{\Psi}}_j \rceil^e$ are interpolation function matrix for the j-th node of the element Ω^e, given in [16].

Substituting element displacement field Eq.(6) in to governing equation (4), the following finite element equations are obtained:

$$[\mathbf{K}]^e \cdot \{\mathbf{d}\}^e = \{\mathbf{f}\}^e \tag{7}$$

where stiffness matrix is given in [16], while force vectors are:

$$\{\mathbf{f^0}\}^e = \iint_{\Omega^e} [\mathbf{H}_i^e]^T \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} [\mathbf{Q}]^{(k)} \{\alpha\}^{(k)} \Delta T dz \, dx \, dy$$

$$\{\mathbf{f^I}\}^e = \iint_{\Omega^e} [\overline{\mathbf{H}}_i^e]^T \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} [\mathbf{Q}]^{(k)} \{\alpha^{\mathbf{I}}\}^{(k)} \Delta T \, dz \, dx \, dy \tag{9}$$

where $\{\alpha^{\mathbf{I}}\}^{(k)} = \{\alpha_{xx}\Phi^I \;\; \alpha_{yy}\Phi^I \;\; \alpha_{xy}\Phi^I \;\; 0 \;\; 0\}^{(k)}$.

4. EXAMPLE

To validate the results of the present theory the following material properties of laminated composite and sandwich plates have been used:

Material (1) laminated plate 0/90/0:

$$E_L / E_T = 25, G_{LT} / E_T = 0.5, G_{TT} / E_T = 0.2, \nu_{LT} = \nu_{TT} = 0.25, \alpha_T / \alpha_L = 1125 \tag{10}$$

Material (2) laminated plate 0/90:

$$E_L / E_T = 15, E_T = 10\,\text{GPa}, G_{LT} / E_T = 0.5, G_{TT} / E_T = 0.3356, \nu_{LT} = 0.30, \nu_{TT} = 0.49, \alpha_L / \alpha_0 = 0.015, \alpha_T / \alpha_0 = 0.015 \tag{11}$$

Material (3) sandwich plate 0/core/0:

(a) Face sheets (h/5 x 2)

$$E_L = 200\,GPa,\ E_T = 8\,GPa,\ G_{LT} = 5\,GPa,\ G_{TT} = 2.2\,GPa,\ \nu_{LT} = 0.25,\ \nu_{TT} = 0.35,$$
$$\alpha_L = -2\times10^{-6}\,/K,\ \alpha_T = 50\times10^{-6}\,/K \tag{12}$$

(b) Core material (3h/5)

$$E_L = 1\,GPa,\ E_T = 2\,GPa,\ G_{LT} = 0.8\,GPa,\ G_{TT} = 3.7\,GPa,\ \nu_{LT} = 0.25,\ \nu_{TT} = 0.35$$
$$\alpha_L = \alpha_T = 30\times10^{-6}\,/K$$

The plates with simply supported boundary conditions are studied subjected to temperature field:

$$\Delta T(x,y,z) = \frac{2T_0}{h} z \cdot \sin\left(\frac{\pi x}{a}\right) \cdot \sin\left(\frac{\pi y}{b}\right) \tag{13}$$

Transverse displacement is given in the following nondimensional form: $\overline{w} = \dfrac{w}{h\alpha_L T_0 (a/h)^2}$ for plate 090/0, and $\overline{w} = \dfrac{w}{h\alpha_L T_0}$ for laminated plate 0/90 and sandwich plate. Results of the present model are compared with available ones from the literature and are given in Table 1.

Table 1. Normalized transverse displacement for laminated composite and sandwich plates subjected to thermal loading

	$\overline{w}$			
	a/h	Present	3D[3]	HSDT
0/90[13]	5 10 20	1.138 4.317 15.423	0.997 4.120 16.600	1.070 4.171 16.567
0/90/0[14]	2 4 10	100.463 44.608 16.014	98.121 42.020 16.890	96.790 42.690 17.390
0/core/0[9]	4 8 12	10.097 18.050 23.350	9.816 19.870 26.960	8.793 18.780 25.640

5. CONCLUSION

In this paper laminated layerwise finite element model for thermo elastic analysis of laminated composite and sandwich plates is presented. Finite element solution is incorporated into an original MATLAB computer program, which is used to analyze laminated composite and sandwich plates with different lamination schemes and different plate thickness. Results of the analysis have shown that the present layerwise model gives acceptable results for laminated composite plates, while the error is increased for sandwich plates, when compared to three dimensional and higher order plate theories. The reason for this may be neglecting transverse normal strains, which become an important factor especially for highly anisotropic sandwich plates subjected to thermal environment. Finally, the presented results have shown the applicability and constrains of present layerwise finite element model for thermal load analysis, which may serve as a benchmark for future investigations.

Acknowledgements The authors are thankful for the financial support received by the Ministry of Education and Science of the Republic of Serbia, Project TP 36048.

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