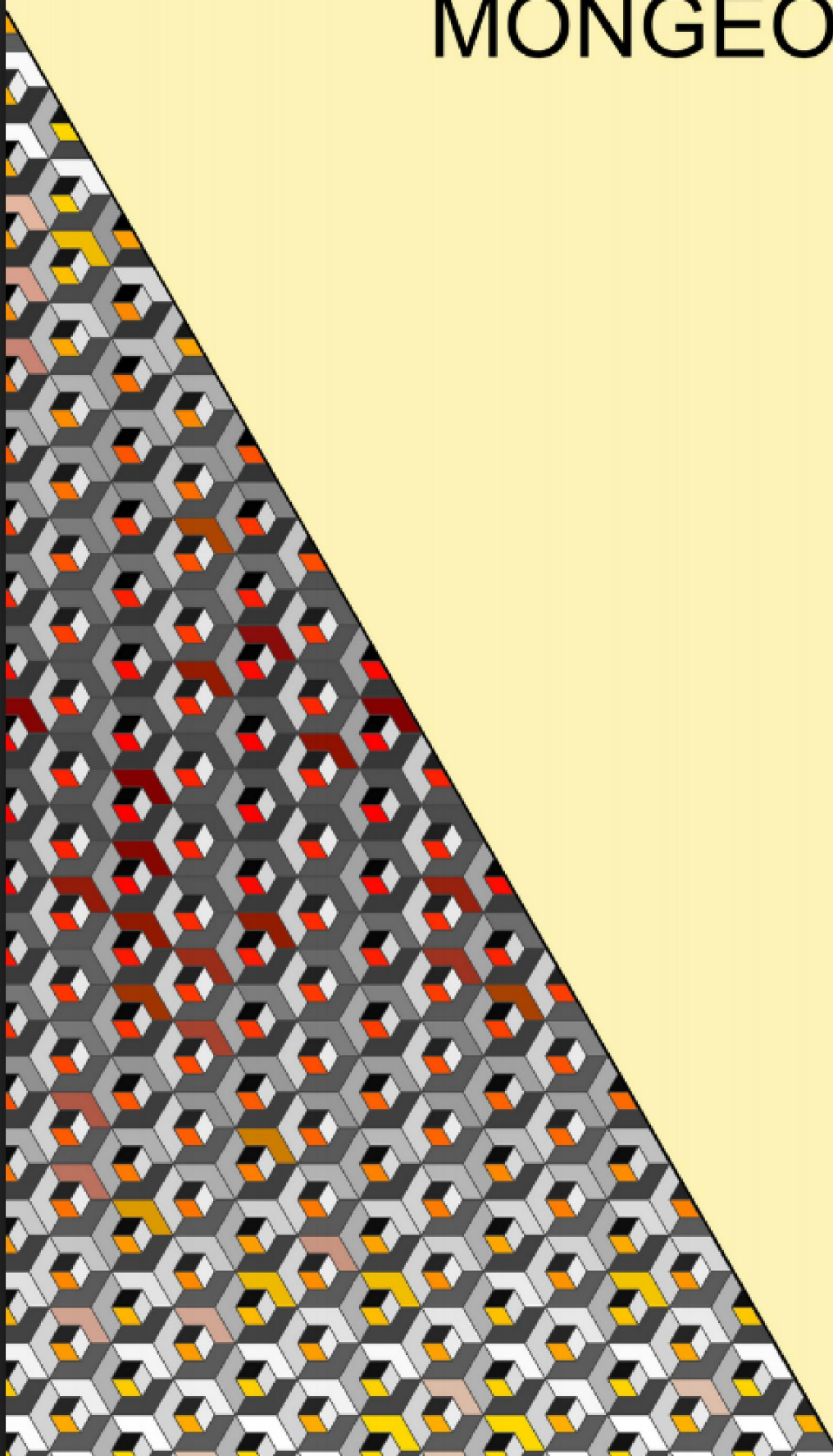


6th International Conference
on Geometry and Graphics

MONGEOMETRIJA 2018

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GEOMETRY BEHIND THE POSITION OF STAIRS: BALANCE IN THE MIND

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Abstract

In this paper we examine the link between the human body and architectural design. For this purpose, we combine Cognitive Metaphor Theory (CMT) and geometry. We start from the hypothesis that the organization of space and the positions of stairs within the building floor plan is the result of the embodiment of human mind, which implies that abstract aspects of human thought are grounded in physical aspects of human body. CMT claims that BALANCE is a prototypical schema consisting of countervailing forces acting on a target. This mental structure arises from living in symmetrical bodies which are able to balance two equal halves and maintain erect posture. Having established the metaphorical mapping between the building and the human body (and, consequently, between the stairs and human spine) we examined the position of stairs in the building. We geometrically analyzed floor plans of a set of buildings applying the theoretical framework of Projective geometry and methods of Computational geometry. Our findings reveal that stairwells tend to stretch along the line which divides the building basis in two equal halves.

Keywords: conceptual metaphor, balance, centre of gravity, area bisector, symmetry, stairs.

1 INTRODUCTION

In this study we use geometrical analysis to explore the link between building design and conceptual metaphor. Focusing on a single element – the stairway – we will examine the geometrical aspect of the position of stairs using the theoretical framework of Cognitive Metaphor Theory (CMT).

On one hand, the staircase, as an architectural element, has a role that is purely functional: its practical purpose is to connect two floors. On the other hand, it often occupies a central position in the building, which suggests that designers tend to assign it a particularly prominent role. This is understandable, given its geometric (and design) specificities in comparison to the other elements of the building, and its unique character of a guide through its levels. Furthermore, in terms of its symbolism, the staircase is generally associated with upward motion. Across cultures it symbolically represents a climb towards knowledge, deep cognition and transformation [4]. This indicates that there is a deeper meaning to this architectural element. Seeking to explore the profound sense of importance associated with the stairs, in this paper we look at this architectural part of the building through the prism of conceptual metaphors. It is our aim to establish what lies behind the central position of the stairs in a building.

Cognitive (or Conceptual) Metaphor Theory (CMT) postulates that metaphor is more than a poetic device. According to cognitive linguists it resides both in the language and the mind, as a tool humans use to understand and interpret the world. Metaphor is ubiquitous and is expressed not only in the language, but also in mathematics [3], gesture, drawings and objects (including architecture [8]). CMT defines metaphor as understanding one thing (typically more abstract and diffuse) in terms of another (usually concrete) [10]. This link between the concrete (or source) and abstract (or target) domains is established as a set of source-to-target mappings. Source domains include concrete, physical, or tangible concepts (such as plants, food, or journey), which are used to make sense of abstract concepts (like organizations, time, or emotions). In other words, human experiences with the physical world are used as a natural and logical starting point for the comprehension of more abstract domains. Conventionally, conceptual domains in CMT are given in small capitals (e.g. FOOD, TIME), while metaphorical links between source and target domains are formulated as X IS Y (e.g. LOVE IS A JOURNEY).

To illustrate, let's use the metaphor TIME IS SPACE. TIME is a domain of abstract experience which is difficult to fathom in itself. CMT postulates that in metaphorical understanding, it becomes the target domain which acquires the structure of a more familiar, concrete domain – SPACE. Human experience

with spatial movement is rich, deep and broad. Furthermore, it is predominantly accompanied by some time lapse: as we move through the physical world, we observe changes of dawn into daylight, day into night. Movement through space and passing of time become experientially associated in the mind, and linguistic realizations of this metaphorical mental link are numerous: we leave the past *behind*, wait for future events (that lie *ahead*) to come, time has certain *length*, if we are early/late, we are *ahead of/behind* time. Metaphors like this one, CMT claims, are what makes abstract conceptualization and reasoning possible.

When it comes to architecture, there are several aspects that have been examined through the lens of CMT. Goatly [8] interprets the tower of Babel, an edifice whose height threatened the deity, and Twin Towers of WTC, as respective realizations of metaphors IMPORTANT IS UP and POWER IS UP. In addition, tall buildings, which defy gravity and annihilate restrictions of natural physical movement, can be seen as realization of the metaphor FREEDOM IS A SPACE TO MOVE.

Some source domains, however, are image schematic [5]. Image schemas (e.g. SCALE, LEFT-RIGHT, CENTRE-PERIPHERY, CONTAINER, PART-WHOLE or BALANCE) are mental structures arising from perception, bodily movements, manipulation of objects, or experience of force. They are the result of early (pre-verbal) observations of how external objects and our own bodies interact with the world. In time, these patterns develop into an indispensable tool for structuring less concrete, or even entirely abstract aspects of experience. BALANCE image schema, for example, structures mental and psychological states (emotional *imbalance*), argumentation (*equal weight* of arguments), or justice (*restoring* the balance *disturbed* by offender's behaviour). What we aim to explore here is its link with architectural design.

Evidence that humans tend to transfer their symbols and metaphors onto blueprints lies scattered over the history of mankind. From ancient civilizations to present-day, the stairway was a necessary construction element which made it possible to master the distance between the different interior levels of a building. Its symbolism, as we have mentioned above, revolves around upward spiritual movement. It represents ascent to knowledge, which may pertain to the visible, natural world or the realm of the divine (including the occult) [4].

The position of the stairway is often such that it serves an esthetic and symbolic purpose, in addition to the functional. This may be attributed to the fact that the stairwell is an instantly recognizable, purpose-built space, whose function is difficult to change. Except for some, predominantly recent, constructional regulations, there are no strict conventions determining the position of the stairway in the floor plan of a building. If there were clear rules (other than those pertaining to the functionality of the interior) governing this area, we would be able to predict the exact position of the stairs with absolute certainty. Although this is not possible, there is, as will be demonstrated in the following sections, a pronounced tendency to position the stairs so that they 'maintain' the balance of the areas in relation to the building's floor plan.

2 GEOMETRICAL EVIDENCE OF METAPHORICITY OF STAIRS IN ARCHITECTURE

Stairs are often positioned in the central zones of the building – quite understandably, given the practical need to minimize the distance between them and some remote corners. Furthermore, stairs are commonly associated with balance, proportion and climbing. But, why is this so? In our opinion, it can be explained by means of the BALANCE image schema. Image schemas are abstract conceptual representations that derive from sensory and perceptual experience. They structure our bodily experience, as well as our nonbodily experience – via metaphor. Cognitive linguistics claims that BALANCE schema is a prototypical schema consisting of countervailing forces acting on a target. It is based on human bodily experience, such as maintaining an erect posture. Stairs, as a physical manifestation of the BALANCE schema, should consequently occupy a central position within a structure, so that the rest of the structure is composed around them.

In terms of geometry, this implies that stairs should be located on the line which metaphorically represents the backbone of the building. The axis of symmetry, gravity line (plumbline) and bisectors, each in its own way – depending on the case – divide the basis in equal halves and guarantee that the balance between the sections is established. In terms of human anatomy, the backbone or spine, which allows us to achieve and maintain the standing position, is a column that follows the axis of symmetry and the gravity line of the whole body (Fig. 1).

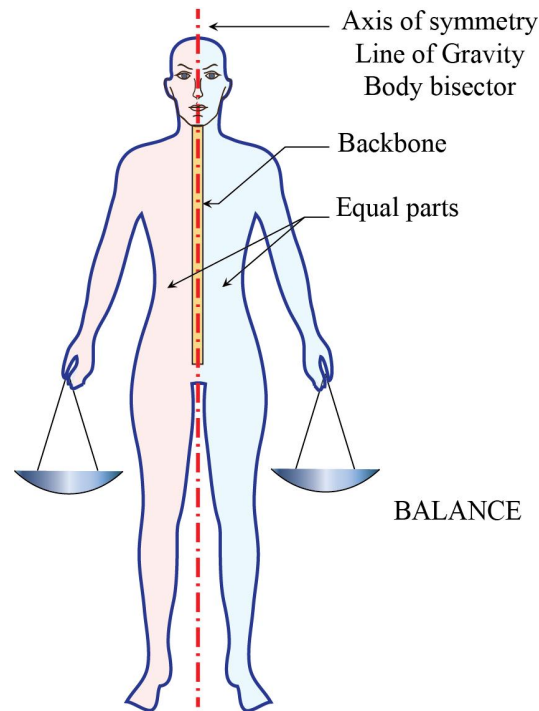


Fig. 1. Human body symmetrical to the backbone maintaining the BALANCE of the equal parts

The mapping between the stairwell and the backbone can be interpreted as the result of metaphorical conceptualisation of a building as human body (BUILDING IS BODY). Although this metaphor has not been explored in CMT, it is not difficult to find linguistic evidence for it. In English, buildings *stand* or *sit* on streets, they are *tall* or *erect*, they *face* other buildings, their windows *look out on* or *overlook* the outside world. Furthermore, the mental metaphorical mapping between the stairwell and the backbone is realized not only in linguistic, but in visual metaphors, as well. Some authors, including the English architect Phillip Watts [15], [26], have expressed this metaphor through the very form of the staircase. Figure 2 presents an example of a sculptural staircase (for a private client) in Northampton, England.

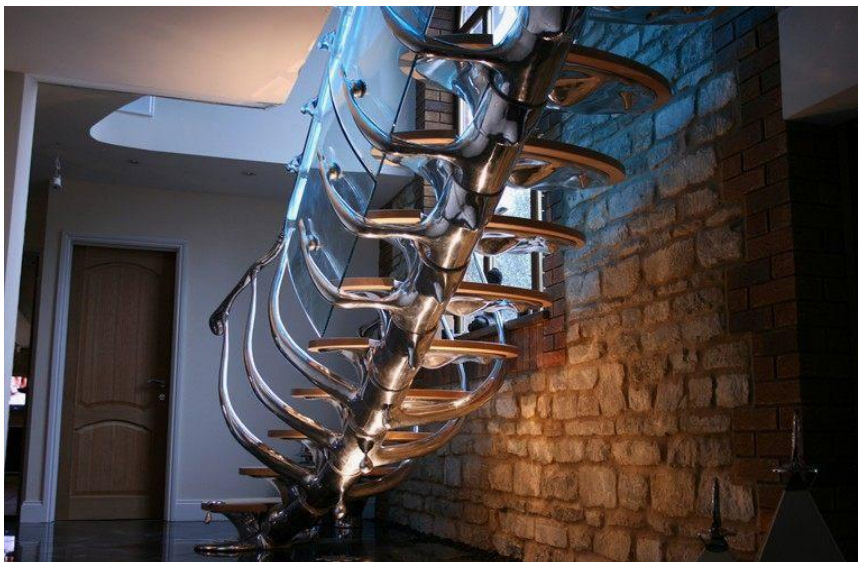


Fig. 2. An example of metaphorical mapping between the backbone and a staircase in a sculptural design of Phillip Watts [26]

As we have already pointed out, CMT claims that BALANCE is a prototypical schema consisting of countervailing forces acting on a target. Stairs, as a physical manifestation of the balance schema,

should consequently be positioned so as to mimic the support of the entire building, i.e. their position in regard to the building's floorplan should correspond to its static stability. To examine this, we hypothesize that their position is in most cases such that it relates to establishing the balance of the base, often dividing it in half. It can be symmetrical, centrally or axially, and where the axis of symmetry is absent, the staircases tend to be on the line that divides the building basis in halves, thereby establishing the balance between the parts of the building either side of this line.

2.1 The Geometrical Basis of the Procedure

As one of the fundamental aesthetic concepts, symmetry has always been an important element in the design of the architectural space [9]. Stairs, consequently, readily comply with the requirements of symmetrical structure. So, when the base of the building is symmetrical, one is led both by intuition and by experience¹ to expect to find the staircase either on the axis of symmetry of the floor plan, or if there are more stairs, in bilaterally symmetrical positions in relation to the axis. The symmetry axis, by its definition, divides the area of the planar figure in two equivalent, identical halves that reflect each other. There are numerous examples of the staircases in the buildings of the symmetrical bases, especially public ones, positioned so that the axis of symmetry passes through the stairwell.

Naturally, such a model of organization is not a universal rule, because the design process is open and creative. It can produce unconventional solutions, just like art itself tends to. However, building design is more than a creative act. It requires successful solutions pertaining to the organization of space, so that future users do not have difficulties getting around the building and staying oriented inside it. Consequently, the very process of design is also a quest for the balance between artistic and aesthetic requirements on the one hand and functional and engineering on the other. This leads us to the following question: what determines the position of the stairs in those buildings whose bases are not symmetrical, as well as in those with symmetrical basis in which the stairs do not lie on the axis of symmetry? In this study we tried to examine, using geometric analysis, whether there is a pattern which the positions of stairs follow and, if so, how this pattern is related to the BALANCE schema. In other words, we hypothesized that if there is a metaphorical mapping between the stairwell of a building and the backbone of the human body, the position of the stairs will reflect the BALANCE schema – it will be on the lines dividing the areas into equal parts. Apart from the axis of symmetry, such lines also include gravity lines and area bisectors.

Although gravity lines and area bisectors are easily confusable, geometrically, there is a difference between them, most obvious on polygons with an odd number of sides [24]. The centre of gravity of the planar figure (polygon) is the arithmetic mean position of all the vertices of the polygon. We can set an infinite number of lines of gravity through the centre of gravity, but not all of them will divide the area of the polygon into equal halves (Pancake theorem, 2-dimensional case of Ham Sandwich Theorem [11]). Area bisectors are the lines that divide the polygon into equal halves and do not necessarily intersect at the centre of gravity [7], [24]. In the rectangle or rhombus, e.g. the area bisector will pass through the centroid of the polygon; it can be a diagonal, a median line, or another line passing through the centroid, as well as the center of mass, or the center of gravity. However, for the polygons with the odd number of sides, such as a triangle, the situation will be different (Fig. 3).

As one can notice, area bisectors do not necessarily pass through the centroid. To illustrate this more clearly, we will present a short explanation of how gravity lines and area bisectors are determined.

In regular polygons, the centre of gravity (G) overlaps with the centroid (C): the point in which median lines intersect. In more complex polygons, this procedure can be conducted in different ways, from the Archimedes procedure, which uses the plumbines [1], to the procedure [12] which is also based on Archimedes Lema [2], which utilizes lines of intersection of local polygons (Fig. 4 a). To simplify the procedure and avoid painstaking constructions to identify the centre of gravity, we applied the Computational geometry method and identified the centre of mass via AutoCAD application, and its MASSPROPP function (Fig 4. b).

All the lines that pass through the centre of gravity are gravity lines, and they are infinite in number. Some of them will divide the surface of the polygon in two equal sections. The shape of the polygon determines which and how many of them will do so. The axes of symmetry are also gravity lines. However, not all gravity lines are axes of symmetry.

¹ Statistically, this is almost impossible to prove, because of the excessive number of buildings (worldwide and throughout history), but empirically it is almost implied.

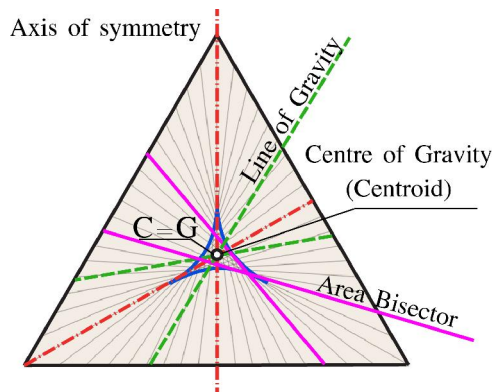


Fig. 3. a) Axis of symmetry, lines of gravity and the area bisectors inside the polygon (the case of a triangle) As one can notice, area bisectors do not necessarily pass through the centroid. To illustrate this more clearly, we will present a short explanation of how gravity lines and area bisectors are determined.

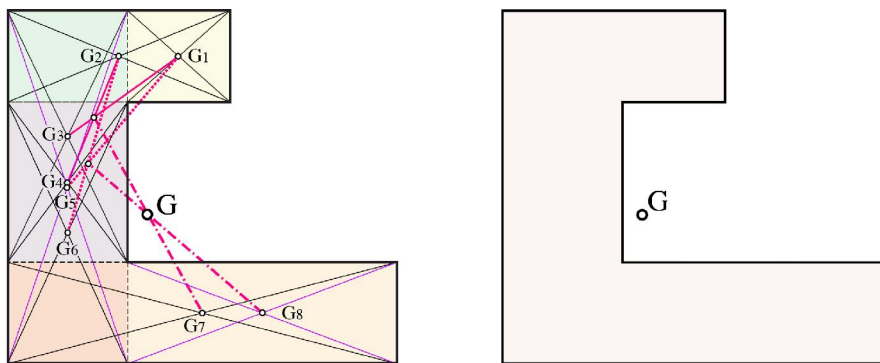


Fig. 4. a) Archimedes Procedure for Finding the Centre of Gravity for the Composite Polygon [12],
 b) The Centre of Gravity Found by AutoCAD Application, Operation: MASSPROPP

As for area bisectors, in terms of construction their identification is more complex than the identification of the centre of gravity and gravity lines. In the case of the triangle, the bisector is a line that is tangent to one of the three hyperbolas, whose asymptotes are determined by the straight lines which contain the sides of the triangle [7]. The hyperbolas are also tangential to the triangle's median lines. If we have the triangle ABC, we can apply the constructive procedures of Projective geometry, the Brianchon's Theorem [6] (Fig. 5), to easily construct tangential hyperbolas for all area bisectors. A bisector can be determined by a given point (P) within or outside of the triangle, as well as by the division's direction. For the points outside of the triangle, there will be only one bisector for a given point. For the points within the triangle, there can be more than one, which depends on the position of the point within the triangle. The area bisector can, in a special case, coincide with the gravity line or the axis of symmetry.

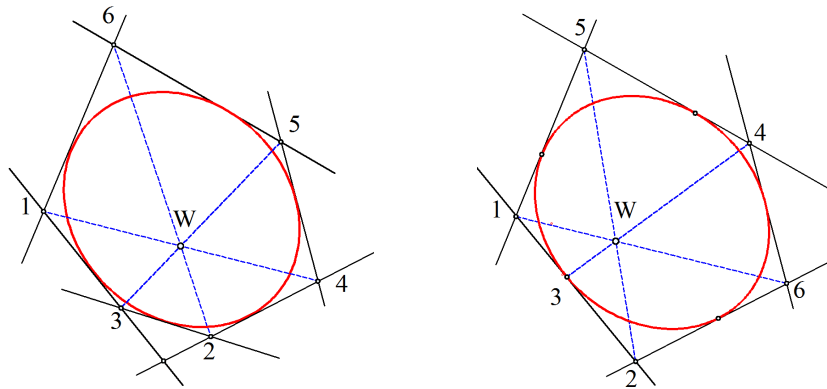


Fig. 5. Graphical procedure based on Brianchon-s Theorem for finding curve's touching point

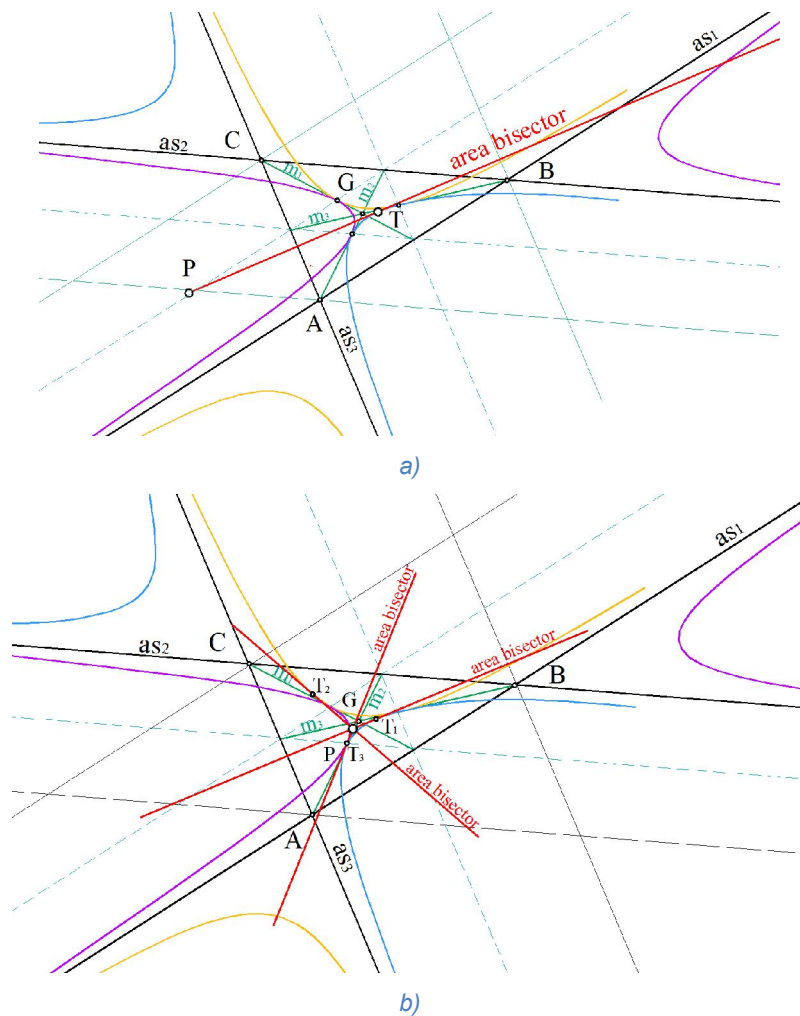


Fig 6. Finding area bisector for the triangle ABC: a) for the given point P outside the triangle ABC, b) for the given point P inside the triangle ABC

An interactive demonstration of this problem is available on several web-pages, such as Wolfram [30] or GeoGebra [21]. Similarly to the centre of gravity (G) determination, instead of the cumbersome construction (which is partly presented in Fig. 6 for reasons of conciseness), in this study we applied AutoCAD LISP utility *SplitArea* [19]. This method produces precise results for area bisectors of a closed polygon, be they determined by a point of by a direction (Fig. 7).

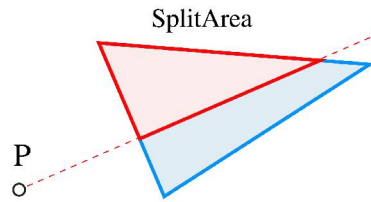


Fig. 7. SplitArea AutoCAD LISP utility, finding area bisector through the given point P

To geometrically confirm that stairs are predominantly given the position which establishes and reflects the balance of the structure, we determined the centre of gravity (centre of mass, centroid), gravity lines and area bisectors for over a dozen floorplans treated as polygons. This set of randomly chosen architectural structures of different purpose contained both those with symmetrical and those with asymmetrical floorplans. We hypothesize that, if a building is a manifestation of the BALANCE image schema, and if the stairwell corresponds to the spine, the stairwell will be positioned in such a way to reflect the balance between the floor plan sections on either side of the area bisector. This line should, consequently, pass through the stairwell space. Since it is clear that the architects do not consciously calculate the exact position of the stairwell to follow this model, we allow that the area bisector does not have to pass precisely through the gravity centres of stairwell areas, if there is more than one. In other words, we will allow for certain deviations. This means that it is sufficient that the area bisector should pass through the stairwell area to indicate the tendency of placing the stairs so as to mark the equal division of the base (Fig. 8).

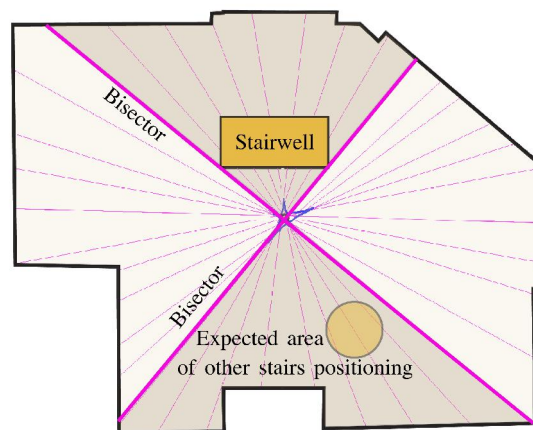


Fig. 8. The position of the stairs within the building floor plan: bisector-related balance

We used the aforementioned criteria on case-by-case basis to divide the floorplan polygon, in order to check whether the area bisector passes through more (or all) stairwells within the floorplan. Despite their infinite number (there being an area bisector for every point in or outside the polygon), they do not necessarily have to pass through more than a single staircase – yet they show a clear tendency to do so.

3 ILLUSTRATIONS AND EXPERIMENTAL EVIDENCE OF THE HYPOTHESIS

For our analysis we selected a set of buildings which serve different purposes, come from various parts of the world and belong to different periods, although modern buildings prevailed due to the availability of floor plans.

The example in Fig. 9 presents a building for collective living [28], whose floor plan is axially symmetrical (if we disregard some minor deviations). As we hypothesized, one of the median lines passes right through the both flights of stairs.

But, what happens when we move away from the symmetrical patterns? If we take one step in that direction and test our hypothesis on asymmetrical floor plans, we will see that the model still works for a large number of cases.

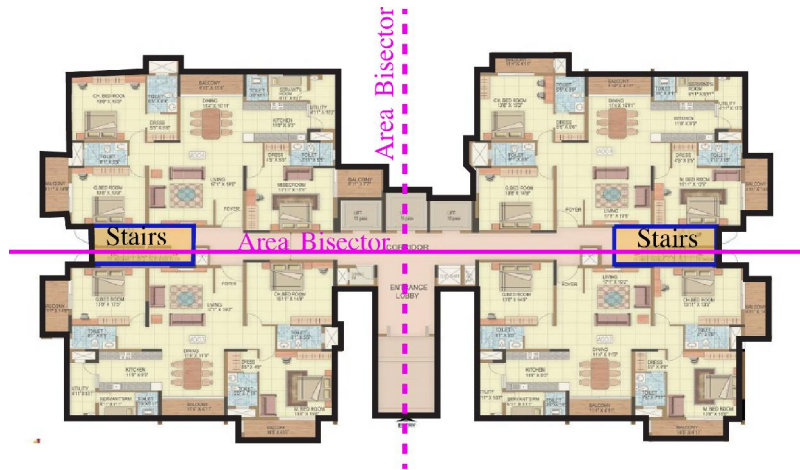


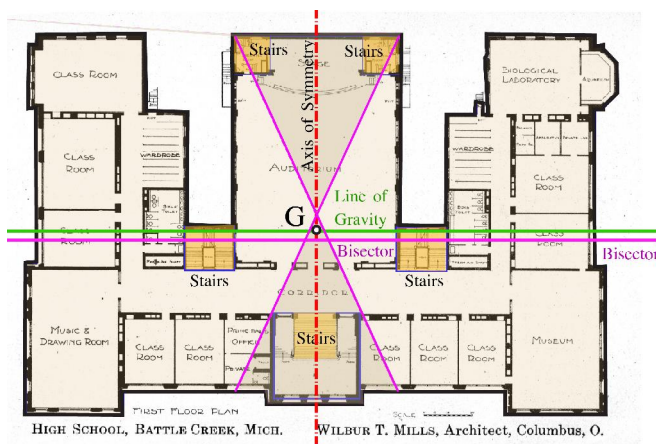
Fig. 9. A floor plan of a building of collective housing: Salarpuria Magnifica Amenities, Bengaluru, India² [28]

Our study suggests that stairwell areas tend to lie along the bisector, which is a universal line – a general principle that applies to both symmetrical and asymmetrical floor plans – the metaphorical backbone of a building. In addition, the BALANCE scheme is more noticeable in larger, especially public buildings.

Regardless of whether a building contains a single or multiple flights of stairs, it is very likely that they will be positioned in such a way to respect the line (or point) that guarantees the balance of the building, dividing the area of the floor plan into two equal parts. The lines that establish and reflect the balance of the building are as follows:

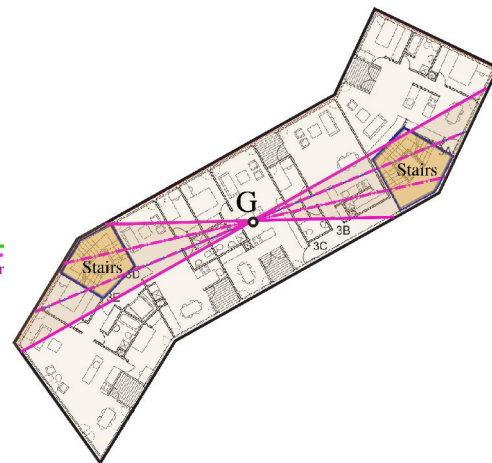
1. symmetry axis of the floor plan polygon, if the polygon is symmetrical (Fig. 10 a);
2. the lines of gravity (Fig. 10 b);
3. the area bisector, the most general case of the line dividing the surface of the polygon into halves (Fig. 11 a, b).

In the first case, the bisector overlaps with both the axis of symmetry and the gravity line, while in the second case it overlaps just with the gravity line, because the axis of symmetry is not present. This means that the area bisector is the universal balance line, present in every case.



High School Battle Creek, Michigan³ [23]

a)



Zoka Zola Drenova Typical Floor Plan⁴ [15]

b)

Fig. 10. Symmetrical Floor plans; a) Axially Symmetrical, b) Point Symmetrical

² <http://www.propreview.in/bangalore/salarpuria-magnifica/>

³ <https://goodbyecynthia.com/school-floor-plans-how-to-build-an-efficient-school/high-school-floor-plans/>

⁴ <https://www.archdaily.com/571837/affordable-housing-at-the-edge-of-the-city-zoka-zola-architecture-urban-design>

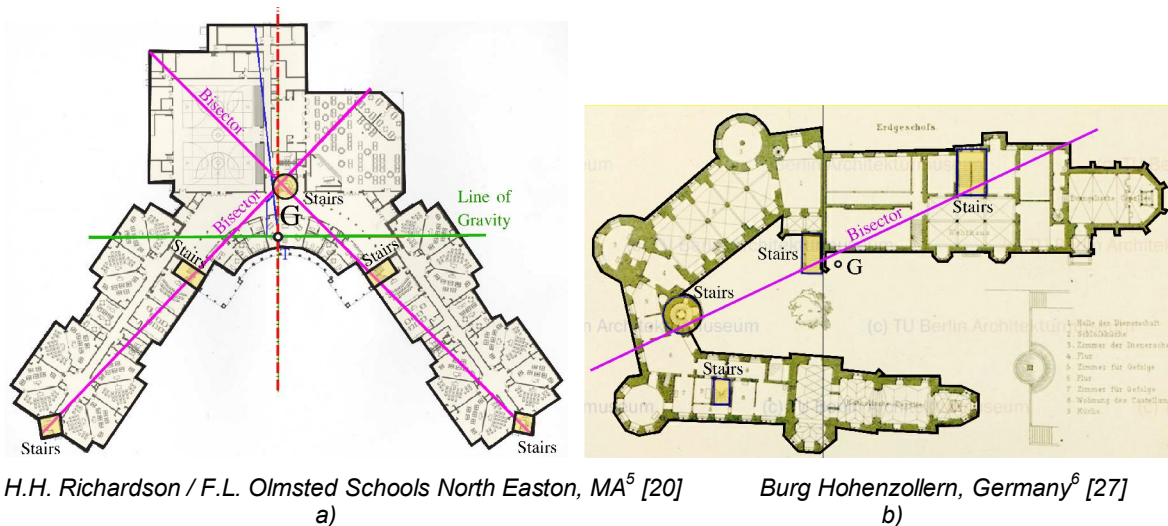


Fig. 11. Examples of stairs organized along the area bisectors in the buildings with a) (almost) symmetrical and b) asymmetrical floor plan polygon

Staircases manifest the BALANCE schema even when they are positioned in such a way to be the points of support for the floor plan’s shape, as can be seen in the example of the triangular base of the famous Schenley High School, Pittsburgh, Pennsylvania [29] (Fig. 12). This case illustrates that balance can be established by positioning the auxiliary stairs along area bisectors. The sense of balance is additionally reflected in the bilateral symmetry of the stairs in the floorplan.

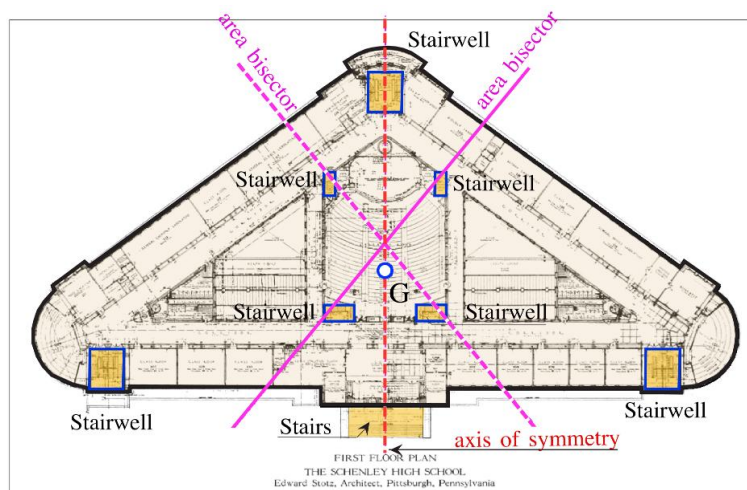


Fig. 12. Triangular, axially symmetrical floor plan of the first Floor of the Schenley High School, Pittsburgh, Pennsylvania⁷ [29]

We present several other selected examples which confirm that the position of a stairwell within a building manifests BALANCE schema. It is noticeable that the position of the stairs respects area bisector lines for bases which are symmetrical as well as for those that are not. This holds even for bases which deviate from orthogonal modular matrix, and also for those that represent wholes within a more complex floor plan, such as building wings in larger complexes (castles, for example).

Figure 13 a) shows the plan of the first floor of the library in Dun Laoghaire, Ireland [13]. There are five stairwells, four of which are placed within the base so that the area bisector lines pass through them.

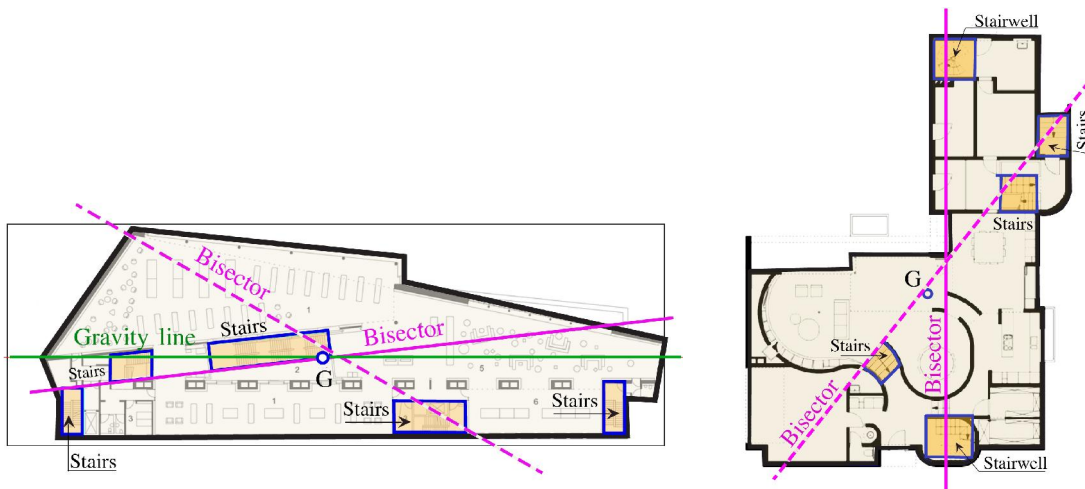
⁵ http://www.designshare.com/ElementaryLibrary/esrichardson/ESRichardson3_PL.htm

⁶ <http://www.picrevise.net/edit.php>

⁷ https://commons.wikimedia.org/wiki/File:Schenley_High_School,_1916,_First_Floor_Plan.png

What is interesting in this example, however, is that the main one-flight stairway literally follows the area bisector line which overlaps with its handrail.

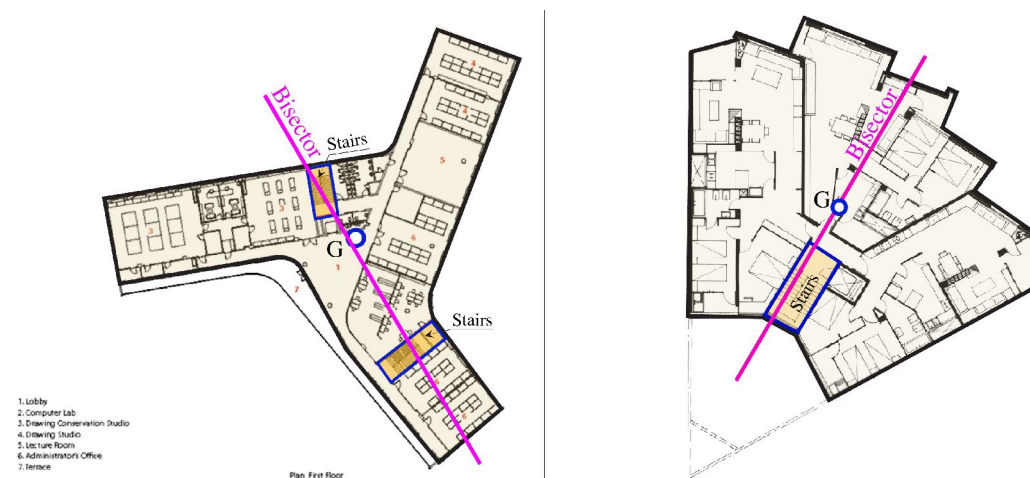
Figure 13 b) illustrates that even when the floor plan is asymmetrical, with apparently random placement of stairs, it is still possible to observe that they are aligned with the area bisectors.



a) Library in Dun Laoghaire, Ireland⁸ [13]

b) Casa N, Hasselt, Bélgica - MASSARCHITECTS⁹ [17]

Fig. 13. Some examples of stairs organization within asymmetrical floor plans



a) Arts and Crafts Studio, Korea¹⁰ [16]

b) Residential building in Barcelona by R. Bofill¹¹ [22]

Fig. 14. Some examples of stairs organization within the buildings floor plans

The example in Fig 14 a) indicates that the stairs follow the placement pattern along area bisectors even when the floor plan of a building observes rotational symmetry, while the one in Fig. 14 b) shows that even when axial symmetry is not complete, the stairs still adhere to the area bisector line.

Figure 15 shows the floor plan of the remains of the Slains Castle in Scotland [14]. It confirms that stairwell areas were distributed so as to reflect the balance between the halves of the basis, while

⁸ <https://www.dezeen.com/2014/12/04/carr-cotter-naessens-granite-clad-library-dlr-lexicon-dun-laoghaire-harbour-ireland/>

⁹ <https://www.archdaily.com/617080/house-n-hasselt-massarchitects>

¹⁰ <https://www.archdaily.com/346471/art-and-crafts-studios-poly-m-ur>

¹¹ <https://habitatgecollectiu.wordpress.com/2014/12/15/assolellament-dels-habitatges-a-leixample-de-barcelona/>

each area bisector passes through at least two stair areas within the basis. This implies that one can find manifestations of this deeply rooted image schema independently of the period to which a building belongs.

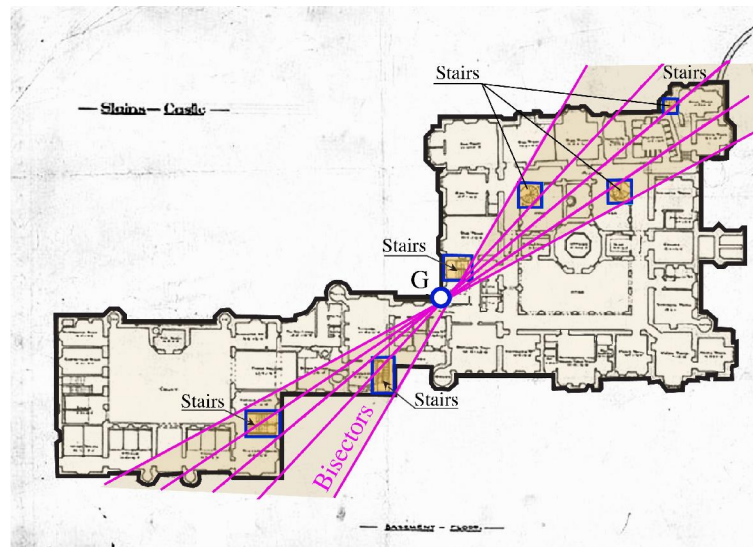


Fig. 15. Slains Castle (1597), Scotland – Basement Floor¹² [14]

These findings suggest that a stairwell placed along the bisector, which metaphorically represents the building's backbone, demonstrates and guarantees the balance of the halves. In cognitive linguistic terms, this is an example of embodied mind. The embodiment hypothesis in this framework serves to overcome the dualism between the body and the mind. It firmly grounds the aspects of human mind in aspects of human body. BALANCE schema, thus, is not a disembodied abstraction, but a knowledge structure that emerges directly from pre-conceptual embodied experience. Our research shows that it largely determines the position of stairs, and the organization of a building's interior space.

3 CONCLUSIONS

In this study, we examined an aspect of architectural design which carries highly symbolic value. To understand and interpret the symbolism of stairs, we started from the metaphor BUILDING IS BODY, which enabled us to map the stairs onto the backbone and formulate the metaphor STAIRWELL IS BACKBONE. To provide geometrical support for this mapping, we examined several lines (the axis of symmetry, gravity lines and area bisectors) whose role corresponds to that of the backbone (in the sense that they divide a given area into two equal sections). We geometrically analyzed positions of staircases in relation to the area bisector line and came to the conclusion that, regardless of the geographical location and historical period, the stairs within the building tend to be positioned in such a way to promote the balance between the halves of the base.

It is common knowledge that architects receive no formal instruction (except for the restrictions concerning construction regulations, which vary from one country to another, and possible explicit wishes of the investor) as to where to place the stairwells in their designs. In addition, the calculations involved in determining area bisector are challenging and time-consuming. Yet, there is a clear pattern that indicates architects tend to position staircases in such a way as to ensure the balance of a building by placing stairwells along area bisectors. Furthermore, our analysis reveals that no other architectural elements (except for hallways, in which the stairs are located) can be found on area bisectors with such consistency. In our opinion, this suggests that BALANCE schema is so deeply rooted in human mind that it can influence building design - a highly abstract aspect of thought. Consequently, the position of stairs can be interpreted as a non-linguistic realization of the BALANCE image schema.

¹² <http://www.presidentsmedals.com/Entry-14600>

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REFERENCES

- [1] Andre K.T. Assis: Archimedes, the Center of Gravity, and the First Law of Mechanics, Apeiron, Montreal, 2010. pp. 76.
- [2] Archimedes. (2009). On the Equilibrium of Planes, Book I. In T. Heath (Ed.), The Works of Archimedes: Edited in Modern Notation with Introductory Chapters (Cambridge Library Collection - Mathematics, pp. 189-202). Cambridge: Cambridge University Press.
- [3] Aubry, Mathieu, Metaphors in Mathematics: Introduction and the Case of Algebraic Geometry (September 26, 2009). Available at SSRN
- [4] Chevalier, J. & Gheerbrant A. (2003). Rječnik simbola. Banja Luka: Romanov d.o.o.
- [5] Clausner, T. & Croft, W. (1999). Domains and image schemas. *Cognitive Linguistics*, 10(1), 1-31.
- [6] Coxeter, H.S.M. (1987). Projective Geometry (2nd ed.). Springer-Verlag. Theorem 9.15, p. 83
- [7] Dunn, J., & Pretty, J. (1972). Halving a Triangle. *The Mathematical Gazette*, 56(396), 105-108
- [8] Goatly, A. (2007). *Washing the Brain – Metaphor and Hidden Ideology*. Amsterdam / Philadelphia: John Benjamins Publishing Company.
- [9] Kim Williams, Symmetry in Architecture, Visual Mathematics (1999) Volume: 1, Issue: 1, page 0-0 ISSN: 1821-1437
- [10] Lakoff G. & M. Johnson (2003 [1980]). *Metaphors We Live By*. Chicago and London: University of Chicago Press.
- [11] Stone, A. H. and Tukey, J. W. "Generalized 'Sandwich' Theorems." *Duke Math. J.* 9, 356-359, 1942.
- [12] Tom M. Apostol, Mamikon A. Mnatsakanian: Finding Centroids the Easy Way, *Math Horizons*, September 2000.
- Web Sources:**
- [13] Amy Frearson: Granite-clad library by Carr Cotter & Naessens Faces out over the Irish Sea, *DeeZeen* magazine, 2014. <https://www.dezeen.com/2014/12/04/carr-cotter-naessens-granite-clad-library-dlr-lexicon-dun-laoghaire-harbour-ireland/>
- [14] Andrew McBride: Among the Ruins of Slains Castle - Is there a future for Scotland's 'Dracula Castle'? RIBA President's Medals (2015). <http://www.presidentsmedals.com/Entry-14600>
- [15] Arch Daily (12. 12, 2014.): Affordable Housing at the Edge of the City /Zoka Zola Architecture + Urban Design <https://www.archdaily.com/571837/affordable-housing-at-the-edge-of-the-city-zoka-zola-architecture-urban-design>
- [16] Arch Daily: Arts and Crafts Studios / poly.m.ur (19. March 2013.) <https://www.archdaily.com/346471/art-and-crafts-studios-poly-m-ur>
- [17] ArchDaily: House N Nasselt / MASSARCHITECT (09 April 2015) <https://www.archdaily.com/617080/house-n-hasselt-massarchitects>
- [18] Archilovers (13/3/2015): A stair is the backbone of every architecture, Siller Stairs, Principal Architect. <http://www.archilovers.com/projects/151649/a-stair-is-the-backbone-of-every-architecture.html>
- [19] CAD Studio a.s.: SplitArea for AutoCAD
- [20] Design Share: Floor Plans http://www.designshare.com/ElementaryLibrary/esrichardson/ESRichardson3_PL.htm
- [21] GeoGebra: Pentagon Bisection: <https://www.geogebra.org/m/h4uy2dYv> (2016/ 07/ 17).

- [22] Habitate col·lectiu 1 / Collective housing: Assolellament dels habitatges a l'exemple de Barcelona (14. December 2014.)
<https://habitatgecollectiu.wordpress.com/2014/12/15/asselellament-dels-habitatges-a-leixample-de-barcelona/>
- [23] Home interior plans ideas (09.09. 2015.): High School Floor Plans
<https://goodbyecynthia.com/school-floor-plans-how-to-build-an-efficient-school/high-school-floor-plans/>
<https://www.cadstudio.cz/en/apps/splitarea/>
- [24] Lessard, L. "Cutting polygons in half" in "Book Proofs – A blog for mathematical riddles, puzzles, and elegant proofs". Created 7/9/2016. Retrieved 1/25/2018. URL:
<http://www.laurentlessard.com/bookproofs/cutting-polygons-in-half/>
- [25] Marvel Buliding (2011): Unique Sculptural Staircase in Metallic Backbone Shape by Philip Watts
<http://www.marvelbuilding.com/unique-sculptural-staircase-metallic-backbone-shape-philip-watts.html>
- [26] Phillip Watts Design: Staircase, Nothampton,
<https://www.philipwattsdesign.com/bespoke/staircase-northampton>
- [27] Picrevise: Antechamber Architecture Floor Plan Burg Hohenzollern
<http://www.picrevise.net/edit.php>
- [28] PropPreview: Salarpuria Magnificia <http://www.propreview.in/bangalore/salarpuria-magnificia/> last modified 2016/10/07.
- [29] Wikimedia Commons: Schenely High School, 1916, First Floor Plan. png.
https://commons.wikimedia.org/wiki/File:Schenley_High_School,_1916,_First_Floor_Plan.png
- [30] Wolfram Demonstation Project: Triangle Area Bisector
<http://demonstrations.wolfram.com/TriangleAreaBisectors/>