25th National and 2nd International Scientific Conference

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# THE COMPLEMENT OF THE HUGELSCHAFFER'S CONSTRUCTION OF THE EGG CURVE

**Maja Petrović**[1]
**Marija Obradović**[2]

**RESUME**

*Hügelschäffer's construction, based on the distortion of the ellipse construction, provides an egg-shaped curve. This curve is a mixed cubic curve, the cubic hyperbolic parabola of type A. Curve is a three-branched and except the oval arising from mentioned construction, it contains two more branches which converge towards two asymptotes: one linear and one parabolic asymptote. Since the Hügelschäffer's construction does not give a solution for this part of the curve, we discussed the possibility of amendments to this construction, so that the entire course of the curve could be graphically processed. We came to a solution using Cartesian hyperbole complementary to the circles from Hügelschäffer's construction.*

***Key words: Hügelschäffer's construction, egg curve, asymptote, circle, hyperbola***

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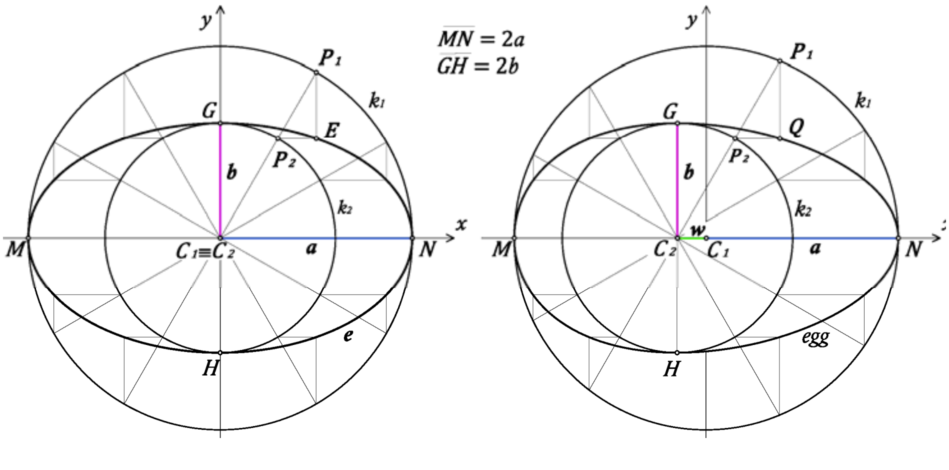
## 1. INTRODUCTION: EGG CURVE OBTAINED BY ELLIPSE DISTORTION

The construction of egg-shaped curve which was conducted by German mathematician Fritz Hügelschäffer in 1948., with two non-concentric circles, is actually a transposed well known ellipse construction using concentric circles (***Figure 1 - a***).

The first circle, $k_1$ with the center $C_1$ is of the radius equal to the ellipse large parameter, and the other circle, $k_2$, centered in $C_2$ has a radius equal to the small parameter of the ellipse. Its center $C_2$ is shifted from the first center $C_1$ for the parameter $w$.

Let the centers $C_1$ and $C_2$ be set on the $x$-axis; then the major parameter of the ellipse and the major axis – the axis of symmetry of the egg-shaped curve, will also lie on the $x$ axis. Center $C_2$ of the minor circle $k_2$ which is located on the $y$-axis is the center of the pencil of radial rays of lines, that spread to the points of another, major circle $k_1$. From the intersection points $P_1$ and $P_2$ of each radial straight line (of the pencil with the apex in $C_2$) with the given circles, we set rays: from the point $P_2$ of the minor circle, parallel to the $x$-axis, and from the point $P_1$ of the major circle $k_1$, parallel to the $y$-axis. The intersection point $Q$ of these rays will represent the point that belongs to a cubic egg curve (***Figure 1 - b***).




*a)* *b)*

***Figure 1****: a) Construction Of An Ellipse By Method Of Concentric Circles*
*b) Hügelschäffer's Construction Of An Egg Curve*

### 1.1. Hyperbolism

One of the curves' transformations which were investigated by Newton – hyperbolism - now will be constructively and analytically elaborated. Newton starts with the planar curve $\Gamma_o$ and with the given point $S\ (x_S,\ y_S)$ and the straight line $m\ (x = x_m)$ in the same plane he obtains a new curve $\Gamma$.

For each point $P\ (x_P,\ y_P)$ of the curve $\Gamma_o$ (***Figure 2***), there will be uniquely associated point $Q\ (x_Q,\ y_Q)$ of the curve $\Gamma$ as follows:

- The straight line ***SP*** intersects the line $m$, at the point $M\ (x_m,\ y_M)$;
- On the parallel line to the line $m$, through the point $P$, projected from the point $M$, the point $Q$ of the newly obtained curve $\Gamma$ will occur.

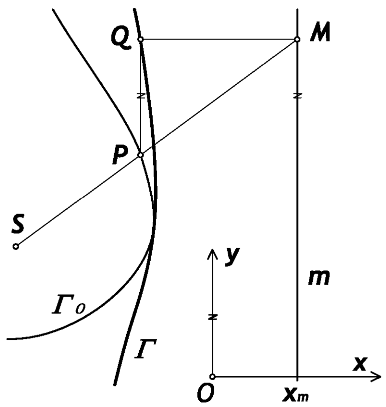



***Figure 2*** *– Hyperbolism (Newton's Transformation Of A Planar Curve By Dint Of A Given Straight Line And A Point)*

If the curve is given analytical as the equation in the implicit form $f\ (x,\ y) = 0$ then the hyperbolism is the curve: $f\ (x,\ xy\ /\ x_m) = 0$

For a given parametric curve equation $x = f\ (t)$, $y = g\ (t)$ hyperbolism is the curve: $x = f\ (t)$, $y = x_m\ g\ (t) /\ f\ (t)$

This transformation is quadratic, so it transforms algebraic curve of degree $n$ to an algebraic curve of degree $\leq 2n$.

The Witch of Agnesi is an example of the curve obtained by hyperbolism, using a circle, a straight line and a point that lies on the circle.

The inverse transformation is caled antihyperbolism: the initial curve with equation of the implicit form $f\ (x,\ y) = 0$ by this transformation transgresses to the curve of form: $f\ (x,\ x_m\, y/x) = 0$ , and for the parametricaly assigned curve $x = f\ (t)$ , $y = g\ (t)$ antihyperbolism is the curve of the form: $x = f\ (t)$ , $y = f\ (t)\, g\ (t)/\, x_m$ .

If, instead of the straight line $m\ (x = x_m)$ we take a new curve in the plane ***Oxy*** then we get a generalized Newton's transformation (***Figure 3***). For a given point $S\ (x_S,\ y_S)$ and two curves $\Gamma_1$ and $\Gamma_2$ in the plane ***Oxy*** will get a new curve $\Gamma$, by the hyperbolism.

Each straight line set through the point ***S*** will intersect the curve $\Gamma_1$ at the point $P_1\ (x_1,\ y_1)$ and the curve $\Gamma_2$ at the point $P_2\ (x_2,\ y_2)$. The point $Q\ (x_Q,\ y_Q)$ of the curve $\Gamma$ will be set in the intersection of the parallel line to ***Ox*** from the point ***P,*** and the parallel line to ***Oy*** from the point $P_2$.

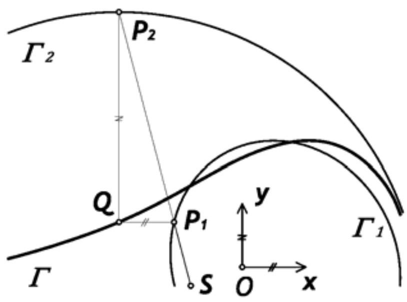



***Figure 3*** – *Hyperbolism (The Generalized Newton's Transformation)*

### 1.2 The Hyperbolism Of Two Circles

Let the first curve $\Gamma_1$ be the circle $k_1$ ***(1)*** of the radius ***a*** and the center $C_1$ at the origin ***O***, and let the second curve $\Gamma_2$ be the circle $k_2$ ***(2)*** of radius ***b*** with center $C_2$ also in ***O***, and let the point $C_2$ be the point ***S*** of the Newton's transformation - by hyperbolism, we get an ellipse (***Figure 1 - a***) centered at the origin ***O***, with the major axis ***a***, and the minor axis ***b***.

$k_1 :\ P_1\ (x_1,\ y_1)$ , $C_1\ (0,\ 0)$, $r_1 = a$, $x_1^2 + y_1^2 = a^2$ ***(1)***

$k_2 :\ P_2\ (x_2,\ y_2)$ , $C_2\ (0,\ 0)$, $r_2 = b$, $x_2^2 + y_2^2 = b^2$ ***(2)***

Each point $E\ (x_1, y_2)$ of the newly obtained curve - ellipse ***(e)*** - will meet the condition ***(3)***:

$$y_1 : y_2 = x_1 : x_2 \qquad (3)$$

By squaring the condition ***(3)*** and substituting ***(1)*** and ***(2)*** equations, we get the equation of an ellipse ***(e):***

$$(e): \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Now, let the first curve $\Gamma_1$ be the circle $k_1$ ***(4)*** of the radius $a$ and the center $C_1$ at the origin $O$, let the second curve $\Gamma_2$ be the circle $k_2$ ***(5)*** of radius $b$ with the center $C_2$, and let the point $S$ of the Newton transformation be the center $C_2$, shifted to $w > 0$ by $x$-axis, then by the hyperbolism we acquire the egg-shaped cubic curve (***Figure 1 - b***).

$$k_1 : P_1\ (x_1, y_1)\ , \ C_1\ (0, 0), \ r_1 = a, \qquad x_1^2 + y_1^2 = a^2 \qquad (4)$$

$$k_2 : P_2\ (x_2, y_2)\ , \ C_2\ (-w, 0), \ r_2 = b, \qquad (x_2 + w)^2 + y_2^2 = b^2 \qquad (5)$$

Each point $Q\ (x_1, y_2)$ of the newly obtained egg shaped curve - (***egg***) - will meet the condition ***(6)***:

$$y_1 : y_2 = (x_1 + w) : (x_2 + w) \qquad (6)$$

By squaring the condition ***(6)*** and substituting ***(4)*** and ***(5)*** of the circle equation, we will get the egg-shaped curve of the equation ***(egg):***

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\left(1 + \frac{2wx + w^2}{a^2}\right) = 1$$

The newly obtained curve (***egg***) is an algebraic cubic curve and its equation can be writen in the form ***(7)***, i.e. as the linear distortion ***(8)*** of an ellipse ***(e)***.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\, g(x) = 1 \qquad (7)$$

$$g(x) = 1 + \frac{2wx + w^2}{a^2} \qquad (8)$$

For $w=0$ then $g(x)=1$, we will obtain the ellipse ***(e)*** equation.

## 2. THE COMPLEMENT OF THE HÜGELSCHÄFFER'S CONSTRUCTION

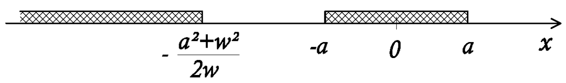
### 2.1 The Complete Cubic Curve As A Graph Of A Function

By examining the flow of the function ***y* = *f* (*x*)** of the cubic curve ***(egg)***

$$f(x) = \sqrt{\frac{b^2(a^2 - x^2)}{a^2 + 2wx + w^2}}$$

and by drawing the function's graph, we notice that its domain of definition is:

$$\left(-\infty, -\frac{a^2 + w^2}{2w}\right) \cup [-a, a]; \quad a, w > 0$$




The curve has two asymptotes: one linear, parallel to the ***y*** axis:

$$x = -\frac{a^2 + w^2}{2w}$$

set on the negative side of the ***x*** axis, and a parabolic one:

$$y^2 = -\frac{b^2}{2w}\left(x - \frac{a^2 + w^2}{2w}\right)$$

Using mathematical software[3] and applets[4], we got a graph that shows the whole cubic curve which consists of one convergent (closed) egg shaped oval part, and two opened branches. Curve is symmetrical with respect to the ***x***-axis and is limited in regard to the two asymptotes.

In ***Figure 4,*** there is shown a cubic curve assigned with the following parameters:

***a=5, b=3, w=2***

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[3] Maple

[4] A&G Grapher

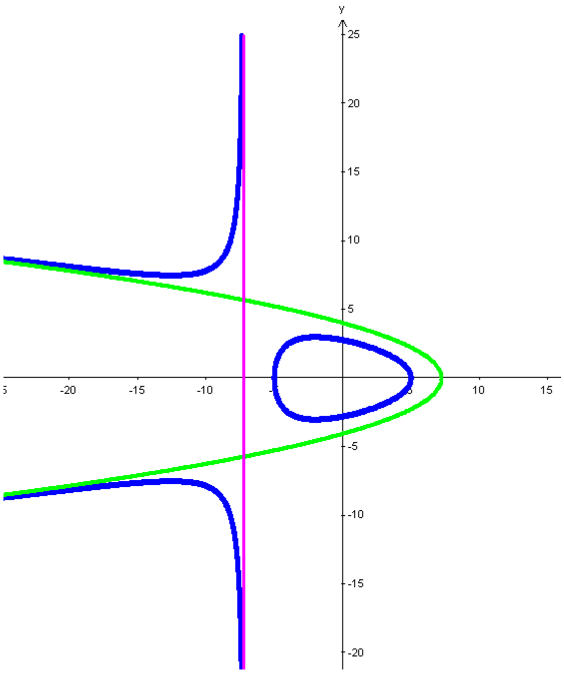


*Figure 4 - **The** Cubic Egg Curve And Its Asymptotes*

$$b^2x^2 + (a^2 + w^2)y^2 + 2wxy^2 - a^2b^2 = 0$$

$$2wx + a^2 + w^2 = 0$$

$$4w^2y^2 = b^2(a^2 + w^2 - 2wx)$$

### 2.2 Complementarity Of The Circle And The Cartesian Hyperbola

Observing the curve assigned in the previously described manner, we tried to find a constructive solution for the part of the curve which is located on the interval:

$$x \in \left(-\infty, -\frac{a^2+w^2}{2w}\right) \qquad \textbf{(9)}$$

By definition, two curves are mutually projectively complementary, regarding a pencil of straight lines, if they induce opposite but equivalent involution on each of the straight lines.

In order to obtain a curve complementary to the circle, for intersection by the set of lines parallel to the ***y*** axis, it is enough to replace the double points of induced elliptical involution on the circle's passing lines, by equivalent double points of hyperbolic involution (***Figure 5***).

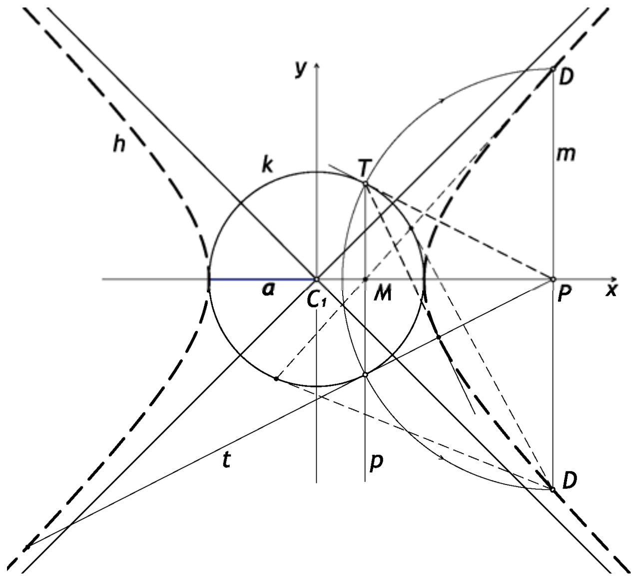


***Figure 5*** *- The Complement Of Circle In Regard To The Set Of Parallel Lines Of Y-Direction*

Rectangular hyperbola which is the complement of the circle $x^2+ y^2 = a^2$ in regard to the set of parallel lines of ***y***-direction, will have the equation $x^2 - y^2 = a^2$, while the equation of the hyperbola for the set of parallel lines of ***x***-direction, requires just a change of the sign in front of member $x^2$ in the circle equation.

### 2.3 Hyperbolism Of Two Hyperbole

Let the first curve $\Gamma_1$ be the rectangular hyperbola $h_1$ ***(10)*** of the radius $a$ with the center $C_1$ at the origin $O$, and let the second curve $\Gamma_2$ be hyperbola $h_2$ ***(11)*** of radius $b$ with center $C_2$, and let the assigned point $S$ of the Newton transformation be the point $C_2$ shifted for $w > 0$ along the ***x***-axis, then we obtain by hyperbolism a part of cubic curve which is located on the interval ***(9)***.

$h_1$ : $P_1\ (x_1, y_1)$ , $C_1\ (0, 0)$, $r_1 = a$, $x_1^2 - y_1^2 = a^2$ *(10)*

$h_2$ : $P_2\ (x_2, y_2)$ , $C_2\ (-w, 0)$, $r_2 = b$, $-(x_2 + w)^2 + y_2^2 = b^2$ *(11)*

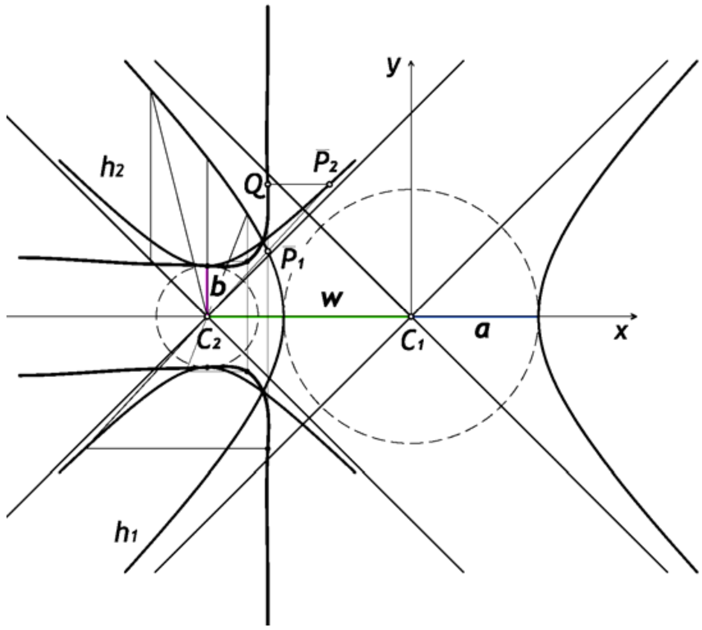


***Figure 7.*** *– Hyperbolism Of Two Rectangular Hyperbole*

The newly obtained curve (***c***) is a cubic algebraic curve and its equation can be written in the following form:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2}\left(-1 - \frac{2wx+w^2}{a^2}\right) = 1$$

It is a cubic hyperbolic parabola of type **A**, and can be observed as a linear distortion of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} G(x) = 1 \qquad (12)$$

$$G(x) = -1 - \frac{2wx+w^2}{a^2} \qquad (13)$$

For ***w=0*** then ***G(x)= -1***, we will obtain the ellipse ***(e)*** equation.

## 3. CONCLUSIONS

In this paper we have graphicaly and mathematicaly solved the problem of describing the whole curve, cubic hyperbolic parabola of type A, which oval part can be presented by Hügelschäffer's construction.

We proved that the complementary curves for the circles in the Hügelschäffer's construction – the rectangular hyperbole - will complete the construction for the integer curve.

Using the Newton's method of hyperbolism, we explained the construction's origin.


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### REFERENCES


1. Sir Isaac Newton (1643 - 1727): *Enumeration of Lines of the third order*, H. G. Bohn, London, p. 62,63, 1860
2. Dovniković L.: *Harmonija sfera, relativistička geometrija harmonijskih ekvivalenata,* Matica srpska, Novi sad, str. 11-16, 346-358, 1999.
3. Dovniković L.: *Nacrtno-geometrijska obrada i klasifikacija ravnih krivih 3. reda,* Zbornik radova Matice srpske za prirodne nauke 73, Novi Sad, str 107-130, 1987.
4. Dovniković L.: *The Principle of the Projective Complementariness of Points, Curves and Surfaces*, Journal of Theoretical Graphics and Computing, Vol. 4, No. 1, USA, 1991.
5. Obradović M. , Mišić S.: *Krive jajastog oblika u Nacrtnoj geometriji,* MonGeometrija 2004, Zbornik radova, Beograd, str. 147 – 160, 2004.
6. Obradović M., Petrović M.: *The spatial interpretation of Hügelschäffer's construction of the egg curve*, MonGeometrija 2008, Proceedings, Vrnjačka Banja, str. 222 – 232, 2008.
7. Meyer J.: *Projections of the twisted cubic*, The Teaching of Mathematics, Vol X, p. 58, 59, 2007.
8. Transformation de Newton, 2009: http://www.mathcurve.com/courbes2d/hyperbolisme/hyperbolisme.shtml