VEHICLE SPEED INFLUENCE ON THE DYNAMIC AMPLIFICATION FACTOR OF BRIDGES

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Summary: *The paper presents an analysis of the influence of vehicle speed movement along bridge on dynamic amplification factor for the case of simply supported, continuous and clamped beam structural system. The analysis was conducted analytically and numerically. Theoretical expressions was developed for the case of movement a concentrated force over the bridge over simply supported beam structural system, while other cases were analyzed numerically. Two cases, movement of concentrated force and system of concentrated forces were considered. The comparative results for the most unfavorable case, with the valid domestic and foreign regulations, compared and presented. The results showed that it is a more unfavorable case when vehicles are moving at higher speeds on short-span bridges, as well as that one concentrated force produces greater dynamic effects than a system of concentrated forces, as well as the simply supported beam system is the most sensitive on dynamic loads respect to other structural systems.*

Keywords: *Dynamic Amplification Factor, vehicle speed movement, bridges structures*

1. INTRODUCTION

Moving load on over bridge structures is dynamic nature and a dynamic calculation is required during analysis. Usually, this analysis carried out considering the bridge static load effect, and the dynamic effect calculates multiplying the load effects with the dynamic amplification factor λ (DAF). In the research the value of λ is analyzed in as function of the vehicle speed movement (concentrated load or systems of concentrated loads) on various static systems of bridge structures. The value (λ) is determined as the ratio of dynamic and corresponding static influence due to dynamic disturbing load. This influence can be deflection, stress, strain, etc. The presented research considered ratio of dynamic and static displacement, while interaction between the surface and moving force

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is not take into account. Also, additional mass which corresponds to concentrated moving load is neglected.

2. ANALYTICAL SOLUTION OF THE TRANSVERSE VIBRATION OF SIMPLY SUPPORTED BEAM UNDER THE MOVING CONCENTRATED FORCE

The problem of determining the transverse vibrations of the simply supported beam due to moving a constant intensity concentrated load, *P(t) = const* with constant speed *V(t) = const* is considered.

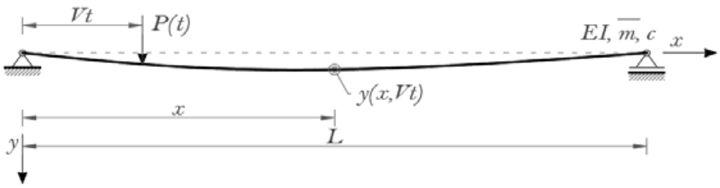

Figure 1: Simply supported beam deflection due to moving concentrated load

In Figure 1 is shown layout of considered simply supported beam with: constant bending stiffeness EI, distributed mass $\overline{m}$, damping coefficient c and span L. Product Vt defines the position of concentrated load $P(t)$ over the structure. Deflecton of structure for considered case is discribed by partial differential equation (1):

$$EI\frac{\partial^4 y(x,t)}{dx^4}+\overline{m}\frac{\partial^2 y(x,t)}{dt^2}+c\frac{\partial y(x,t)}{dt}=\delta(x-Vt)P(t) \tag{1}$$

The moving load is defined by the *Dirac function*, and solution of equation (1), according to [2], is defined by expesion (2):

$$\begin{aligned} y(x,t) = y_0\sum_{n=1}^{\infty}\frac{1}{n^2[n^2(n^2-\alpha^2)^2+4\alpha^2\mu^2]}[n^2(n^2-\alpha^2)\sin n\overline{\omega} \\ -\frac{n\alpha[n^2(n^2-\alpha^2)-2\mu^2]}{\sqrt{n^4-\mu^2}}e^{-\varepsilon_n t}\sin\omega_{d,n}t-2n\alpha\mu(\cos n\overline{\omega}t-e^{-\varepsilon_n t}\cos\omega_{d,n}t)]\sin\frac{n\pi x}{L}\end{aligned} \tag{2}$$

Solution represented by expresion (2) is in the form of infinite sum of products of position trigonometric functions and functions which depends on time t. The coefficients ω_n, $\overline{\omega}$, α and μ are n circular modal frequency of free vibrations, the “circular load frequency“, the speed parameter and damping parameter respectively, as follow:

$$\omega_n = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EI}{\overline{m}}}, \overline{\omega} = \frac{\pi V}{L}; \alpha = \frac{\overline{\omega}}{\omega_1} = \frac{VL}{\pi}\sqrt{\frac{\overline{m}}{EI}} = \frac{V}{V_{cr}}; \mu = \frac{\varepsilon_n}{\omega_1} = \frac{\varepsilon_n L^2}{\pi}\sqrt{\frac{\overline{m}}{EI}} \quad (3)$$

The parameter $\overline{\omega}$, “circular load frequency“ is parameter which depends of speed of movement and length of the beam *L*. The critical speed V_{cr} is included as in the speed parameter (3). When the force moves at critical speed, the *frequency of the movement of the load* and frequency of free vibration of the beam coincides. The coefficient y_0 is equal to $2PL^3 / \pi^4 EI$, and ε_n , $\omega_{d,n}$ are the n^{th} damping coefficient of ratio between viscous damping and distributed mas, while the n^{th} is circular frequency of the damped oscillation of the beam, respectively (4).

$$\varepsilon_n = \frac{c}{2\overline{m}}; \ \omega_{d,n}{}^2 = \omega_n{}^2 - \varepsilon_n{}^2 \quad (4)$$

The solution of the differential equation (1) takes into account all the modes of free vibrations of the system (ω_n , $n \to \infty$) so it represent the exact solution.

It is important to emphasize that the solution (2) represent displacement of an arbitrary point of the simply supported beam at time *t* while the concentrated load moves from the start to the end point of the beam, and when it reaches the end of the beam it returns to the initial – start point with the opposite sign and thus again. Therefore, the solution (2) applies over the time $t \in [0, \mathrm{L/V}]$ i.e. just over the time while force crosses the beam. The analysis of the beam after the first crossing of the concentrated load requires solving modified equation (1) with function on right-hand side is equal to zero, with initial conditions (displacement and velocity) defined by solution of original equaion (1) at the time *t=L/V*.

3. DYNAMIC AMPLIFICATION FACTOR OF SIMPLY SUPPORTED BEAM UNDER MOVING CONCENTRATED LOAD

A case of analytical and numerical computation of the dependence of the dynamic amplification factor respect to the speed of the movement of the concentrated load over the bridge with the following characteristics is considered:

- Bridge span $L = 10m$
- Constant bending stiffeness $EI = 4.717\,kNm^2$
- Moving concentrated load $P = 100\,kN$, mass $m_p = 10.2\,kN / (\mathrm{m/s^2})$
- Relative damping ratio $\zeta = 0.05$

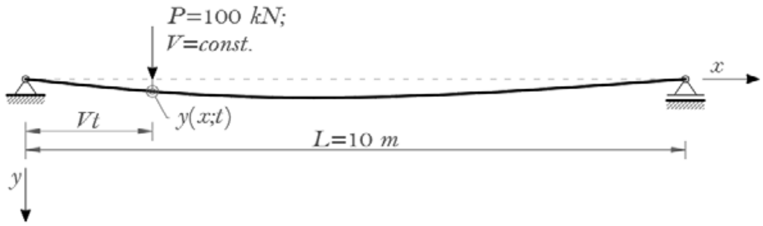

Figure 2: Simply supported beam under concetrated moving load

The DAF is defined as the maximum ratio between dynamic and static displacement at mid-span of the beam, Figure 2. The analytical solution computed using *Matlab programming language* based on expression (2), taking into the account the first *50* natural modes, with the time step (Δt) of *0,001 s*. Using the structural analyzing software SAP2000, the $\lambda = \lambda\,(V)$ dependence is also computed, with the beam model formed from 10 beam finite elements length of *1 m* using concentrated masses at the ends of elements. So, there are 9 degrees of freedom of vertical movement of the beam and *Linear Time History analysis* was performed.

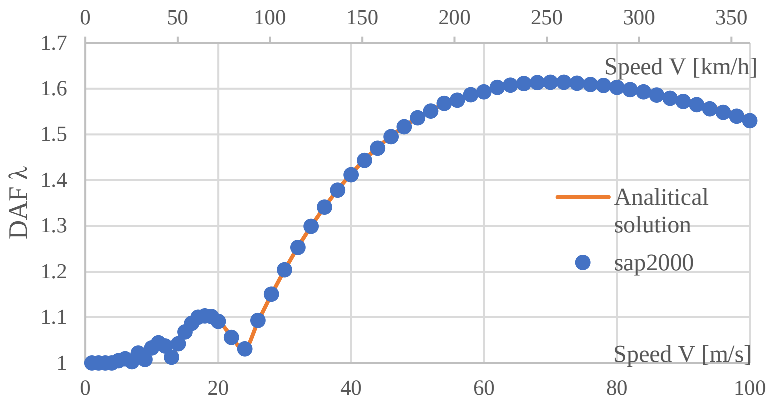

Figure 3: Dynamic amplification factor as function of load speed movement

Figure 3 shows good matching of the results with the maximal deviation of *0.23%*. It occurs because the model developed in *SAP2000 software* has just nine concentrated masses, the smaller number of degrees of freedom taken into the account, as well as due to the fact the longer time step of integration was applied.

In Figure 4 is shown dependence of $\lambda = \lambda\,(V)$ the initial part of the diagram shown in Figure 3, just for the analytical solution case. The critical speed in the particular case is $V_{cr} = 119\ m/s = 428\ km/h$, but the maximum value of λ achieved at the speed of aprox. for *60%* V_{cr}, i.e. *70 m/s*.

Absence of significant peak of movement for the speed of concentrated load at speed $V = V_{cr}$ is consequence of fact that concentrated load passes the beam length just one time, and excite it to start to oscillate freely until it calms down.

If the expression (2) will be considered durring the longer period $t > L/V$, while the concetrated load crosses n times, as already described, then there would be expressive peaks at speeds $V = V_{cr}$, $3V_{cr}$, $5V_{cr}$... which corespond to modal forms with maximum deflection at mid-point of the beam, 1st, 3rd, 5th, ...

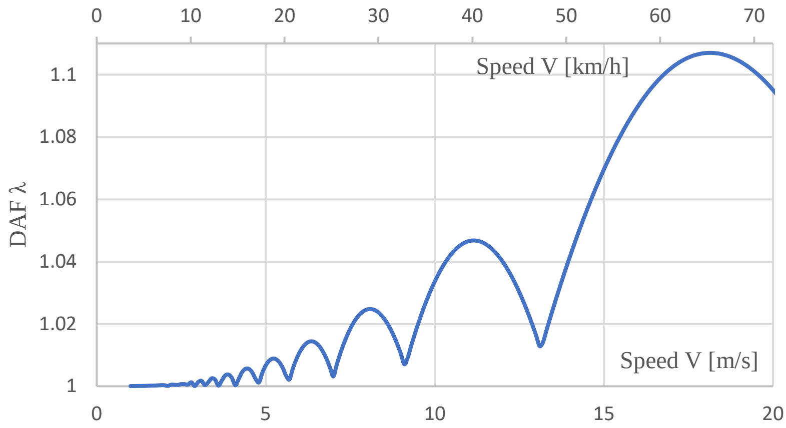

Figure 4: The starting part of the diagram in Figure 3

Resonance at the odd number multipliers of critical velocitiy would always arise because it corresponds to the odd modal frequencies/forms because the mid-point with maximal deflection. So, only the odd natural modes contribute to the total deflection/displacement. Further, will be consider the speeds that can be realistically achieved, in the range of $V = [0,130]\ km/h$, just using the results computed by the SAP2000 structural analysis software package, because the analytical solutions for other considered cases are complex.

4. DYNAMIC AMPLIFICATION FACTOR OF SIMPLY SUPPORTED BEAM UNDER MOVING CONCENTRATED LOAD SYSTEM

The case of determining the dependence of a dynamic amplification factor of speed of the moving concentrated load system with internal distance of *1 m* is considered.
In the case of moving concetrated load system with more forces at lower speeds, up to *8 m/s*, could be expected the same or greater DAF than it produced by movement of system with just one force, as it is shown in Figure 5. Increasing speed of movement, the influence of inertial effects of the beam became the most significant during the movement of one concentrated force.

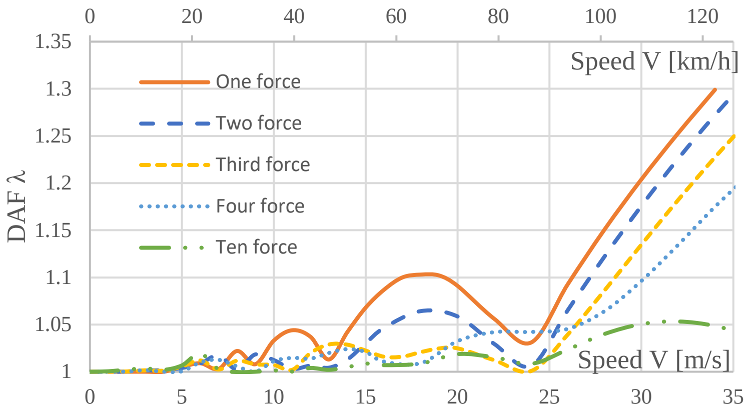

Figure 5: Dynamic Amplification Factor depedance of speed V for different moving concentrated load systems

5. DYNAMIC AMPLIFICATION FACTOR OF DIFERENT STRUCTURAL SYSTEMS UNDER MOVING LOAD

In this section, beams the geometrical and material properties defined in section 3, consider for the cases of different structural systems. The simply-supported beam is identical as it is shown in Figure 2, the continuous beam with two and three equal spans of *10 m* length, and the both ends clamped beam with length of span of *10 m*. The reason for the analysis of the both ends clamped beam is often structural system of short bridges (*10* to *15 m*) with the deck clamped on abutments. In the case of continuous beam bridges, dynamic amplification factor was observed at the point on distance of *4.5 m* from the first support.

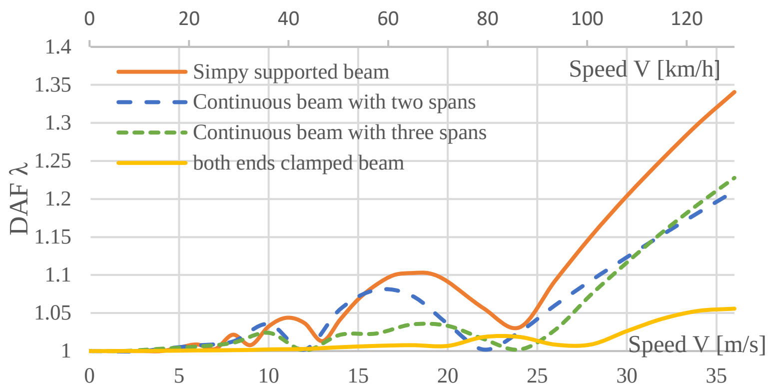

Figure 6: Dynamic Amplification Ffactor depedance of speed V for different structural systems

In Figure 6 could be clearly seen that the simply-supported beam structural system is the most sensitive to the movement of one concentrated force. Although, the frequencies of the first modes of the sample beam and the continuous beam with the span lengths equal to span of the simply-supported beam are the same. The both sides clamped beam is the least sensitive in the sense of DAF at normal speeds, with significant amplification at high speeds, which is consequence of higher stiffness of mid-span deflection.

6. DYNAMIC AMPLIFICATION FACTOR RELATED TO DOMESTIC AND EUROPEAN REGULATIONS

In this section are compared dynamic amplification factors of simply supported beam structural system shown in Figure 2 at a speeds of *80 km/h* and *130 km/h* according to domestic [3] and European [4] regulations. According to domestic regulation [3], DAF (λ) for multiplication static influences of traffic load for road and railway bridges is defined by equeations (5),

$$\lambda = 1.4 - 0.008 \cdot L \geq 1; \qquad \lambda = \frac{1.44}{\sqrt{L} - 0.2} + 0.82 \tag{5}$$

for the corresponding (effective) span length of the beam defined by parametar L, see [3]. The European standard [4] also includes dynamic amplification factor for road bridges, but not explicitly, it is taken into account in load definition for design.

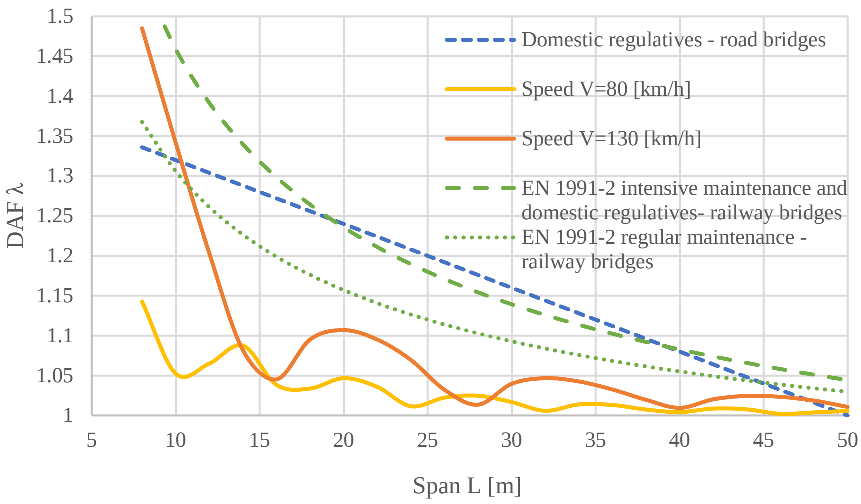

Figure 7: Dynamic factor in the function of the relevant bridge span

The load effects of railway bridges are multiply by the dynamic coefficient explicitly in order to include dynamic effects, equivalently as it is defined in domestic regulations [3]:

$$\Phi_2 = \frac{1.44}{\sqrt{L_\Phi} - 0.2} + 0.82; \quad \Phi_3 = \frac{2.16}{\sqrt{L_\Phi} - 0.2} + 0.73 \tag{6}$$

The coefficients Φ_2 and Φ_3 are valid for an intensive and standardly maintained railways, whil L_Φ is the relevant span of the bridge element of brige, see [4].

7. CONCLUSION

Based on the analysis so far, it can be concluded that the dynamic amplification factor λ depends on load type, structural system of beam, and the speed parameter α, i.e. the ratio of load frequency property and structural modal properties, as well as damping of the structure. The most unfavorable case is when a concentrated force moves along the simply-supported beam structural system for a speed in the range of *60%* of critical velocity V_{cr}, Figure 8, for different relative damping values of the symple-supported beam structure loaded by single concentrated moving force. The real speed of vehicle is significantly less than the computed critical speed.
Figure 7 shows that prescribed the values of DAF by the domestic and European standards are significantly larger than computational values, because in regulations are included rougnesses road surfaces and railway tracks, as well as vehicle – structure interaction.

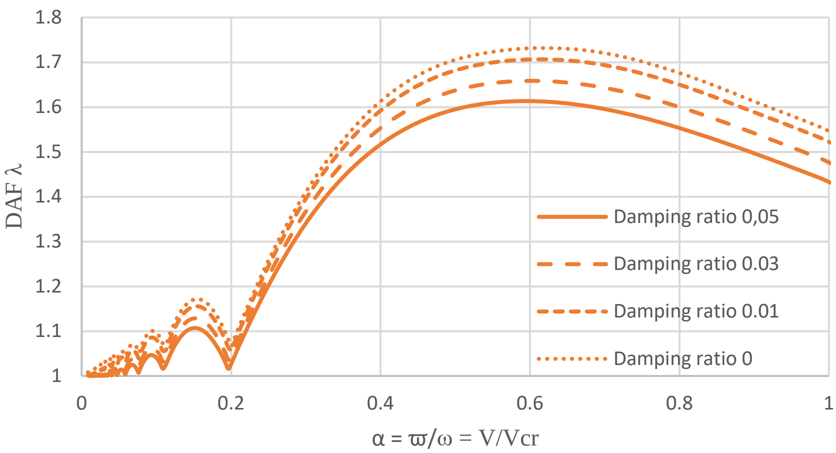

Figure 8: Dynamic Amplification Factor in the function of the speed parameter α

The expression for λ for rail bridges is the same for the domestic and European[4] regulations. The same figure shows that λ is higher than the recommended value by the domestic standard for road bridges. But, should be noted that the presented computation

[4] For an intensively maintained railways

conducted for load speed of *V = 130 km/h* and one concentrated force passing the bridge span of *10 m* is not a real case. Real vehicles, which could produce significant load effects, shold be represented with more than one axes (moving concentrated forces) and the real values shold be less. Also, the real speed of such vehicles is up to *80 km/h*, and as it is shown in Figure 7, DAF function for such speed are significantly less, beside the fact that the vehicle is treated as one concentrated load system.

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УТИЦАЈ БРЗИНЕ КРЕТАЊА ВОЗИЛА НА ДИНАМИЧКИ ФАКТОР МОСТОВА

Rezime: *У овом раду је приказана анализа утицаја брзине кретања возила на динамички фактор за случај мостовске конструкције конструктивног система просте греде, континуалног носача и обострано укљештене греде. Анализа је спроведена аналитички и нумерички. Развијен је теоријски израз за случај кретања једне концентрисане силе преко моста конструктивног система просте греде, а остали случајеви су анализирани нумерички. Разматран је случај покретне концентрисане силе и покретног система концентрисаних сила. Приказан је упоредни резултат најнеповољнијег случаја са важећом домаћом и иностраном регулативом. Резултати су показали да је неповољнији случај када се возила крећу већим брзинама на мостовима краћих распона, као и да једна концентрисана сила изазива веће динамичке ефекте од система концентрисаних сила, а да систем просте греде најинтензивније реагује на динамичка оптерећења од осталих разматраних поменутих статичких система.*

Кључне речи: *Динамички фактор мостова, брзина кретања возила, мостовке конструкције*