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EC2: DESIGN OF REINFORCED CONCRETE FOR SHEAR[2]

Summary

An overall comparison of the shear design according to EC2 (prEN 1992-1-1, July 2002.) with Yugoslav PBAB 87 is presented. Required amount of the reinforcing steel is analyzed through several levels of stressing. Changes of the first published ENV 1992-1-1 (December 1991.), related to the shear design, are also reviewed.

Key words: concrete structures, design, shear, Eurocode 2, PBAB 87

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[2] This paper arise from research on project of Serbian Ministry of science, technology and development: "Introduction of structural Eurocodes into national regulations"

1. Introduction

The final draft of EC2 – prEN 1992-1-1, issued in july 2002, contains a number of parameters that are left open for national choice (NDP – Nationally Determined Parameters) and should be established by local authorities in the National annex (NA), for use within the national territory. National annexes accommodate EC standards to the local conditions including environmental and economic aspects.

Following text offers a comparison of certain numerical results of the shear design of reinforced concrete elements preformed according to Yugoslav PBAB 87 and EC2. The recommended values for NDP (supplied by prEN 1992-1-1: 2002) are adopted for the design according to EC2.

Several parameters are varied through the analysis. The most relevant parameter is the shear stress. Shear design in EC2 is based on the shear force rather than the shear stress as it is with PBAB 87. However, the stress is more convenient for dimensionless analysis, and results obtained according to EC2 are recalculated into 'stress'. This approach may be useful for choice of NDP.

2. Shear design in former ENV 1992-1-1: 1991

The former EC2 (1991) provided two methods for the design of the shear reinforcement:

- the standard method which assumes inclination 45^o of compression struts and
- the variable strut inclination method (VSI), with the free choice of the angle of struts within limits.

The standard method was easier to use and included so-called secondary shear resistance, provided by concrete in the compression zone, friction in the inclined shear cracks and dowel effects. Shear resistance according to the standard method was the sum of resistance of the shear reinforcement and the secondary shear resistance, while the VSI method had only resistance of the shear reinforcement whenever the shear capacity of concrete was exceeded.

The latest EN 1992-1-1: 2002 omits the standard method and uses only the VSI method. Besides that, it is more conservative in estimation of the design shear capacity of elements without the shear reinforcement, in particular for higher concrete grades.

3. Review of the design parameters

Quality of concrete is the principal parameter relating the design shear resistance of a member without shear reinforcement, in both standards. The quality is stated through the compressive strength, being measured by the concrete grade (MB) in PBAB 87 and the strength class (C) in EC2. Quality of steel relates the shear resistance of members provided with the shear reinforcement. The quality is defined by the yield stress (s_v in PBAB 87; f_{yk} in EC2).

EC2 also involves two parameters not being treated by BAB 87, namely: the structural depth of a section (d) and the amount of reinforcing steel for flexure ($r_1 = A_s /bd$).

4. Parameters correlation

Notation of the strength class C in EC2 is followed by numerical values of the cylinder strength and the cube strength (C $f_{ck,cyl}$ /$f_{ck,cube}$). The size of test cube is 150 mm with statistical limit of 5%, so that, for a usual dispersion, the conversion factor for 200-mm cube and 10 % limit (PBAB 87) is near 1,0. This means that the second number in a strength class is close to the concrete grade (for example, C25/30 ≈ MB30). The yield stress of reinforcing steel has the same definition in both standards ($s_v = f_{yk}$).

Partial factor of safety for permanent actions (loads) is 1,35 (EC2) and 1,60 (PBAB 87, for dominant bending), while for variable actions equals 1,50 (EC2) and 1,80 (PBAB 87). Table 1 shows values of the average factor of safety for several ratios of permanent (g) and variable ($q \sim p$) loads.

Table 1: Average values of safety factor for loads

$g = (g_g\, g + g_q\, q)/(g + q)$	g = 1,0 q	g = 2,0 q	g = 3,0 q	g = 4,0 q	g = 5,0 q
g_{EC2}	1,425	1,400	1,388	1,380	1,375
$g_{PBAB\ 87}$	1,700	1,667	1,650	1,640	1,633
$g_{PBAB\ 87} / g_{EC2}$	1,193	1,190	1,189	1,188	1,188

It may be seen from Table 1 that the average safety factor according to EC2 is lower than one that follows PBAB 87. But, additional partial safety factors for the material properties are used when a design according to EC2 is performed: $g_s = 1{,}15$, for steel, and $g_c = 1{,}50$, for concrete. That gives similar value of the safety factor on the whole. For further analysis the value $g_{PBAB\,87}/g_{EC2} = 1{,}19$ is adopted.

5. Scope of the analysis

Three values of the structural depth (d = 300, 500, 700 mm) are combined with three strength of concrete (C25/30 ≈ MB30, C35/45 ≈ MB45, C50/60 ≈ MB60), with use of GA240/360 and RA400/500 reinforcing steel, when links are required. Objectives of this text are:

- the design shear resistance of the member without shear reinforcement;
- design value of the maximum shear force which can be sustained by the member;
- minimum value of the shear reinforcement ratio and its shear resistance;
- amount of the reinforcing steel (links) required to sustain shear force.

6. Shear resistance of the member without shear reinforcement

The design value for the shear resistance in EC2 is given by expression (6.2a). (Numeration of formulas from EC2 is according to prEN1992-1-1: July 2002.) The shear force $V_{Rd,c}$ (N) that can be resisted without shear reinforcement is:

$$V_{Rd,c} = \left[C_{Rd,c}\, k (100 r_l\, f_{ck})^{1/3} + k_1 s_{cp} \right] b_w d \qquad (6.2a)$$

but not less than:

$$V_{Rd,c} = \left[v_{min} + k_1 s_{cp} \right] b_w d \qquad (6.2b)$$

where

f_{ck} is the concrete strength in MPa (cylinder strength – strength class C);

$k = 1 + \sqrt{\frac{200}{d}} \leq 2{,}0$ with the structural depth d in mm;

$r_l = \frac{A_{sl}}{b_w d} \leq 0{,}02$ is the reinforcement ratio for the longitudinal reinforcement. The tensile reinforcement that can be included into area A_{sl} (mm^2) should extend beyond the section considered for a specified distance (explained on Figure 6.3 of prEN1992-1-1; not shown here);

b_w is the smallest width of the cross-section in the tensile area (mm);

s_{cp} is the section stress due to axial force, (neglected for further analysis).

The recommended value for $C_{Rd,c} = 0{,}18/g_c = 0{,}18/1{,}5 = 0{,}12$ is adopted here and

$$v_{min} = 0.035\, k^{3/2} f_{ck}^{1/2}. \qquad (6.3N)$$

Calculated values of the shear resistance (6.2a) and (6.2b) for strength classes C25/30, C35/45 and C50/60 are given in Table 2, 3 and 4 respectively.

Table 2: Shear resistance of C25/35 member without shear reinforcement

C25 ≈ MB30	$V_{Rd,c} / b_w d$ (MPa)				V_{min} (MPa)
$100\, r_1$ =	0,2	0,5	1,0	2,0	
d = 300 mm	0.373	0.506	0.637	0.803	0.428
d = 500 mm	0.335	0.455	0.573	0.722	0.365
d = 700 mm	0.315	0.427	0.538	0.678	0.333

Table 3: Shear resistance of C35/45 member without shear reinforcement

C35 ≈ MB45	$V_{Rd,c} / b_w d$ (MPa)				V_{min} (MPa)
$100\, r_1$ =	0,2	0,5	1,0	2,0	
d = 300 mm	0.417	0.566	0.713	0.898	0.507
d = 500 mm	0.375	0.509	0.641	0.807	0.432
d = 700 mm	0.352	0.478	0.602	0.759	0.394

Table 4: Shear resistance of C50/60 member without shear reinforcement

C50 ≈ MB60	$V_{Rd,c} / b_w d$ (MPa)				V_{min} (MPa)
100 r_1 =	0,2	0,5	1,0	2,0	
d = 300 mm	0.470	0.637	0.803	1.012	0.606
d = 500 mm	0.422	0.573	0.722	0.909	0.516
d = 700 mm	0.397	0.538	0.678	0.855	0.470

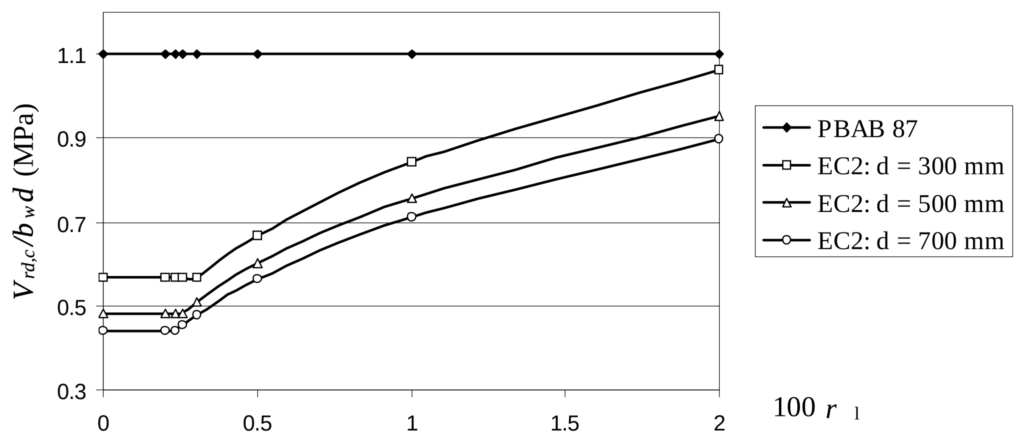
In the clause 89 PBAB 87 specifies the design values of shear resistance t_r of concrete. The corresponding shear force is obtained multiplying shear stress t_r by the lever arm z of internal forces and the smallest width b of the tensile zone. Since the multiplier in Expression (6.2a) is $d \approx z/0.9$ and the ultimate load in EC2 is approximately 1,19 lower, the values from Tables 2-4 should be multiplied by 1,19/0,9 = 1,32 for reasons of comparison with t_r. Results are summarized in Table 5. The limits for EC2 are obtained from the shaded values of Tables 2-4.

Table 5: Shear capacity of the concrete without the shear reinforcement (stress)

Class	$EC2_{min}$ (MPa)	$EC2_{max}$ (MPa)	t_r (MPa)	Grade
C25/30	0,44	1,06	1,10	MB30
C35/45	0,52	1,19	1,40	MB45
C50/60	0,62	1,34	1,60	MB60

The values from Table 5 show that EC2 introduces shear reinforcement at the significantly lower level

Figure 1: Shear resistance of C25/30 (MB30) member without shear reinforcement



of shear than PBAB 87 does. In diagrams on the Figure 1 all parameters are plotted for C25/30 (MB30).

7. Maximum shear force

The design value of maximum shear force that can be sustained by the member, limited by crushing of the compression struts, is given by Expression (6.9) of EC2. For elements with vertical shear reinforcement and the inclination of compression struts of 45°, it gives:

$$V_{Rd,max} = 0{,}5 a_c\, b_w\, z\, n\, f_{ck} / g_c \qquad (6.9)$$

where:

$$n = 0{,}6\left[1 - \frac{f_{ck}}{250}\right], \quad f_{ck} \text{ in MPa} \qquad (6.6)$$

and the recommended value for a_c is 1, for non-prestressed structures (6.11aN). Comparison of ($V_{Rd,max}/zb_w$) multiplied by the ultimate load factor 1.19, with $5t_r$ of PBAB 87 is presented in Table 6.

Table 6: Maximum shear stress

		C25/30	C35/45	C50/60
EC2	$(V_{Rd,max} / z\, b_w) \times 1{,}19$	5,36	7,16	9,52
PBAB87	$5t_r$	5,50	7,00	8,00
		MB30	MB45	MB60

The values by EC2 and by PBAB 87 in Table 6 are very close, except for the high strength (MB60). Reduced values of all concrete strength parameters for the high-grade concrete result from a precautionary approach at the time of introduction of PBAB 87.

9. Minimum area of shear reinforcement

Both EC2 and PBAB 87 introduce the minimum area of shear reinforcement for beams, whenever the shear capacity of concrete is exceeded. PBAB 87 states (clause 94) that the ratio of shear reinforcement should not be less than 0,2 %. The recommended value of minimum ratio in EC2 is given by:

$$r_{w,min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yk}} \qquad \text{(9.5N)}$$

Calculated values of $r_{w,min}$ for RA 400/500 (f_{yk} = 400 MPa) and GA 240/360 (f_{yk} = 240 MPa) are presented in the Table 7.

Table 7: Minimum values of shear reinforcement ratio by EC2

$r_{w,min}$	C25/30	C35/45	C50/60
RA 400/500	0,100 %	0,118 %	0,141 %
GA 240/360	0,167 %	0,197 %	0,236 %

The minimum amount of shear reinforcement is in the most cases significantly smaller in EC2 than one in BAB 87. The value of shear that can be sustained with the minimum area of shear reinforcement is

$$\frac{V_{Rd,s,min}}{b_w z} = r_{w,min}\, f_{yd} = r_{w,min}\frac{f_{yk}}{g_s} = 0{,}0696\sqrt{f_{ck}} \qquad (f_{ck} \text{ in MPa}),$$

If we multiply previous value by the ratio $z/d \approx 0{,}9$ (for easier comparison with the values from Tables 2-4) it comes to the result shown in Table 8.

Table 8: Shear capacity of minimum shear reinforcement $V_{Rd,s,min}$ and concrete $V_{Rd,c,min}$ (EC2)

	C25/30	C35/45	C50/60
$V_{Rd,s,min} / b_w d$ (MPa)	0,313	0,370	0,443
$V_{Rd,c,min} / b_w d$ (MPa) d = 300 mm	0,428	0,507	0,606
$V_{Rd,c,min} / b_w d$ (MPa) d = 500 mm	0,365	0,432	0,516
$V_{Rd,c,min} / b_w d$ (MPa) d = 700 mm	0,333	0,394	0,470

It is apparent that minimum shear reinforcement does not cover the shear capacity of concrete.

10. Design of shear reinforcement

Required area of the vertical links by EC2 is slightly smaller than one by PBAB 87 when the shear stress exceeds value $3\tau_r$. Partial safety factor for steel g_s = 1,15 combined with the ultimate load ratio 1,19 gives the total ratio (EC2 : PBAB 87) = 1,15/1,19 = 0,966. But, with the shear ranging from t_r to $3t_r$, PBAB 87 includes shear resistance of concrete, while EC2 takes into account only the resistance of shear reinforcement. Due to the reduced value of shear stress sustained by the reinforcement (t_{Ru}) PBAB 87 requires less reinforcement than EC2 in the range $t_r \div 3t_r$.

Calculated values of the shear reinforcement ratio r_w (%, vertical links) for several levels of the shear load ($0{,}5t_r$ - $3t_r$) are presented in Tables 9 and 10 for C25/30 (MB30) and C50/60 (MB60) respectively.

Table 9: Shear reinforcement ratio r_w (%) for C25/30 » MB30

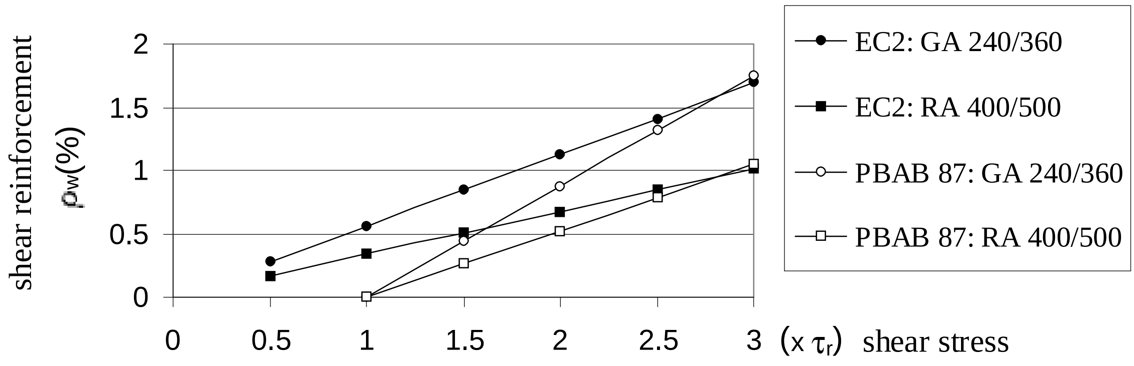
C25/30 ~ MB30 (t_r = 1,1 MPa)			GA 240/360		RA 400/500	
Shear load	t (MPa)	t_{Ru} (MPa)	$r_{w,BAB\ 87}$	$r_{w,EC2}$	$r_{w,BAB\ 87}$	$r_{w,EC2}$
0,5 t_r	0,55	0	-	0,222	-	0,133
1,0 t_r	1,10	0	-	0,444	-	0,266
1,5 t_r	1,65	0,825	0,344	0,666	0,206	0,399
2,0 t_r	2,20	1,650	0,688	0,888	0,413	0,532
2,5 t_r	2,75	2,475	1,032	1,110	0,619	0,664
3,0 t_r	3,30	3,300	1,375	1,332	0,825	0,797

Table 10: Shear reinforcement ratio r_w (%) for C50/60 » MB60

C50/60 ~ MB60 (t_r = 1,6 MPa)			GA 240/360		RA 400/500	
Shear load	t (MPa)	t_{Ru} (MPa)	$r_{w,BAB\ 87}$	$r_{w,EC2}$	$r_{w,BAB\ 87}$	$r_{w,EC2}$
0,5 t_r	0,80	0	-	0,322	-	0,193
1,0 t_r	1,60	0	-	0,644	-	0,387
1,5 t_r	2,40	1,20	0,500	0,966	0,300	0,587
2,0 t_r	3,20	2,40	1,000	1,289	0,600	0,773
2,5 t_r	4,00	3,60	1,500	1,611	0,900	0,966
3,0 t_r	4,80	4,80	2,000	1,933	1,200	1,160

Required area of shear reinforcement for C35/45 (MB45) is given in the Figure 2.

Figure 2: Required area of links for C35/45 (MB45)



11. Conclusion

- EC2 involves more parameters and appears to be more conservative in estimation of the shear capacity of concrete member without the design shear reinforcement than PBAB 87.
- Maximum shear that can be sustained by the concrete member is similar for EC2 and PBAB 87 with the exception of high concrete grades, where PBAB 87 is more conservative.
- The minimum area of shear reinforcement (links) required by EC2 is significantly lower than one by PBAB 87 and, in most cases, is lower than design capacity of concrete member without the shear reinforcement.
- The overall factor of safety for design of links is slightly lower by EC2 than PBAB 87 (3,4 %). But, once the shear reinforcement is required, EC2 does not include any resistance of the concrete. It results in more shear reinforcement by EC2 than PBAB 87 in the range of shear stress from t_r to $3t_r$. In the range of shear stress from $3t_r$ to $5t_r$ EC2 gives 3,4 % less of shear reinforcement.
- National annex should provide some accommodations of the recommended values for NDP.

12. Literature

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