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ELASTIČNA KRITIČNA SILA REBRA I-NOSAČA OPTEREĆENIH LOKALIZOVANIM OPTEREĆENJEM

Rezime:

U radu se razmatra elastična kritična sila rebra čeličnih I nosača pod dejstvom lokalizovanog opterećenja u ravni rebra. Izvršena je analiza uticaja dužine opterećenja i debljine pojasa na elastičnu kritičnu silu kod nosača bez poprečnih i podužnih ukrućenja u zoni unošenja opterećenja. Određeni su koeficijenti izbočavanja za rebra I nosača i upoređeni su sa vrednostima koeficijenta izbočavanja izolovanih uklještenih ploča. Dat je predlog za poboljšanje proračuna granične nosivosti prema Evrokodu EN1993-1-5.

Ključne reči: elastična kritična sila, koeficijent izbočavanja, lokalizovano opterećenje

ELASTIC CRITICAL LOAD OF THE WEB OF I-GIRDERS SUBJECTED TO PATCH LOADING

Summary:

The paper considers the elastic critical (buckling) load of steel I-girders subjected to patch loading in the plane of the web. The analysis of the influence of the load length and thickness of the flange on the elastic critical load of the girder without transverse and longitudinal stiffeners in the load application zone was performed. The buckling coefficients for the web of the I-girders were determined and compared with the values of the buckling coefficient of isolated clamped plates. Suggestions for improving the EN1993-1-5 patch loading resistance model are given.

Key words: elastic critical load, buckling coefficient, patch loading

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1. INTRODUCTION

A particular problem of the stability and ultimate load behavior of steel plate girders, which has been attracting attention in recent decades, relates to the behavior of I-girders under a localized load on the flange in the plane of web. The influence of the patch load (or partially distributed load) on the behavior of I-girders without vertical stiffener in the zone of load introduction was intensively investigated. This problem has got the importance with a general trend to avoid vertical stiffeners, except at supports and also in the case of moving loads, e.g., crane girders loaded by crane wheels and launching phase of multi-span steel plate girder bridges during construction over temporary or permanent supports.

In order to describe the behavior of patch loaded girders, it is necessary to determine the distribution of the local direct stresses under the load in the web, elastic critical load of the web panel, and the ultimate load of the girder. No complete insight into the solution to the problem has been provided. The development of this field has increased sharply with the improvement of computer programs based on the finite element method, which has enabled a wide range of numerical experiments necessary to link a large number of variables.

The development of Eurocodes for the design of civil engineering structures has brought a new approach to harmonize solutions to the problem of patch load with other stability problems. The main points in determining the ultimate load are elastic critical load, yield load, and resistance function. The determination of the elastic critical (buckling) load got the importance again, as it is needed to determine the slenderness parameter.

The results for the ultimate load given in Eurocode EN1993-1-5 [1] are significantly conservative compared to the results obtained experimentally [2]. An attempt is made in this paper to give some improvements to that procedure. Eurocode EN1993-1-5 [1] provides a simplified expression for the calculation of the buckling coefficient. For a plate girder under patch load applied on the top flange, between two vertical stiffeners on a distance *a*, the buckling coefficient should be obtained using Eq. (1):

$$k_F = 6 + 2\left(\frac{h_w}{a}\right)^2 \tag{1}$$

As it could be observed, the given expression in Eq. (1) does not take into account the length of an applied patch load. This expression for the determination of the buckling coefficient is adopted by simplifying the expression proposed by Lagerqvist [3]:

$$k_F = \left(1 + \frac{s_s}{2h_w}\right)\left(5.3 + 1.9\left(\frac{h_w}{a}\right)^2 + 0.4\sqrt[4]{\beta}\right), \ \beta = \frac{b_f \cdot t_f{}^3}{h_w \cdot t_w{}^3} \tag{2}$$

The following notations are used in this paper to describe the problem of patch loading, compare the results, and derive conclusions: h_w – web depth, t_w – web thickness, t_f – flange thickness, b_f – flange width, a – distance of vertical stiffeners, s_s – patch load length, ν – Poisson's ratio, E – modulus of elasticity, k_F – buckling coefficient, which can be seen in Fig. 1.

Unlike Eq. (1), expression (2) includes the ratio between the flange and web stiffness and a ratio between the load length and the web depth.

The results and conclusions given in the paper are a continuation of the previous research [4]. Influence of the patch load length was investigated. The buckling coefficients of I-girders were determined and compared with the corresponding buckling coefficients of the isolated web of the girder by varying boundary conditions [4]. It was concluded that the behavior of I-girders best corresponds to the behavior of clamped plates. The results were compared with the expressions of Eurocode EN1993-1-5. Suggestions for improving existing terms were given.

Influence of the patch load length and flange thickness is investigated in this paper. Following numerical analysis is based on the finite element method. The commercial software Abaqus [5] was used in this research as the most popular computation tool for application in this field.

2. NUMERICAL SIMULATION

The purpose of this paper is the determination of the critical loads of an I-girder and its comparison with the elastic critical loads of an isolated web plate (that was modeled as a plate clamped along all edges), using FE analysis. The plate and I-girder with a web panel aspect ratio $a/h_w = 1$ and plate (web) thickness 4 mm (schematically presented in Fig. 1), were investigated. Patch load length s_s and flange thickness t_f were varied. The boundary conditions for the plate were set according to the clamped plate – that is, degrees of freedom 2 and 5 are only constrained in the vertical edges, degree of freedom 4 in the horizontal edges, while degree of freedom 3 is constrained in all edges.

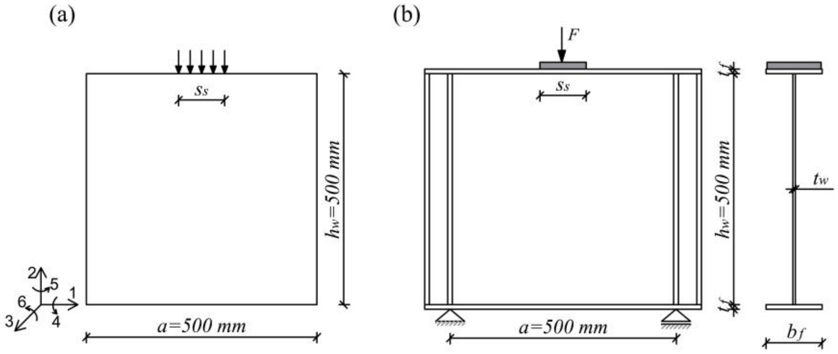

Figure 1 – (a) Plate under patch load; (b) I-girders under patch load

For the I-girder considered, the flange width was set to 120 mm, while the flange thickness was set to 8, 16, and 24 mm. The considered material was homogenous with an elastic modulus of 205 GPa and Poisson's ratio of 0.3.

For the FE analysis, a general-purpose four-node quadrilateral shell element with reduced integration and six degrees of freedom per node (S4R) from the Abaqus element library was used. Finite element size of 5 mm was adopted for all numerical runs.

3. RESULTS AND DISCUSSION

The results of numerical analysis in the form of buckling coefficients for the I-girder are given in this chapter and compared to the buckling coefficients for the clamped plate. Previously derived expressions in [4] are now associated with another variable – t_f. In that way, the influence of the flange on the calculation of the elastic critical buckling force is taken into account. New expressions for determining the buckling coefficient, as a function of patch load length and flange thickness, are presented. The purpose of the proposed expressions is to improve the ultimate load calculated using the procedure given in Eurocode EN1993-1-5.

3.1. BUCKLING COEFFICIENT OF THE I-GIRDER AS A FUNCTION OF PATCH LOAD LENGTH AND FLANGE THICKNESS

Table 1, Table 2, and Table 3 show a comparison summary of the buckling coefficient obtained by numerical analysis, by Eurocode EN1993-1-5 - Eq. (1), and according to Lagerqvist's expression - Eq. (2), for various flange thicknesses. The analysis of clamped plate (hereinafter referred to as CC plates) was chosen because it has been shown that the buckling coefficient for CC plates corresponds better to the buckling coefficient of I-girders than the buckling coefficients obtained varying boundary conditions of plates, e.g., simply supported plates and plates simply supported on the vertical and clamped on the horizontal edges [6].

Table 1 – Buckling coefficient for flange thickness 8 mm

s_s [mm]	s/h_w	k_F			
		CC	I-girder	EC3	Lagerqvist
0	0.00	8.00	8.98	8.00	7.67
25	0.05	8.02	9.06	8.00	7.86
50	0.10	8.10	9.23	8.00	8.05
75	0.15	8.20	9.44	8.00	8.25
100	0.20	8.35	9.70	8.00	8.44
125	0.25	8.52	10.01	8.00	8.63
150	0.30	8.74	10.37	8.00	8.82
200	0.40	8.97	11.25	8.00	9.21
250	0.50	9.26	12.37	8.00	9.59

A graphical representation of the results is shown in Fig. 2. There are significant differences between the results obtained according to Eurocode EN1993-1-5 and those obtained numerically for I-girders. They are more noticeable for large values of s_s. It could be observed in Fig. 2 that the Eurocode expression gives constant values of the buckling coefficient (i.e., independent of the flange thickness and patch load length), while Lagerqvist's expression has a linear trend line. However, the proposed Eq. (2) does not give a good fit to the numerically obtained results for I-girders.

Table 2 – Buckling coefficient for flange thickness 16 mm

s_s [mm]	s/h_w	k_F			
		CC	*I-girder*	*EC3*	*Lagerqvist*
0	0.00	8.00	10.10	8.00	7.99
25	0.05	8.02	10.19	8.00	8.19
50	0.10	8.10	10.40	8.00	8.39
75	0.15	8.20	10.68	8.00	8.59
100	0.20	8.35	11.01	8.00	8.79
125	0.25	8.52	11.40	8.00	8.99
150	0.30	8.74	11.86	8.00	9.19
200	0.40	8.97	13.00	8.00	9.59
250	0.50	9.26	14.52	8.00	9.99

Table 3 – Buckling coefficient for flange thickness 24 mm

s_s [mm]	s/h_w	k_F			
		CC	*I-girder*	*EC3*	*Lagerqvist*
0	0.00	8.00	11.00	8.00	8.27
25	0.05	8.02	11.09	8.00	8.48
50	0.10	8.10	11.34	8.00	8.69
75	0.15	8.20	11.69	8.00	8.89
100	0.20	8.35	12.11	8.00	9.10
125	0.25	8.52	12.60	8.00	9.31
150	0.30	8.74	13.18	8.00	9.51
200	0.40	8.97	14.65	8.00	9.93
250	0.50	9.26	16.68	8.00	10.34

As Eq. (1) and Eq. (2) have obvious shortcomings for predicting the buckling coefficient for the analyzed I-girder models, they need to be corrected.

The expression for the buckling coefficient as a function of patch load length - Eq. (3) is defined in earlier research to fit the obtained discrete values of k_F for the I-girders with a constant flange thickness [4]. In the attached Table 1, Table 2, and Table 3, it can be observed that the buckling coefficient depends not only on the patch load length s_s, but also on the flange thickness. Accordingly, the correction of Eq. (3) was performed by introducing the influence of the flange thickness on the buckling coefficient of the I-girder, given in Eq. (4). A similar form of the previously derived equation is retained.

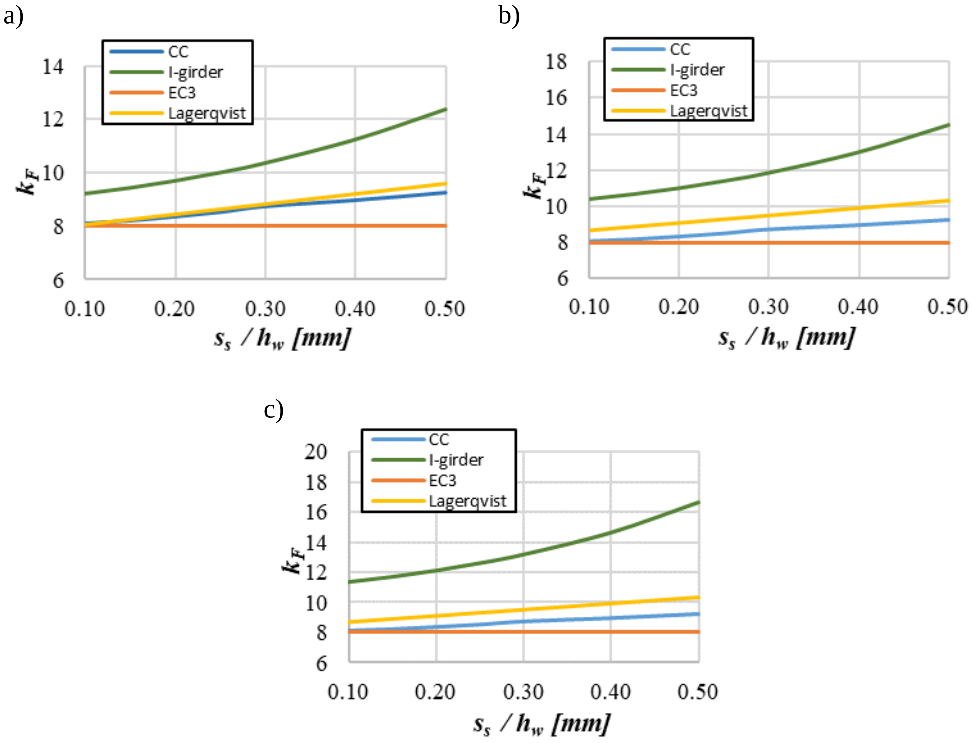

Figure 2 – Comparison of the buckling coefficient for flange thickness (a) t_f= 8 mm; (b) t_f= 16; mm (c) t_f= 24 mm

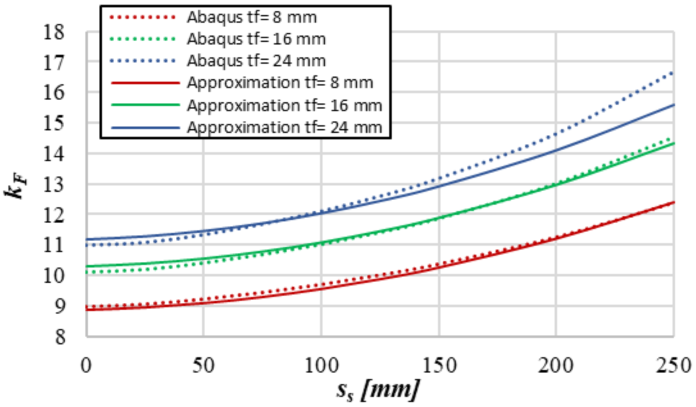

Figure 3 – k_F values for I-girders obtained in Abaqus and using the proposed expression (4)

$$k_F = 4.7 \cdot 10^{-5} s_s^{\,2} + 2 \cdot 10^{-3} s_s + 8.7 \qquad (3)$$

$$k_F = t_f^{\,0.21} \cdot (3.1 \cdot 10^{-5} s_s^{\,2} + 1.32 \cdot 10^{-3} s_s + 5.74) \qquad (4)$$

The correlation between the buckling coefficient of the I-girder and the coefficient approximated by the proposed expressions is shown in Fig. 3, for all flange thickness analyzed. All derived conclusions are determined only for the cases of the analyzed geometry. A detailed parametric study is necessary to obtain a unique general expression also as a function of other geometric parameters.

3.2. COMPARISON OF THE BUCKLING COEFFICIENT OF THE I-GIRDER WITH THE BUCKLING COEFFICIENT OF AN ISOLATED CLAMPED WEB PLATE

Further analysis aims to find the relationship between the buckling coefficient for I-girders and CC plates. It has been numerically shown, that the buckling coefficient of I-girders can be represented by the expression for a CC plates, if the fact is taken into account that the flange plate enables spreading the loading length into the web [4]. On girders, the patch load is introduced on the flange plate, and the angle of stress distribution through the flange is defined in the design standard as 45˚ [1]. As k_F values for the I-girder are larger than k_F values for the CC plate for the same patch load length, it is believed that this difference comes as a result of load distribution. If the stress distribution with a slope in the function of flange thickness is assumed instead of the defined slope 1:1, the corresponding distributed stress length is s_s'.

Table 4 – Relation between the buckling coefficient for CC plates and I-girders for $t_f = 8$ mm

s_s [mm]	s/h_w	k_F	$k_F\,(s_s)$	$s_s'=s_s+2\cdot 8\,t_f$ [mm]	$k_F\,(s_{s'})$	k_F	k_F (I-girder) / $k_F\,(s_{s'})$-Eq. (6)
		CC	Eq. (5)		Eq. (6)	I-girder	
0	0.00	8.00	8.00	128.00	8.49	8.98	1.06
25	0.05	8.02	8.02	153.00	8.70	9.06	1.04
50	0.10	8.10	8.08	178.00	8.95	9.23	1.03
75	0.15	8.20	8.17	203.00	9.24	9.44	1.02
100	0.20	8.35	8.30	228.00	9.56	9.70	1.02
125	0.25	8.52	8.47	253.00	9.92	10.01	1.01
150	0.30	8.74	8.68	278.00	10.32	10.37	1.01
200	0.40	8.97	9.20	328.00	11.23	11.25	1.00
250	0.50	9.26	9.88	378.00	12.29	12.37	1.01

Table 5 – Relation between the buckling coefficient for CC plates and I-girders for t_f =16 mm

s_s [mm]	s/h_w	k_F	$k_F\ (s_s)$	$s_s'=s_s+2\cdot 7t_f$ [mm]	$k_F\ (s_{s'})$	k_F	k_F (I-girder)/ k_F ($s_{s'}$)-Eq. (6)
		CC	Eq. (5)		Eq. (6)	I-girder	
0	0.00	8.00	8.00	224.00	9.51	10.10	1.06
25	0.05	8.02	8.02	249.00	9.86	10.19	1.03
50	0.10	8.10	8.08	274.00	10.25	10.40	1.01
75	0.15	8.20	8.17	299.00	10.68	10.68	1.00
100	0.20	8.35	8.30	324.00	11.15	11.01	0.99
125	0.25	8.52	8.47	349.00	11.65	11.40	0.98
150	0.30	8.74	8.68	374.00	12.20	11.86	0.97
200	0.40	8.97	9.20	424.00	13.39	13.00	0.97
250	0.50	9.26	9.88	474.00	14.74	14.52	0.98

Table 6 – Relation between the buckling coefficient for CC plates and I-girders for t_f = 24 mm

s_s [mm]	s/h_w	k_F	$k_F\ (s_s)$	$s_s'=s_s+2\cdot 6t_f$ [mm]	$k_F\ (s_{s'})$	k_F	k_F (I-girder)/ k_F ($s_{s'}$)-Eq. (6)
		CC	Eq. (5)		Eq. (6)	I-girder	
0	0.00	8.00	8.00	288.00	10.49	11.00	1.05
25	0.05	8.02	8.02	313.00	10.94	11.09	1.01
50	0.10	8.10	8.08	338.00	11.43	11.34	0.99
75	0.15	8.20	8.17	363.00	11.95	11.69	0.98
100	0.20	8.35	8.30	388.00	12.52	12.11	0.97
125	0.25	8.52	8.47	413.00	13.12	12.60	0.96
150	0.30	8.74	8.68	438.00	13.76	13.18	0.96
200	0.40	8.97	9.20	488.00	15.14	14.65	0.97
250	0.50	9.26	9.88	538.00	16.68	16.68	1.00

The buckling behavior of the CC plate with an applied load length s_s', gives a very good match with the behavior of the I-girder with an applied load length s_s. The corresponding distributed stress length s_s' could be calculated as: $s_s + 2\cdot t_f \cdot (9-0.125t_f)$.

$$k_F = 3\cdot 10^{-5} s_s^{\,2} + 8 \tag{5}$$

$$k_F = 8 + 3\cdot 10^{-5}\cdot\left[s_s + 2t_f\cdot\left(9-0.125t_f\right)\right]^2 \tag{6}$$

This is presented in Table 4, Table 5, and Table 6 (for various flange thicknesses), where for calculation of k_F for the CC plate with an applied load length s_s', previously given in Eq. (5) is used. The presented values can be compared to the k_F values for I-girder, which are obtained in numerical simulation. Based on the values shown, it is noticed that the length of the distribution depends on the flange thickness and it varies as a function of t_f. Eq. (6) is proposed for the calculation of the buckling coefficient, obtained by a comparative analysis of the behavior of the clamped plate and the girder of the varied flange thickness. The ratio of the buckling coefficient derived by numerical models and using the proposed equation is given in Table 4, Table 5, and Table 6 within the last column.

3.3. COMPARISON OF THE ULTIMATE LOADS NUMERICALLY OBTAINED AND FROM EUROCODE

Values of some characteristic numerically obtained ultimate loads F_u^{FEA} from [6] and the corresponding ultimate loads according to the procedure applied in Eurocode EN1993-1-5 (using values for the buckling coefficient given in Eurocode F_u^{EC} and using the proposed values in this paper $F_u^{EC\,p}$ – Eq. (4)) are compared in Table 7. Fig. 4 shows a case for flange thickness $t_f = 16$ mm.

Table 7 – Comparison of the ultimate loads for various approaches

t_f [mm]	s_s [mm]	F_u^{EC} [kN]	$F_u^{EC\,p}$ [kN]	F_u^{FEA} [kN]	F_u^{EC} / F_u^{FEA}	$F_u^{EC\,p} / F_u^{EC}$
8	0	105.47	111.14	128.94	0.82	1.05
	100	131.25	143.53	203.05	0.65	1.09
	200	152.74	180.81	263.25	0.58	1.18
16	0	125.23	141.93	196.55	0.64	1.13
	100	147.60	173.59	251.93	0.59	1.18
	200	167.00	212.61	308.60	0.54	1.27
24	0	145.34	171.88	254.93	0.57	1.18
	100	165.01	202.51	308.00	0.54	1.23
	200	174.68	222.04	401.17	0.44	1.27

It can be concluded that the proposed expression for the buckling coefficient Eq. (4) improves results for the ultimate load obtained with Eurocode EN1993-1-5, especially with the increased load length and flange thickness. The results could be improved up to 27% for the cases considered.

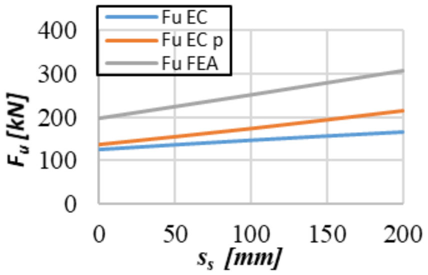

Figure 4 – Comparison of the ultimate loads for various approaches

4. CONCLUSIONS

The elastic critical load of I-girders under patch loading is considered in this paper. The expression for the buckling coefficients of I-girders as a function of patch load length and flange thickness is proposed – Eq. (4).

The buckling coefficients of I-girders and isolated plate corresponding to the web of I-girders are compared for various patch load lengths and flange thicknesses. It was found that the flange enables spreading the loading into the web of I-girder as a function of flange thickness, and independent of loading length.

Comparison of the ultimate loads determined (a) numerically, and according to Eurocode EN1993-1-5 (using (b) values for k_F given in EN1993-1-5 and (c) values for k_F proposed in this paper) is made. The present Eurocode EN1993-1-5 gives rather conservative values for the ultimate loads, but significant improvement can be obtained by application of the here proposed expression for buckling coefficients k_F.

The presented results and conclusions referred to the concrete geometry of the girder and observed variables. There are a need and a lot of space for further research on this topic. For a complete analysis, various parameters describing their buckling behavior must be taken into account, as the web thickness, web depth, web and flange slenderness, length zone between vertical stiffeners as well as different relationships of the mentioned quantities (for example, the ratio of web and flange thickness). The presented results enable a valuable background for the continuation of parametric analysis, where a significant amount of numerical tests is necessary to draw comprehensive conclusions.

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