ELASTIC CRITICAL LOAD OF STIFFENED I-GIRDERS SUBJECTED TO PATCH LOADING

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Summary: *The paper considers the elastic critical load of the isolated web plate and steel I-girders subjected to patch loading. The girder without transverse stiffeners in the load application zone, but with longitudinal stiffeners was analyzed. The influence of the load length, flange thickness and stiffener thickness on the buckling coefficient is obtained. The buckling coefficients of clamped plate and I-girder are compared, and suggestions for improving the EN1993-1-5 patch loading resistance model are presented.*

Keywords: *elastic critical load, buckling coefficient, patch loading*

1. INTRODUCTION

The stability problem and ultimate load behaviour of steel plate girders have been attracting attention in recent decades. The I-girders' behaviour without vertical stiffener in the zone of load introduction, under the localized load on the flange in the plane of the web, was intensively investigated. This problem has got the importance with a general trend to avoid vertical stiffeners (except at supports) for moving loads, e.g., crane girders loaded by crane wheels and during the incremental launching of multi-span steel bridges over temporary or permanent supports. The distribution of the local direct stresses under the load in the web, the elastic critical load of the web panel, and the ultimate load of the girder are necessary to define the behaviour of the patch loaded girder. Computer programs based on the finite element method have enabled a wide range of numerical experiments. Still, no complete insight into the solution to the problem has been provided. The research subject in this paper are girders with longitudinal stiffeners as a continuation of previous research related to unstiffened girders [2].

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The development of Eurocodes for the design of civil engineering structures has brought a new approach to harmonize solutions of the patch load problem with other stability problems. The ultimate patch load determination is closely related to the elastic critical load, yield load, and reduction factor. The buckling coefficient of the steel plate girders obtained using the procedures given in the Eurocode EN1993-1-5 [3] leads to conservative values of the ultimate load. An attempt is made in this paper to give some improvements to that procedure. Eurocode EN1993-1-5 [3] provides a simplified expression for the calculation of the buckling coefficient. For a plate girder under patch load applied on the top flange with longitudinal stiffeners, between two vertical stiffeners on a distance a, the buckling coefficient should be obtained using Eq. (1) and Eq. (2):

$$k_F = 6 + 2\left(\frac{h_w}{a}\right)^2 + \left[5.44\frac{b_1}{a} - 0.21\right]\cdot\sqrt{\gamma_s} \tag{1}$$

$$\gamma_s = 10.9\frac{I_{sl1}}{h_w \cdot t_w^{\,3}} \le 13\cdot\left[\frac{a}{h_w}\right]^3 + 210\left[0.3 - \frac{b_1}{h_w}\right] \quad , \tag{2}$$

where I_{sl1} - second moments of area of the stiffener closest to the loaded flange including contributory parts of the web

Generally, the moment of inertia of the longitudinal stiffener is calculated for a cross section according to Eurocode EN1993-1-5 [3] with respect to its centroidal axis parallel with the web plane. The effective cross section consists of the stiffener itself and an effective portion of the web plate having a width of $15\varepsilon t_w$ on each side of the stiffener weld.

As it could be observed, the given expression in Eq. (1) does not take into account the length of an applied patch load and influence of flange.

The following notations are used in this paper to describe the problem of longitudinally stiffened steel plate girders subjected to the patch loading, to compare the results, and to derive conclusions: h_w – web depth, t_w – web thickness, t_f – flange thickness, b_f – flange width, a – distance between vertical stiffeners, b_1 – depth of the loaded subpanel taken as the clear distance between the loaded flange and the stiffener, s_s – patch load length, ν – Poisson's ratio, E – modulus of elasticity, k_F – buckling coefficient, γ_s - relative flexural rigidity of the stiffener, γ_s^* - limit value of relative flexural rigidity of the longitudinal stiffener .

Previous studies [2] have shown the influence of the flange thickness and patch load length on the buckling coefficient of steel plate girders without longitudinal stiffeners. The buckling coefficients of I-girders and isolated plates corresponding to the web of I-girders were compared. Improved expressions for calculating the buckling coefficient were given, and the shortcomings of the Eurocode EN1993-1-5 [3] were pointed out [2].

In this paper, the influence of load length ss, flange and longitudinal stiffener thickness (tf, ts) on the critical load of the I-girder is given. In particular, the isolated web of the girder and I-girders are analyzed. The results were compared with the expressions in the Eurocode EN1993-1-5. Suggestions for improving existing terms were given. Following numerical analysis is based on the finite element method. The commercial software

Abaqus [4] was used in this research to obtain the elastic critical buckling load. All further conclusions refer to precisely defined material and geometric characteristics.

2. NUMERICAL SIMULATION

The purpose of this paper is to show the influence of longitudinal stiffeners on the elastic critical load of an isolated web plate (that was modelled as a plate clamped along all edges) and I-girder, using FE analysis. The plate and I-girder with a web panel aspect ratio a/h_w = 1 and plate (web) thickness of 4 mm (schematically presented in Fig. 1) were investigated. Since it has been shown [1] that the clamped plate (hereinafter referred to as CC plates) best suits the behaviour of the girder, the boundary conditions are set as follows: degrees of freedom 2 and 5 are only constrained in the vertical edges, degree of freedom 4 in the horizontal edges, while degree of freedom 3 is constrained in all edges. The length of an applied uniform load s_s, flange and stiffener thickness (t_f, t_s) were varied.

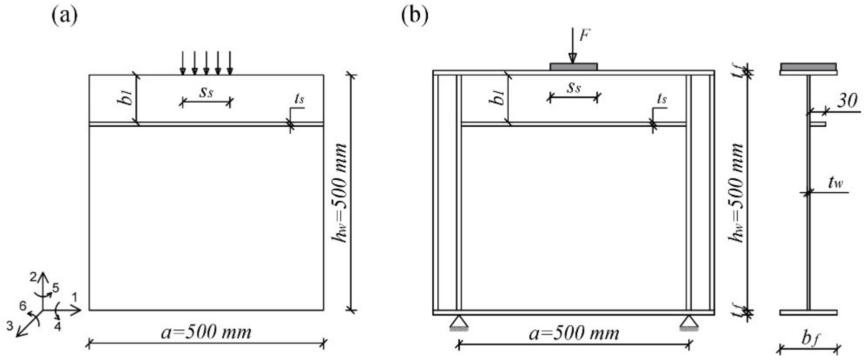

Figure 1. (a) Plate under patch load; (b) I-girders under patch load

The flange width of the I-girder was set to 120 mm, while the flange thickness was set to 8, 16, and 24 mm. The longitudinal stiffener thickness corresponds to the flange thickness. Thus, the derived conclusions apply only to ratio t_f/t_s = 1. The material considered was homogenous with an elastic modulus of 205 GPa and Poisson's ratio of 0.3. A general-purpose four-node quadrilateral shell element with reduced integration and six degrees of freedom per node (S4R) from the Abaqus element library was used for the FE analysis. All numerical runs were performed with a 5 mm finite element mesh size.

3. RESULTS AND DISCUSSION

As a result of the numerical analysis of an isolated web plate and I-girder, the buckling coefficient k_F was obtained for different flexural rigidity of the stiffener and flange thicknesses. The results are given as a function of patch load length. The influence of stiffener on the buckling coefficient of the clamped plate was analyzed first, and then the contribution of the flange. Compared with the propositions of Eurocode EN1993-1-5, the expressions for improving the existing procedures are derived. Finally, the influence of the proposed equations on the ultimate load value is given.

3.1. Buckling coefficient of the isolated web plate as a function of patch load length and stiffener thickness

Table 1 shows the buckling coefficients k_F of isolated clamped web plates. The values are given as a function of the patch load length s_s, and stiffener thickness t_f, varying from 0 (unstiffened plate) to 24 mm. The boundary conditions corresponding to the clamped plate were chosen for analysis. Previous studies have shown that the CC plate corresponds better to the behaviour of the I-girder than simply supported plates and plates simply supported on the vertical and clamped on the horizontal edges [1].

Table 1. Buckling coefficient k_F of isolated clamped web plates, as a function of patch load lengths ss and stiffener thicknesses t_f

t_f[mm] / s_s [mm]	0	2 γ_s=6.87	4 γ_s=12.39	8 γ_s=21.21	16 γ_s=34.2	24 γ_s=44.27
0	8.00	10.70	11.55	12.83	14.14	14.54
25	8.02	10.84	11.73	13.00	14.32	14.72
50	8.10	10.88	11.77	13.02	14.32	14.72
75	8.20	10.92	11.80	13.03	14.30	14.70
100	8.35	11.12	12.01	13.23	14.49	14.88
125	8.52	11.31	12.23	13.41	14.66	15.05
150	8.74	11.61	12.53	13.72	14.97	15.36
200	9.26	12.23	13.17	14.36	15.60	15.99
250	9.91	13.03	14.00	15.22	16.49	16.88

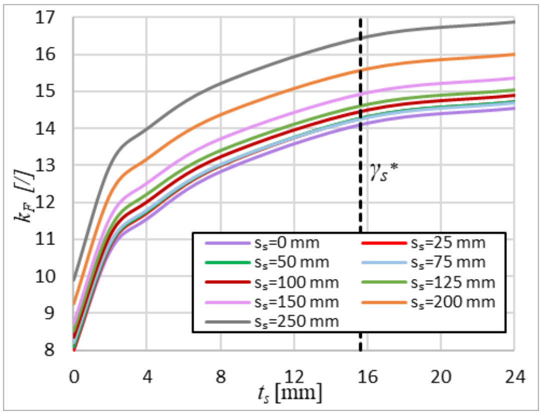

Figure 2. Graphical interpretation of the buckling coefficient for different patch load lengths s_s and stiffener thicknesses t_s.

Figure 2. shows that the buckling coefficient increases with the patch load length. For small ss values, these increases are smaller, while with an increase in ss, the differences in the values of the coefficient kF also increases. It has been noticed that stiffeners with

small flexural rigidity contribute a lot to the elastic critical load values. For larger stiffener thicknesses, the increase in the buckling coefficient diminishes. The contribution of the stiffener is significantly reduced when the limit value γ_s^* is reached (Eq. 2.). This finding means that the limit value of flexural rigidity set by the Eurocode EN1993-1-5 is satisfactory.
In earlier research [1], the expression for the buckling coefficient k_F is given for unstiffened plate as a function of patch load length s_s ($k_F = 3 \cdot 10^{-5} s_s^2 + 8$). For the analysis of stiffened plates, it is necessary to consider not only s_s, but also the stiffener thickness.

$$k_F = t_s^{0.125} \cdot (2.58 \cdot 10^{-5} s_s^2 + 10) \quad (3)$$

3.2. Buckling coefficient of i-girders as a function of patch load length and thickness of the flange and stiffener

The following part of the analysis refers to the influence of the I-girder's flange on the buckling coefficient k_F. A numerical procedure was performed for a limited number of girders, where the thickness of the flange is equal to the thickness of the stiffener. The values of buckling coefficients obtained numerically, by Eurocode EN1993-1-5 and by proposed Eq. (3) are shown and compared in Tables 2-4 for various thicknesses of flange and stiffener. The stiffener significantly increased the buckling coefficient. The flange's effect on the elastic critical load is essential, but the longitudinal stiffener influence is more remarkable.
As the flange thickness increases, the coefficient k_F increases in a linear trend (Fig. 3). On the other side, the plate showed stagnation of increasing coefficient k_F with stiffener getting thicker (Fig. 2) after reaching a limit value γ_s^*. The results of the Eurocode EN1993-1-5 are even more conservative when the influence of the flange is taken into account.

Table 2. Comparison of the buckling coefficient for CC plates and I-girders obtained by Eurocode EN1993-1-5, Eq. (3) and numerically – t_f =8 mm (t_f=t_s)

	Unstiffened plate	*Unstiffened I-girder*	*Stiffened plate*	*Stiffened I-girder EC*	*Stiffened I-girder FEA*	*Stiffened I-girder Eq.4*
s_s [mm]	t_s = 0 mm	t_f = 8 mm	t_s = 8 mm	t_f = 8 mm	t_f = 8 mm	t_f = 8 mm
0	8.00	8.98	12.83	12.04	13.04	13.00
25	8.02	9.06	13.00	12.04	13.12	13.08
50	8.10	9.23	13.02	12.04	13.26	13.22
75	8.20	9.44	13.03	12.04	13.47	13.42
100	8.35	9.70	13.23	12.04	13.72	13.69
125	8.52	10.01	13.41	12.04	14.03	14.02
150	8.74	10.37	13.72	12.04	14.39	14.41
200	9.26	11.25	14.36	12.04	15.31	15.38
250	9.91	12.37	15.22	12.04	16.50	16.60

Table 3. Comparison of the buckling coefficient for CC plates and I-girders obtained by Eurocode EN1993-1-5, Eq. (3) and numerically – t_f =16 mm (t_f=t_s)

	Unstiffened plate	*Unstiffened I-girder*	*Stiffened plate*	*Stiffened I-girder EC*	*Stiffened I-girder FEA*	*Stiffened I-girder Eq.4*
s_s [mm]	t_s = 0 mm	t_f = 16 mm	t_s = 16 mm	t_f = 16 mm	t_f = 16 mm	t_f = 16 mm
0	8.00	10.10	14.14	13.13	15.54	15.46
25	8.02	10.19	14.32	13.13	15.64	15.55
50	8.10	10.40	14.32	13.13	15.87	15.72
75	8.20	10.68	14.30	13.13	16.17	15.96
100	8.35	11.01	14.49	13.13	16.53	16.28
125	8.52	11.40	14.66	13.13	16.96	16.67
150	8.74	11.86	14.97	13.13	17.47	17.14
200	9.26	13.00	15.60	13.13	18.76	18.29
250	9.91	14.52	16.49	13.13	20.49	19.74

Table 4. Comparison of the buckling coefficient for CC plates and I-girders obtained by Eurocode EN1993-1-5, Eq. (3) and numerically – t_f =24 mm (t_f=t_s)

	Unstiffened plate	*Unstiffened I-girder*	*Stiffened plate*	*Stiffened I-girder EC*	*Stiffened I-girder FEA*	*Stiffened I-girder Eq.4*
s_s [mm]	t_s = 0 mm	t_f = 24 mm	t_s = 24 mm	t_f = 24 mm	t_f = 24 mm	t_f = 24 mm
0	8.00	11.00	14.54	13.13	16.75	17.11
25	8.02	11.09	14.72	13.13	16.87	17.21
50	8.10	11.34	14.72	13.13	17.18	17.40
75	8.20	11.69	14.70	13.13	17.58	17.67
100	8.35	12.11	14.88	13.13	18.07	18.02
125	8.52	12.60	15.05	13.13	18.65	18.45
150	8.74	13.18	15.36	13.13	19.34	18.96
200	9.26	14.65	15.99	13.13	21.07	20.24
250	9.91	16.68	16.88	13.13	23.48	21.85

Due to the deviation of the results obtained by Eq. (1) and numerical analysis, there is a need to make some corrections (Table 2-4). Expression 4 was proposed to improve the value of the buckling coefficient of stiffened I-girder. The correlation between the buckling coefficient of the I-girder obtained numerically and approximated by the proposed expressions is shown in Tables 2-4, for all flange thickness analyzed.

$$k_F = t_s^{0.25} \cdot (2.97 \cdot 10^{-5} s_s^{\,2} + 1.13 \cdot 10^{-3} s_s + 7.73) \qquad (4)$$

Further research is recommended to differentiate especially the influence of flange and stiffener in the proposed expression. It is necessary to achieve numerical results for girders with different ratios of flange and stiffener thicknesses.

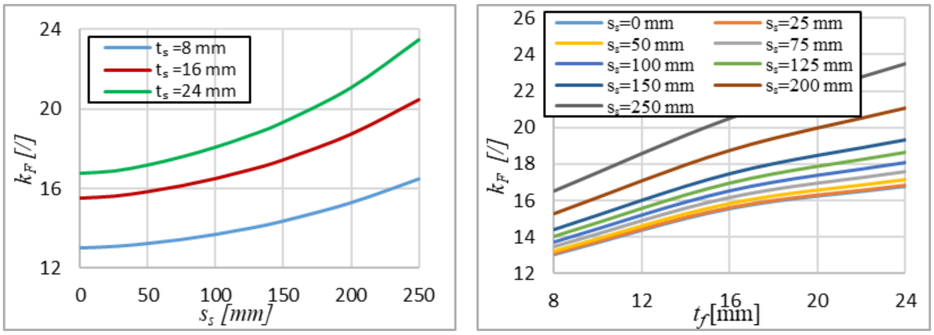

Figure 3. Graphical interpretation of the buckling coefficient of I-girders for different patch load lengths ss and thicknesses of flange and stiffener – (t_s=t_f).

3.3. Comparison of the ultimate loads numerically obtained and from eurocode

Table 5 compares some numerically obtained ultimate loads (F_u^{FEA}) from [5], with the corresponding ultimate loads according to the procedure applied in Eurocode EN1993-1-5 – (F_u^{EC}), and using the proposed expression ($F_u^{EC\,p}$)– Eq. (4). The ultimate loads F_u^{EC} *and* $F_u^{EC\,p}$ were calculated using the appropriate buckling coefficients. Fig. 4 illustrates a case for flange and stiffener thickness of 16 mm.

Table 5. Comparison of the ultimate loads for various approaches

t_f [mm]	s_s [mm]	F_u^{EC} [kN]	$F_u^{EC\,p}$ [kN]	F_u^{FEA} [kN]	F_u^{EC} / F_u^{FEA}	$F_u^{EC\,p} / F_u^{EC}$
8	0	131.00	134.45	134.18	0.98	1.03
	100	163.01	171.70	212.25	0.77	1.05
	200	189.70	211.77	309.27	0.61	1.12
16	0	160.37	174.09	211.36	0.76	1.09
	100	189.02	210.56	290.06	0.65	1.11
	200	213.86	252.49	426.55	0.50	1.18
24	0	186.13	212.55	292.33	0.64	1.14
	100	211.31	247.62	362.27	0.58	1.17
	200	223.69	265.83	547.33	0.41	1.19

The proposed expression for the buckling coefficient Eq. (4) improves results for the ultimate load given by Eurocode EN1993-1-5. With increasing patch load length, flange and stiffener thickness, the improvements are higher and reach a value of up to 19 % for the cases considered.

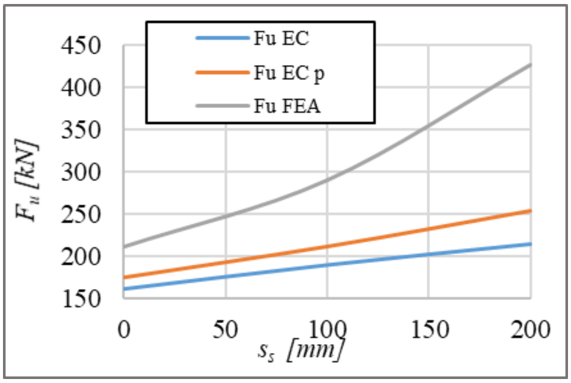

Figure 4. Comparison of the ultimate loads for various approaches

4. CONCLUSIONS

This paper is a continuation of previous research [1,2] related to unstiffened steel plate girders, intending to show that similar dependencies exist in stiffened girders. The elastic critical loads of stiffened isolated clamped web plate and I-girder is considered. Expressions (Eq. (3) and Eq. (4)) as a function of patch load length, flange and stiffener thickness are proposed for the determination of the buckling coefficient. Numerically obtained ultimate loads are compared with the Eurocode EN1993-1-5 and proposed expressions (Eq. (4)). The present design standard Eurocode EN1993-1-5 gives rather conservative values for the ultimate loads. Significant improvement can be obtained by applying the here proposed expression for the buckling coefficient k_F.

To separate the influence of the flange and longitudinal stiffener on the elastic critical load of I-girders, it is necessary to conduct additional numerical tests. Future models should take into account different ratios of flange and stiffener thicknesses. In that way, more comprehensive conclusions will be obtained. The presented results enable a valuable background for the continuation of parametric analysis.

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ЕЛАСТИЧНА КРИТИЧНА СИЛА УКРУЋЕНИХ I-НОСАЧА ОПТЕРЕЋЕНИХ ЛОКАЛИЗОВАНИМ ОПТЕРЕЋЕЊЕМ

Резиме: *У раду се разматра еластична критична сила изоловане плоче и челичног I-носача, оптерећених локализованим оптерећењем. Обрађени су носачи без вертикалних укрућења у зони уношења оптерећења, али са подужним укрућењима. Дат је утицај дужине оптерећења, као и дебљине појаса и укрућења на коефицијент избочавања. Резултати за прорачун коефицијента избочавања укљештене плоче и I-носача су упоређени и дати су предлози за побољшање прорачуна носивости I-носача датог у Еврокоду EN1993-1-5.*

Кључне речи: *еластична критична сила, коефицијент избочавања, локализовано оптерећење.*