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PROCENA TAČNOSTI POSTUPKA PRORAČUNA TEČENJA I SKUPLJANJA PREMA EC4

Rezime

U radu se prikazuje proračun tečenja i skupljanja u spregnutom nosaču od čelika i betona. Koriste se približne metode proračuna AAEM, EM, i postupak proračuna prema Evrokodu 4, kao i tačna metoda koja je zasnovana na integralnoj vezi između napona i deformacije za beton. Za obostrano uklešten spregnuti nosač određene su vrednosti napona u karakterističnim tačkama preseka za stalno opterećenje i skupljanje. Koristeći granične funkcije tečenja betona, određene su gornje i donje granice napona. Na osnovu tih granica, izvršena je procena tačnosti postupka proračuna datog u EC4.

Ključne riječi

Spregnuti nosač, tečenje, skupljanje, proračun, naponi, EC4.

ACCURACY EVALUATION OF CREEP AND SHRINKAGE CALCULATION METHODS ACCORDING TO EC4

Summary

In the paper, the creep and shrinkage calculation methods for composite steel-concrete beams are presented. The approximate methods AAEM, EM and methods according to Eurocode 4, as well as, the exact method based on the integral relation between stress and strain for concrete are used. For doubly-clamped composite beam, stresses at characteristic points of the cross-section for permanent loading and shrinkage effects are determined. Using the creep limit functions for concrete, the upper and lower limits for stresses are determined. According to these limits, the accuracy of the methods for creep and shrinkage calculation given in EC4 is evaluated.

Key words

Composite girder, creep, shrinkage, calculation method, stresses, EC4.

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1. INTRODUCTION

We analyze a general steel-concrete composite structure that consists of steel member (a), concrete slab (c), reinforcement (s) and prestressing steel (p). Because of viscoelastic properties of concrete, i.e. creep and shrinkage, and because of relaxation of prestressing steel, the redistribution of stresses and substantial changes in deformations of a composite structure occur in time. Hence, the analysis should take into account these mentioned effects. The calculation of a composite beam is performed for time $t=t_o$ (time of the first loading) and for time t (time at the considered moment which, usually, corresponds to the final state $t\rightarrow\infty$). Several methods have been developed so far that count for creep and shrinkage of concrete, as well as, relaxation of prestressing steel with different level of accuracy. The accuracy of these methods primarily depends on the adopted stress-strain relationship for concrete and prestressing steel.

Under standard exploitation conditions for the most of composite structures, i.e. when the largest stress in concrete does not exceed 0.4 of the concrete compression strength, it is fully justified to accept the linear relation between concrete creep deformations under constant stress and the principle of superposition for concrete creep deformations due to stress increment. These assumptions lead to the integral relation between concrete stress and strain. Starting from this integral relationship, the exact and approximate calculation methods are established. The exact calculation methods adopt the integral relation between concrete stresses and strains and, therefore, apart from the inevitable approximations related to the rheological characteristics of constituent materials, no other mathematical approximations are introduced. In this paper, the exact method developed by Lazic [1] with linear integral operators will be used. The approximate calculation methods introduce few mathematical simplifications and modify the integral stress-strain relation for concrete into an algebraic relation. The accuracy of the obtained algebraic relation depends on the type of mathematical simplifications. In this paper, the following approximate methods are used: AAEM method, EM method, and calculation method proposed by Eurocode 4 (EC4) [4] which is based on the EM method.

2. EXACT METHOD

The stress strain relationship for concrete is integral and may be symbolically written in the following operator form [1]:

$$\varepsilon-\varepsilon_s=\frac{1}{E_{co}}\widehat{F}'\sigma_c\,,\qquad \sigma_c=E_{co}\widehat{R}'(\varepsilon-\varepsilon_s) \tag{1}$$

Operators $\widehat{F}'$ and $\widehat{R}'$ are inverse. They satisfy the following relation:

$$\widehat{R}'\widehat{F}'=\widehat{F}'\widehat{R}'=\widehat{1}' \tag{2}$$

The concrete creep and relaxation functions F^* and R^* represent the following integrals: $F^*=\widehat{F}'1^*,\quad R^*=\widehat{R}'1^*$.

In the analysis of composite structures, the common assumption is that concrete shrinkage is similar, through time, to creep of concrete [1, 2]. Hence, the shrinkage deformation of concrete ε_s is given with the expression:

$$\varepsilon_s = r(F^* - 1^*), \quad r = \frac{\varepsilon_s(t,t_o)}{F^*(t,t_o) - 1^*} \tag{3}$$

The stress-strain relation for prestressing steel has the form:

$$\sigma_p = E_p \widehat{R}'_p \varepsilon \tag{4}$$

where the relaxation function R_p^* linearly depends on the concrete relaxation function R^*:

$$R_p^* = \widehat{R}'_p 1^* = (1-\rho)1^* + \rho R^*, \quad \rho = \frac{\varsigma'_p(t-t_o)}{1^* - R^*(t,t_o)} \tag{5}$$

where $\varsigma'_p = \varsigma'_p(t-t_o)$ is relaxation of prestressing steel.

Other steel parts, steel member (a) and reinforcement (s) follow the Hook's law:

$$\sigma_k = E_k \varepsilon, \quad k = a,m \tag{6}$$

The analysis of a composite structure is the same as the analysis of the corresponding structure made of homogeneous elastic material with the exception that in composite structures, using the exact method, the integral equation is solved.

Starting from the assumption that, for the composite cross-section, Bernoulli's hypothesis of plain cross section is valid, the equilibrium conditions of external and internal forces in the cross-section together with the Equations (1.b), (4) and (6), the normal strain $\eta = \eta(x,t,t_o)$ and the change in the curvature of the member axis $\kappa = \kappa(x,t,t_o)$ can be obtained:

$$\eta = \frac{1}{E_u A_i}\widehat{F}'_{11}N + \frac{1}{E_u S_i}\widehat{F}'_{12}M, \quad \kappa = \frac{1}{E_u S_i}\widehat{F}'_{12}N + \frac{1}{E_u J_i}\widehat{F}'_{22}M \tag{7}$$

where E_u is the relative modulus of elasticity, A_i and J_i are transformed cross-section area and second moment of inertia of this area, and $S_i = \sqrt{A_i J_i}$. Operators $\widehat{F}'_{11}, \widehat{F}'_{22}$ and $\widehat{F}'_{12}$ are elements of operator matrix $\left[\widehat{F}'_{hl}\right]_{2,2}$, and their principal values are $\widehat{F}'_1$ and $\widehat{F}'_2$.

For convenience, the following operators are also defined: $\widehat{B}'_h = \widehat{R}\widehat{F}'_h$.

Functions B_h^*:

$$B_h^* = \widehat{B}'_h 1^* = \frac{1}{\gamma_h}1^* - \frac{\gamma'_h}{\gamma_h}F'_h, \quad h = 1,2 \tag{8}$$

are the basic functions of the composite section [1]; γ_h are principal values of the matrix of the reduced geometric characteristics. For the known creep function B_h^*, the solutions of the nonhomogeneous integral equation are determined.

With the expression (7) for the section deformations η and κ, and using the principle of virtual forces, the expressions for the generalized displacements can be obtained, and also, the expressions for solving the statically indeterminate structures.

After some transformations, the coordinate stress is:

$$\sigma_u = E_u \varepsilon = E_u(\eta + \kappa y) = \sum_{h=1}^{2} n_h \widehat{F}_h' N + \sum_{h=1}^{2} m_h \widehat{F}_h' M , \quad h = 1,2 \tag{9}$$

where n_h and m_h are constants [3].

Using the expressions (1.b), (4) and (6) the stresses at parts of the composite section can be determined and, for convenience, the following ratios between modules of elasticity are introduced $n_k=E_u/E_k$, $k=a,m,c,p$:

$$\begin{aligned}
\sigma_k &= \frac{1}{n_k} \sum_{h=1}^{2} n_h \widehat{F}_h' N + \sum_{h=1}^{2} m_h \widehat{F}_h' M , \quad k = a, s \\
\sigma_c &= \frac{1}{n_c} \left(\sum_{h=1}^{2} n_h \widehat{B}_h' N + \sum_{h=1}^{2} m_h \widehat{B}_h' M \right) + \sigma_s \\
\sigma_p &= \frac{1}{n_p} \left\{ (1-\rho) \left[\sum_{h=1}^{2} n_h \widehat{F}_h' N + \sum_{h=1}^{2} m_h \widehat{F}_h' M \right] + \rho \sum_{h=1}^{2} n_h \widehat{B}_h' N + \sum_{h=1}^{2} m_h \widehat{B}_h' M \right\}
\end{aligned} \tag{10}$$

Where stress σ_s in concrete due to shrinkage, according to the expressions (1.b), (3) for ε_s and the relation (2), is: $\sigma_s = -E_{co} \widehat{R}' \varepsilon_s = -E_{co} r(1 - R^*)$.

3. APPROXIMATE METHODS

The approximate calculation methods are based on the algebraic relation between stress and strain and their accuracy depends on the adopted expression for this relation. In this paper, the AAEM method, EM method and the method proposed by EC4 [4] are used.

The algebraic relation of the **AAEM** method, proposed by Bazant (1972.), has the following form:

$$\sigma_c = E_{c,aeff}(\varepsilon_c - \varepsilon_{cs}) - \rho_c \sigma_{co}, \quad E_{c,aeff} = \frac{E_{co}}{1 + \chi \varphi_r}, \quad \rho_c = \frac{(1-\chi)\varphi_r}{1 + \chi \varphi_r} \tag{11}$$

where $E_{c,aeff}$ is the corrected effective elastic modulus of concrete (or effective elastic modulus of concrete with reduced aging); $\chi = \chi\ (t,t_o)$ is the aging coefficient whose value is less then 1.0 (0.6-0.9); $\varphi_r(t,t_o)$ is the reduced creep coefficient of concrete. This method is more complex, but also more accurate then other approximate methods since it considers the age of concrete through the aging coefficient χ.

If the aging coefficient is adopted as equal to 1 ($\chi=1$) the algebraic relation for concrete of the **EM** method, proposed by Faber (1927.), is obtained:

$$\sigma_c(t) = E_{c,eff}(\varepsilon_c - \varepsilon_{cs}), \qquad E_{c,eff} = \frac{E_{co}}{1+\varphi_r} \tag{12}$$

This relation is correct only for the creep function of the heritage theory in time $t \to \infty$. The $E_{c,eff}$ is the effective elastic modulus of concrete; φ_r is the reduced creep coefficient of concrete. In time $t=t_o$, $\varphi_r=0$ and, therefore, $E_{c,eff}=E_{co}$. It is evident that, according to this method, the creep of concrete is taken into account simply by reduction of the modulus of elasticity of concrete and, consequently, the analysis in time t is analogues to the analysis in time t_o, with the only difference that in time t_o we use E_{co} for the elastic modulus of concrete, while in time t we use the effective modulus of concrete $E_{c,eff}$. Because of its simplicity, this method, with several modifications, is widely used in practice. Fritz (1961.) proposed the method that has been used in the analysis of a great number of composite structures. His method introduces the correction factor ψ into the effective modulus relation $E_{c,eff} = \frac{E_{co}}{1+\psi\varphi_r}$. The correction factor ψ depends on the type of loading and characteristics of the cross-section and has the value of *1.1* for the calculation of concrete creep effects and the value of *0.52*, for the calculation of shrinkage of concrete.

The **EC4** [4], the contemporary European code for design of steel-concrete composite structures, is based on the limit state design philosophy. Depending on type of structure, a class of cross-section, a limit state and actions that are considered, the EC4 proposes few simple methods for taking into account creeping and shrinkage of concrete. The effects of creep and shrinkage of concrete can be neglected in analysis for verifications of limit states other than fatigue, for composite members with all cross-sections in class 1 or 2 and in which no allowance for lateral-torsional buckling is necessary. In other cases, as well as for serviceability limit states, these effects should be taken into account. When more accurate analysis in time t is necessary, the recommended algebraic equation is the algebraic equation of the EM method with the effective elasticity modulus of concrete $E_{c,eff}$ proposed by Fritz, with the creep multiplier ψ_L used instead of correction factor ψ, and $E_{co}=E_{cm}$:

$$E_{c,eff} = \frac{E_{cm}}{1+\psi_L\varphi_r} \tag{13}$$

where E_{cm} is the secant modulus of elasticity of concrete for short-term loading according to Eurocode 2; the creep multiplier ψ_L depends on the type of loading and takes the following values: *1.10* for permanent loading, *0.55* for shrinkage effects, *1.50* for prestressing by imposed deformations.

For the approximate methods, the expressions for stresses at characteristic points of a composite section will be given next. When the relaxation of prestressing steel is considered, the algebraic stress-strain relation for the prestressing steel in time t may be written as follows:

$$\sigma_p = E_{p,eff}\varepsilon \qquad E_{p,eff} = \varsigma_p E_p, \quad \varsigma_p = 1 - \frac{R}{100}, \quad 0 \le \varsigma_p \le 1 \tag{14}$$

where $E_{p,eff}$ is the effective elastic modulus of prestressing steel. The coefficient ζ_p is calculated from the value of relaxation R given in percent. In time t_o, ζ_p =1 and, therefore, the algebraic relation is $\sigma_p = E_p\varepsilon$. Steel member (a) and reinforcement (s) follow the Hook's law (6).

Determination of stresses and deformations at time t in the case of statically determinate and indeterminate composite girder is analogous to the analysis of the corresponding homogeneous girder made of an elastic material, whose modulus of elasticity is E_u, geometric characteristics are A_i and J_i, and modular ratios are $n_{kt}=E_u/E_{kt}$, $k=a,s,p,c$.

Stresses at time t can be found from the following expressions:

$$\begin{aligned} &\sigma_k = \frac{1}{n_{kt}}\left(\frac{N_\varphi}{A_i} + \frac{M_\varphi}{I_i}z\right) \quad k = a,s; \quad \sigma_p = \frac{1}{n_{pt}}\left(\frac{N_\varphi}{A_i} + \frac{M_\varphi}{I_i}z\right) \\ &\sigma_c = \frac{1}{n_{ct}}\left(\frac{N_\varphi}{A_i} + \frac{M_\varphi}{I_i}z\right) - \rho\sigma_{co} - E_{c,aeff}\varepsilon_{cs} \end{aligned} \tag{15}$$

where N_φ are M_φ fictive forces:

$$\begin{aligned} &N_\varphi = N + n_s + n_\rho, \quad n_\rho = \rho N_{co}, \quad n_s = E_{c,aeff}A_c\varepsilon_{cs} \\ &M_\varphi = M + m_s + m_\rho, \quad m_\rho = \rho(M_{co} + z_c N_{co}), \quad m_s = n_s z_c \end{aligned} \tag{16}$$

N_{co}, M_{co} are forces in concrete part at time $t=t_o$; n_s and m_s are forces that count for the effect of shrinkage of concrete.

4. EXAMPLE

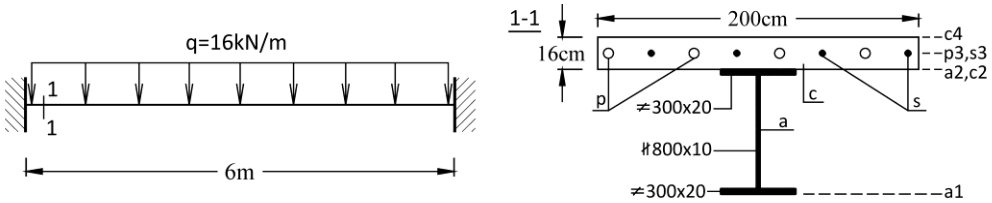

Figure 1. Composite girder and its cross-section

The composite girder shown in Fig. 1 with constant cross section 1-1 is analyzed. The stresses at characteristic points of the cross section at fixed end are calculated for time t_o and time t. Stresses are determined due to given uniform loading and due to shrinkage of concrete in accordance with the exact method, the approximate methods EM and AAEM with χ=0.85, and EC4. The results of the analysis are given in Table 1 and in Figs. 2-3.

Example data are: *Concrete (c)* $E_{co}=E_{cm}=30GPa$, $\varphi_r=3.5$, $\varepsilon_s = -30\cdot 10^{-5}$; *Prestressing steel (p):* $E_p=210GPa$, $A_p=100cm^2$, $\zeta_p=8\%$; *Structural steel (a):* $E_a=200GPa=E_u$; *Reinforcing steel (s):* $E_s=200GPa$, $A_s=80cm^2$.

In the analysis of this girder with the exact method, as is well known, the section forces due to the permanent loading do not depend on time ($N=N1^*$, $M=M1^*$), and stresses (10) can be determined directly from functions B_h^* and F_h^* (8). We used the function of the aging theory under the constant concrete modulus of elasticity, so that the expressions for the basic functions and relaxation are $B_h^* = e^{-\gamma_h' \varphi_r}$, $R^* = e^{-\varphi_r}$.

Table 1. Results of analysis

H	MPa	t_o	EM	EC4	AAEM	Exact
q	$\sigma_{c4}=$	0.220	0.085	0.080	0.071	0.049
	$\sigma_{c2}=$	0.040	0.038	0.036	0.040	0.035
	$\sigma_{s3}=$	0.868	1.840	1.882	1.923	2.099
	$\sigma_{p3}=$	0.912	1.778	1.818	1.857	2.046
	$\sigma_{a2}=$	0.267	1.129	1.166	1.202	1.358
	$\sigma_{a1}=$	-6.042	-6.341	-6.353	-6.369	-6.420
shrinkage	$\sigma_{c4}=$		2.676	4.450	3.097	3.063
	$\sigma_{c2}=$		2.545	4.185	2.935	2.952
	$\sigma_{s3}=$		18.320	24.192	19.933	23.303
	$\sigma_{p3}=$		17.697	23.370	19.250	22.459
	$\sigma_{a2}=$		16.357	21.604	17.798	20.906
	$\sigma_{a1}=$		-4.252	-5.566	-4.616	-5.415

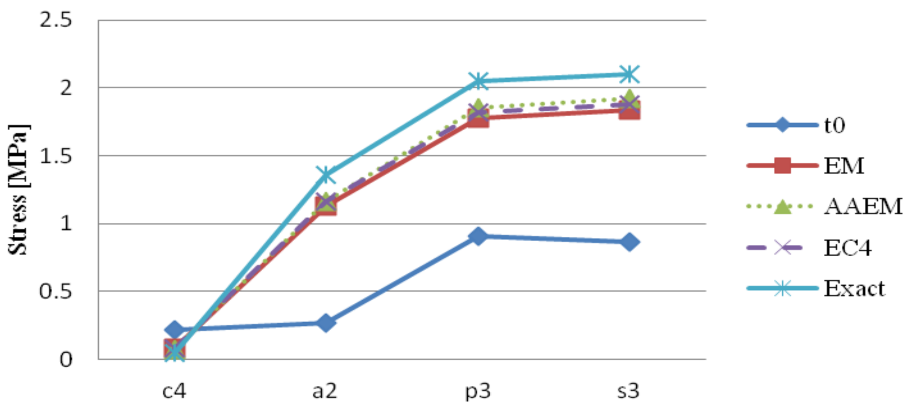

Figure 2. Stresses due to permanent loading q (the cross-section at the fixed end)

The effect of shrinkage is taken into account as the external equilibrium loading with axial force and bending moment and, from the expressions (3) and (1.b), these forces are:

$N_s^* = E_u A_{cr} r(1^* - R^*)$, $M_s^* = E_u A_{cr} z_c r_c (1^* - R^*)$. Then, stresses due to shrinkage according to expressions (10) can be found directly from the functions B_h^*, F_h^* and R^*.

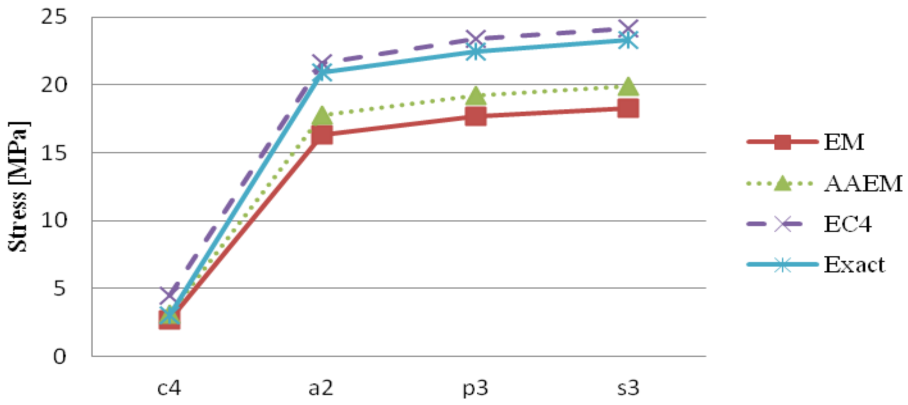

Figure 3. Stresses due to shrinkage (the cross-section at the fixed end)

5. CONCLUSION

Using the creep function in accordance with the ageing theory in the exact method and the creep function in accordance with the hereditary theory in the EM method, the upper and lower bounds are determined and results obtained by other theories should take place between them. As can be seen from Fig.2., the AAEM method fulfill this condition. The method proposed by EC4 gives results that are on the safe side, and since it is modification of the EM method, its results are closer to the lower bound (EM). For shrinkage stresses (Fig.3), the results of the AAEM method are still between the two limits, while the results of the EC4 method show that this method overestimates stresses in concrete and steel member. Though, obtained results are on the safe side, much closer to the exact method solution and with improved accuracy compared to the EM method solution.

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