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MODELIRANJE DVOFAZNOG STRUJANJA (VODE I VAZDUHA) NAKON UTISKIVANJA VAZDUHA U VODU

NUMERICAL MODELING OF TWO PHASE FLOW (WATER-AIR) AFTER AIR INJECTION INTO THE WATER

APSTRAKT

Jedna od raspoložovih tehnologija za poboljšanje kvaliteta vode u jezerima i akumulacijama predstavlja utiskivanje vazduha, odnosno kiseonika, u dublje slojeve vode (hipolimnion), čime se podstiče mešanje vode po vertikalnoj osi i ostvaruje transfer kiseonika iz gasne faze u vodu. U radu se prikazuje razvijeni 3D numerički model za simulaciju dvofaznog strujanja gasa i vode, kojim su obuhvaćeni i uticaji stišljivosti vazduha, kao i transfer mase između vazduha i vode. Model je najpre verifikovan simulacijom nekoliko eksperimenata iz literature, gde su poređeni podaci o distribuciji vazduha i brzine vode nakon utiskivanja vazduha u vodom ispunjen sud sa rezultatima simulacije. Pri tome, simulirani su uslovi stru-janja gde se postiže kvazi-ustaljeno strujanje, kao i eksperimenti u kojima takvo strujanje ne može da se ostvari. U drugom delu, model je korišćen za simulaciju sprovedenih eks-perimenata u kojima je osmatran porast koncentracija rastvorenog kiseonika tokom utiskivanja vazduha. U radu se prikazuju rezultati ovih simulacija, gde se mogu konstatovati zadovolja-vajuća slaganja izmerenih i sračunatih veličina.

Ključne reči: numeričko modeliranje, dvofazno strujanje

ABSTRACT

One of available lake restoration technologies is injection of compressed air or oxygen into a hypolimnion, promoting vertical mixing of water and dissolution of oxygen. A numerical model has been developed for simulation of bubble flow, with consideration of gas compressibility and oxygen dissolution. Developed model is intended to be utilized for simulation of lakes DO recovery. Model is firstly verified by simulation of bubble flow experiments, reported in the literature, where very good quantitative and qualitative agreement between measured and simulated results is observed. Both flow configurations are tested: in which steady state is achieved, and where steady conditions can not develop. In the second part, model is applied for simulation of conducted experiments where oxygen dissolution has been considered. The paper presents simulation results, where good agreement of simulated and measured quantities is observed.

Key words: numerical modeling, two phase flow

1 UVOD

Eutrofikacija predstavlja prirodni proces starenja vodnih tela tokom koga dolazi do prelaza iz nisko produktivnog stanja (oligotrofnog) u visoko produktivno stanje (eutrofno). Kod ovakvog stanja, većina organske mase koja se produkuje u površinskim slojevima se ne razgrađuje u potpunosti, već se akumulira na dnu, gde se obavlja razgradnja. Usled povećanog (veštačkog) unosa nutrijenata, ovaj proces se znatno ubrzava, usled čega dolazi do značajnog smanjenja koncentracija kiseonika pri dnu eutrofnog jezera, a u nepovoljnijim slučajevima i do anaerobnog stanja. Ovo ima za posledicu povećanje koncentracija amonijaka, gvožđa, mangana i drugih materija, kao i pojave vodonik sulfida i metana, što negativno utiče na kvalitet, kako sa stanovišta biotopa, tako i u smislu mogućnosti korišćenja vode.

1 INTRODUCTION

Eutrophication is a natural process of water body aging which occurs during the transition from low-productive state (oligotrophic) to a highly productive state (eutrophic). In such state, most of the organic mass that is produced in the surface layers is not completely disintegrated, but accumulates on the bottom, where decomposition is performed. Due to the increased (artificial) input of nutrients, this process is much faster, which results in significant reduction of oxygen concentration at the bottom of eutrophic lakes, and in unfavorable cases, anaerobic state. The consequence is increased concentration of ammonia, iron, manganese and other substances, as well as the appearance of hydrogen sulfide and methane, which negatively affects the quality, both from the standpoint of biotopes, and in terms of ability to

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Kao jedna od mogućih mera revitalizacije, odnosno usporavanja procesa eutrofikacije, često se koristi tehnologija utiskivanja vazduha ili čistog kiseonika pod pritiskom [1, 2, 3]. Cilj može da bude veštačko mešanje vode u vertikalnom pravcu, koje se javlja usled sile uzgona koja deluje na gasovitu fazu, a koja pokreće težu vodu ka površinskim slojevima. Takođe, u periodu kada nije povoljno vertikalno mešanje, ovim je moguće uticati na povećanje koncentraciju kiseonika u dubljim slojevima, do čega dolazi usled rastvaranja kiseonika iz gasovite faze.

Kao inovativna tehnologija, u okviru ovde prikazanih istraživanja, razmatrana je tehnologija proizvodnje kiseonika elektrolizom, pri čemu je predviđeno da se istovre-meno oslobođeni vodonik sakuplja i koristi u energetske svrhe. U radu se prikazuje numerički model, razvijen u okviru ovih istraživanja, za simulaciju dvofaznog strujanja vode i vazduha. Modelom su obuhvaćeni efekti stišljivosti gasovite faze, kao i proces transfera kiseonika iz gasovite faze u vodu.

2 OPIS MATEMATIČKOG MODELA

Numeričko modeliranje dvofaznog tečenje vode i vazduha, u cilju simulacije tehnologije za poboljšanje kvaliteta vode, se može podeliti na dva problema. Prvi se odnosi na modeliranje rasporeda gasovite i tečne faze, gde uzgon predstavlja osnovnu pokre-tačku silu, dok se drugi problem odnosi na rastvaranje, odnosno transfer mase iz jedne u drugu fazu, a zatim i transport rastvorene materije u tečnoj fazi.

Imajući u vidu stišljivost gasovite faze, kao i prelaz mase jedne faze u drugu, jednačine kontinuiteta za gasovitu, odnosno tečnu fazu se može pisati kao:

$$\frac{\partial(\rho_g \alpha_g)}{\partial t} + \frac{\partial(\rho_g \alpha_g V_{gi})}{\partial x_i} = -G_{gw} \quad (1)$$

$$\frac{\partial \alpha_w}{\partial t} + \frac{\partial(\alpha_w V_{wi})}{\partial x_i} = \frac{G_{gw}}{\rho_w} \quad (2)$$

gde se indeksi g i w odnose na gasovitu fazu i vodu, respektivno, x_i je kooridnatni pravac, t je vreme, ρ je gustina pojedine faze, α zapreminski udeo pojedine faze, V_i komponenta vektora brzine, dok G_{gw} predstavlja transfer mase između pojedinih faza, u jedinici vremena i po jedinici zapremine. Pri tome, neophodno je da uvek bude ispunjen uslov:

$$\alpha_g + \alpha_w = 1 \quad (3)$$

Obzirom da je neophodno voditi računa o stišljivosti vazduha, pretpostavljeno je da važi jednačina stanja:

$$\frac{p_g}{\rho_g} = RT \quad (4)$$

gde je p_g apsolutni pritisak u gasovitoj fazi, T je apsolutna temperatura, i R gasna konstanta.

use water.

As one possible measure of revitalization, or slowing down the eutrophication process, technology of air or pure oxygen injection under pressure is often used [1, 2, 3]. The goal can be artificial vertical mixing of water, which occurs due to the buoyancy force acting on the gaseous phase, which runs water to surface layers. Also, in the period of adverse conditions of vertical mixing, this can affect the increase of oxygen concentration in deeper layers, which occurs due to the dissolution of oxygen in the gas phase.

As an innovative technology, among others researches shown in this paper, the technology of oxygen production by electrolysis was discussed, with simultaneous collecting of released hydrogen which can be used for energy purposes. This paper presents the numerical model developed in this research, simulation two-phase flow of water and air. Model includes gas compressibility effects, and the process of oxygen dissolution.

2 DESCRIPTION OF NUMERICAL MODEL

Numerical modeling of bubble flow, for the purpose of simulation of technology for improving water quality, can be divided into two problems. The first relates to the modeling of gas and liquid phase distribution, where buoyancy is the primary driving force, while the second problem is related to dissolution and mass transfer from one phase to another, and then transport of dissolved matter in liquid phase.

Considering the compressibility of gaseous phase, as well as mass transfer from one phase to another, the equation of continuity for gas or liquid phase can be

$$\frac{\partial(\rho_g \alpha_g)}{\partial t} + \frac{\partial(\rho_g \alpha_g V_{gi})}{\partial x_i} = -G_{gw} \quad (1)$$

$$\frac{\partial \alpha_w}{\partial t} + \frac{\partial(\alpha_w V_{wi})}{\partial x_i} = \frac{G_{gw}}{\rho_w} \quad (2)$$

where indexes g and w are for gaseous and liquid phase, respectively, x_i is coordinate direction, t is time, ρ is density of each phase, α is volume share of individual phase, V_i is component of velocity, while G_{gw} represents the mass transfer between phases, in unit time and per unit volume. Beside that it also requires the following condition:

$$\alpha_g + \alpha_w = 1 \quad (3)$$

Since it is necessary to take into account the gas compressibility, it is assumed that the following equation

$$\frac{p_g}{\rho_g} = RT \quad (4)$$

Transfer kiseonika iz gasovite faze u vodu je modeliran na osnovu sledećeg izraza [1]:

$$G_{gw} = K_{gw} A\left(H p_g - DO\right) \tag{5}$$

gde je K_{gw} koeficijent prenosa mase, A specifična kontaktna površina, H Henry-eva konstanta u dimenzionalnom obliku i DO koncentracija rastvorenog kiseonika. Nave-deni koeficijent je funkcija prečnika mehurića, i to približno linearna funkcija do određene vrednosti prečnika (~ 1,4 mm), dok se za veće vrednosti može aproksimovati konstantnom vrednošću od približno 0,04 cm/s [2].

Tokom kretanja mehurića vazduha, dolazi do promene njihove zapremine usled promene pritiska, kao i usled procesa rastvaranja, na osnovu čega se menja kontaktna površina kroz koju se odvija transfer mase između faza, ali što utiče i na razmenu količine kretanja između faza. Zbog toga je u opisanom modelu uključena i konvektivna jednačina kojom se prati broj mehurića u kontrolnoj zapremini:

$$\frac{\partial N_b}{\partial t} + \frac{\partial\left(N_b V_{gi}\right)}{\partial x_i} = 0 \tag{6}$$

gde je N_b broj mehurića po jedinici zapremine. Na osnovu sračunatog zapreminskog udela gasovite faze i na ovako dobijenog broja mehurića se određuje prečnik mehurića. Drugim rečima, pretpostavljena je ista veličina mehurića u jednoj kontrolnoj zapremini, kao i da ne dolazi do razdvajanja mehurića, odnosno njihovog međusobnog spajanja.

Slede dinamičke jednačine za obe faze, koje se rešavaju ovim modelom:

$$\frac{\partial\left(\rho_g \alpha_g V_{gj}\right)}{\partial t} + \frac{\partial\left(\rho_g \alpha_g V_{gi} V_{gj}\right)}{\partial x_i} =$$

$$= -\alpha_g \frac{\partial p_g}{\partial x_j} - \rho_g \alpha_g g_j - F_{gwj} \tag{7}$$

$$\frac{\partial\left(\alpha_w V_{wj}\right)}{\partial t} + \frac{\partial\left(\alpha_w V_{wi} V_{wj}\right)}{\partial x_i} = -\frac{\alpha_w}{\rho_w}\frac{\partial p_w}{\partial x_j} -$$

$$-\alpha_w g_j + \frac{1}{\rho_w}\frac{\partial\left(\alpha_w \tau_{ij}\right)}{\partial x_i} + \frac{1}{\rho_w} F_{gwj} \tag{8}$$

gde je g_j komponenta gravitacione sile, τ_{ij} tangencijalni napon, gde su uključeni viskoz-ni i efekti turbulencije i *Fgwj* sila „otpora", koja predstavlja razmenu količine kretanja između dve faze. Upravo kroz ovaj član, kao i kroz pritisak, kuplovane su jednačine (7) i (8). Naznačajniju komponentu sile otpora predstavlja sila otpora oblika, koja je ovde računata kao:

$$F_{gwj} = \frac{1}{2} C_d \rho_w \pi \frac{d_b^2}{4}\left|V_g - V_w\right|\left(V_{gj} - V_{wj}\right) \tag{9}$$

gde je d_b prečnik mehurića, dok se koeficijent C_d može odrediti na osnovu izraza u kome figurišu Rejnoldsov i Eotvosov broj [4]:

where p_g is absolute pressure in gaseous phase, T is absolute temperature, and R is gas constant.

Oxygen transfer from the gas phase is modelled by the following expression [1]:

$$G_{gw} = K_{gw} A\left(H p_g - DO\right) \tag{5}$$

where K_{gw} is mass transfer coefficient, A is specific contact area, H is Henry constant k in dimensional form and DO concentration of dissolved oxygen. This coefficient is a function of the diameter of bubbles, and approximately linear function for certain diameter values (~ 1,4 mm), while for higher values it can be approximated with constant value of approximately 0,04 cm/s [2].

During the movement of air bubbles, their volume is changing due to pressure changes, and due to the process of dissolution, which results in change of contact area through which mass transfer takes place between phases, but it also affects the exchange of momentum between stages. Therefore, the described model includes convective equation, which monitors the number of bubbles in the control volume:

$$\frac{\partial N_b}{\partial t} + \frac{\partial\left(N_b V_{gi}\right)}{\partial x_i} = 0 \tag{6}$$

where N_b is number of bubbles per unit volume. Based on the calculated volume share of gas phase and thus obtained the number of bubbles, the diameter of bubbles is determined. In other words, the assumption is the same size of bubbles in a single volume control, as well as non-existence of bubble separation or their mutual connection.

In the following are dynamic equations for both phases, which are solved with this model:

$$\frac{\partial\left(\rho_g \alpha_g V_{gj}\right)}{\partial t} + \frac{\partial\left(\rho_g \alpha_g V_{gi} V_{gj}\right)}{\partial x_i} =$$

$$= -\alpha_g \frac{\partial p_g}{\partial x_j} - \rho_g \alpha_g g_j - F_{gwj} \tag{7}$$

$$-\alpha_w g_j + \frac{1}{\rho_w}\frac{\partial\left(\alpha_w \tau_{ij}\right)}{\partial x_i} + \frac{1}{\rho_w} F_{gwj}$$

$$-\alpha_w g_j + \frac{1}{\rho_w}\frac{\partial\left(\alpha_w \tau_{ij}\right)}{\partial x_i} + \frac{1}{\rho_w} F_{gwj} \tag{8}$$

where g_j is component of gravity, τ_{ij} is tangential tension, which includes the effects of viscosity and turbulence and *Fgwj* is "resistance" force, which represents the exchange of momentum between the two phases. Equations (7) and (8) are obtained using this part and pressure. The resistance force is calculated as:

$$F_{gwj} = \frac{1}{2} C_d \rho_w \pi \frac{d_b^2}{4}\left|V_g - V_w\right|\left(V_{gj} - V_{wj}\right) \tag{9}$$

where d_b is bubble diameter, while C_d coefficient can be determined on the basis of the equation with Reynolds and Eötvös number [4]:

$$C_d = \max\left[\frac{24}{\mathrm{Re}}\left(1+0.15\,\mathrm{Re}^{0.687}\right)\frac{8}{3},\ \frac{Eo}{Eo+4}\right] \quad (10)$$

Prikazane jednačine (1) – (10) su diskretizovane metodom konačnih zapremina, tako da su vektorske veličine definisane na stranicama, dok su skalarne veličine defini-sane u centru kontrolne zapremine. Za računanje vrednosti pritisaka i brzina u svakom vremenskom koraku, korišćen je HSMAC (Highly Simplified Marker And Cell) metod [5], gde je primenjena Adams-Bashforth shema za vremensku integraciju konvektivnih i viskoznih članova u dinamičkim jednačinama. Sama HSMAC shema je u određenoj meri korigovana, kako bi se primenila na dvofazni problem strujanja [6]. Za proračun Rejnoldsovih napona, korišćen je standardni κ–ε μοδελ turbulencije [7], čije su jedna-čine dobro poznate, te se ovde ne navode.

Kod modeliranja višefaznog strujanja, konkretno kod rešavanja jednačina (1), (2) i (6), od izuzetnog značaja predstavlja diskretizacija konvektivnih članova u smislu sprečavanja efekta numeričke difuzije i numeričkih oscilacija. Zbog toga je u modelu korišćena shema trećeg reda tačnosti, uz primenu TVD (Total Variation Diminishing) limitera [6].

3 VALIDACIJA MODELA

3.1 *Simulaciju eksperimenata sa konsantnim utiskivanjem vazduha u dnu suda*

Razvijeni model je najpre primenjen za simulaciju laboratorijskih eksperimenata iz literature [8, 9], u kojima je vazduh utiskivan u rezervoar dimenzija 200 × 50 × 8 cm. Rezervoar je prethodno ispunjen vodom, pri čemu je nivo vode variran za različite eksperimente. Raspored gasovite faze je vizuelno osmatran tokom eksperimenta, uz merenje brzine vode po srednjoj vertikalnoj ravni (4 cm od zida), sa međusobnim rastojanjem mernih tačaka od 10, odnosno 20 cm.

Prva simulacija se odnosi na eksperiment u kome je nivo vode u rezervoaru bio na 50 cm od dna rezervoara, dok je na sredini dna rezervoara utiskivan vazduh protokom od 1.0 L/min. U ovom slučaju, osmotreno je formiranje jednog stacionarnog vrtloga, po čijem obodu se formira tok vazduha ka površini.

Slika 1 prikazuje poređenje formirane vazdušnu struje, osmotrenu eksperimentom i dobijenu simulacijom. Može se videti da je na početku vazdušne struje dobijena nešto šira zona strujanja vazduha, što se može objasniti krupnijom diskretizacijom (domen strujanja je podeljen na kocke sa stranicom dužine 2 cm). Raspored izmerenih i sračunatih brzina vode je prikazan na slici 2.

$$C_d = \max\left[\frac{24}{\mathrm{Re}}\left(1+0.15\,\mathrm{Re}^{0.687}\right)\frac{8}{3},\ \frac{Eo}{Eo+4}\right] \quad (10)$$

Equations (1) - (10) are discretized by the finite volume method, so that the vector and scalar sizes are defined on the sides and in the center of the control volume, respectively. Calculation of pressure and velocity values in each time step used HSMAC (Highly Simplified Marker and Cell) method [5], with application of Adams-Bashforth scheme for time integration of convective and viscous parts in the dynamic equations. HSMAC scheme itself is to some extent corrected, in order to implement the bubble flow problem [6]. Standard κ–ε μοδελ model [7] was used for calculation of Reynolds voltage, with well known equations, which are not listed here.

For modeling of multi-phase flow, specifically at solving equations (1), (2) and (6), of great importance is the discretization of convective arts in terms of preventing the effect of numerical diffusion and numerical oscillations. Therefore, the model used the third order accuracy scheme, with application of TVD (Total Variation Diminishing) limiters [6].

3 MODEL VALIDATION

3.1 *Simulation of experiments with a constant air injection into the water*

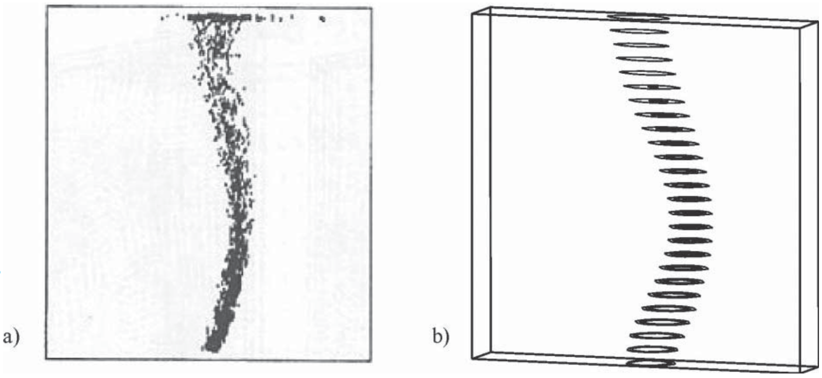
The developed model is applied to simulate laboratory experiments from the literature [8, 9], where the air is injected in the 200 × 50 × 8 cm reservoir. The reservoir was previously filled with water while the water level varied for different experiments. Distribution of gas phase is visually observed during the experiment, with the measuring of water flow by high vertical plane (4 cm from the wall), with a mutual distance between measuring points of 10 and 20 cm.

The first simulation refers to an experiment in which the water level in the reservoir was 50 cm from the bottom, while the gas was injected from the center of the reservoir bottom with air flow of 1.0 L/min. In this case, the formation of a stationary vortex was observed; with air flow forming on its rim streaming to the surface.

Figure 1 shows a comparison of the formed air current, observed during the experiment and obtained by simulation. Slightly wider zone of air flow can be seen at the beginning of air current, which can be explained with larger discretization (flow domain is divided into cubes with side length of 2 cm). Distribution of measured and calculated water velocities is shown in Figure 2.

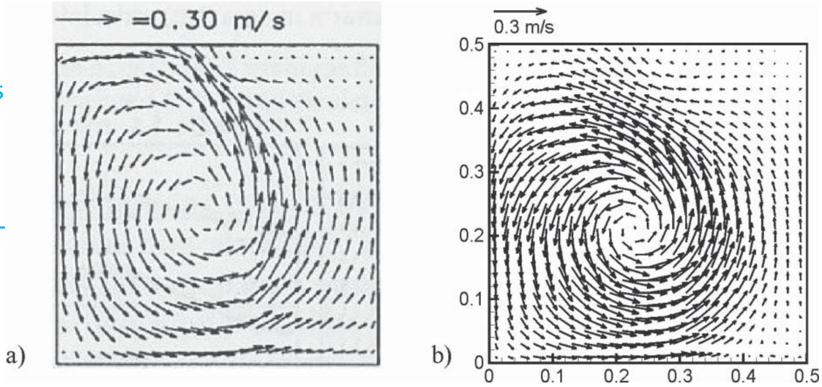
Slika 1. Poređenje osmotrenog i simuliranog rasporeda vazdušne struje pri protoku od 1 L/min: a) eksperiment (Borchers et al., 1999), b) granice vazdušne struje dobijene simulacijom.

Figure 1 Comparison of observed and simulated air flow distribution of 1 L/min: a) experiment (Borchers et al., 1999), b) limits of air flow obtained by simulation.



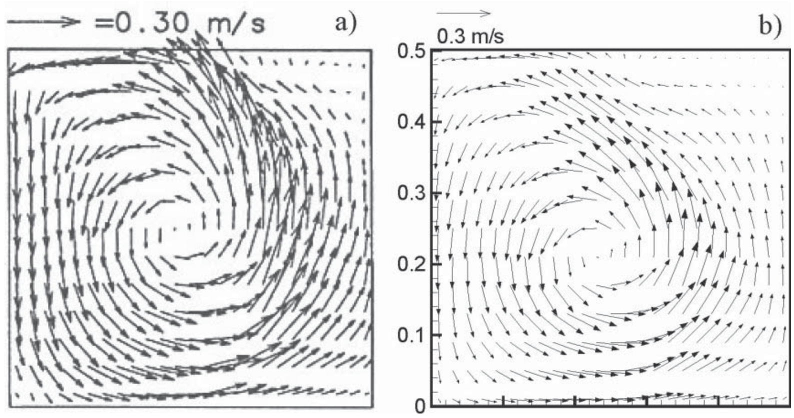
Slika 2. Poređenje izmerene i modelirane strujne slike pri protoku vazduha od 1 L/min: a) izmereni raspored brzina (Borchers et al., 1999), b) simulacija.

Figure 2 Comparison of measured and modeled air flow current of 1 L/min: a) measured velocity distribution (Borchers et al., 1999), b) simulation.



Slika 3. Poređenje osmotrene i modelirane strujne slike pri protoku vazduha od 2 L/min: a) izmereni raspored brzina (Borchers et al., 1999), b) simulacija.

Figure 3 Comparison of observed and modeled air flow current of 2 L/min: a) measured velocity distribution (Borchers et al., 1999), b) simulation.



Slična strujna slika je dobijena i kod protoka vazduha od 2 L/min, što je prikazano na slici 3. Generalno, dobijeno je dobro slaganje eksperimentalnih i simuliranih veličina. Pri tome, treba imati u vidu da prilikom simulacije nije bila poznata početna veličina mehurića, već je usvojena konstantna vrednost, što svakako ne odgo-vara eksperimentalnim uslovima.

Similar air flow current is obtained with air flow of 2 L/min, as it is shown in Figure 3. In general, a good agreement of experimental and simulated values was obtained. Besides, one should bear in mind that during the simulation initial size of bubbles was not known, but a constant value was adopted, which certainly does not correspond to experimental conditions.

3.2 Simulaciju eksperimenata sa merenjem promene koncentracije kiseonika

3.2 Simulation of experiments with measuring of changes in oxygen concentration

Kao što je ranije napomenuto, prikazani model je razvijen u okviru istraživanja mogućnosti primene elektrolize za poboljšanje kvaliteta vodnih tela [10]. Istraživanja su obuhvatila niz laboratorijskih opita (Shinshu University, Matsumoto, Japan), kao i terenskih istraživanja na Biwa jezeru u Japanu. Određeni rezultati ovih ispitivanja su iskorišćeni za validaciju razvijenog modela, od kojih se pojedini prikazuju u nastavku.

Laboratorijski opit se sastoji od posude zapremine

As previously mentioned, displayed model was developed within the research of possibilities for application of electrolysis for improvement of water body quality [10]. Research has included a series of laboratory experiments (Shinshu University, Matsumoto, Japan), and field research on Lake Biwa in Japan. The results of these tests are used to validate the developed model, some of which are shown below.

The laboratory experiment consists of a reservoir, with volume of 90.6 L and 6 pairs of metal (platinum)

90,6 L, na čijem dnu se nalazi 6 parova metalnih ploča (platina) kroz kroje se propušta struja jačine 2.3A. Pre počet-ka eksperimenta, hemijskim putem je smanjena koncentracija rastvorenog kiseonika, tako da je početna koncentracija rastvorenog kiseonika bliska nuli. Osim što je osma-tran raspored gasovite faze u vodi vizuelno, merena je i promena koncentracije rastvo-renog kiseonika u tri tačke duž vertikalne ose koja prolazi kroz centar osnove suda. Merne tačke su se nalazile na 25, 35 i 45 cm od dna suda.

Za numeričku simulaciju domen je izdeljen na elemente 5 × 5 cm u osnovi i sa promenjivom visinom, od 1,2 cm do 5 cm. Na 7,5 cm od dna suda, zadat je fluks me-hurića kiseonika i vodonika konstantnog prečnika od 0.1 mm. Fluks kiseonika je sra-čunat na osnovu Faradej-ovog zakona:

$$\frac{24.8L/mol\cdot 2.3A}{4\cdot 96485As/mol}=0.148\cdot 10^{-3}L/s, \qquad (11)$$

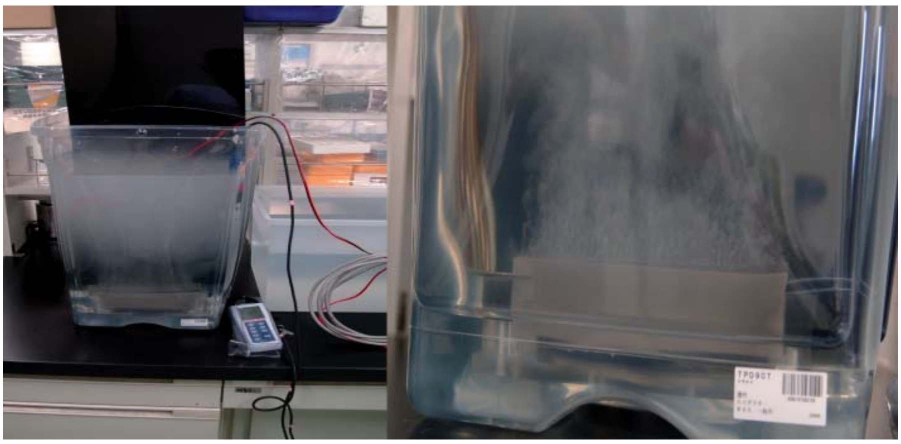
što pri pritisku od 1.05 atm daje 8.46 × 10^{-3} L/min. Usvojeno je da je fluks vodonika jednak dvostrukoj vrednosti za kiseonik. U cilju simulacije rasporeda koncentracije kiseonike, pored navedenog sistema jednačina, neophodno je rešavati i jednačinu kontinuiteta za rastvoreni kiseonik u vodi:

$$\frac{\partial(\alpha_w DO)}{\partial t}+\frac{\partial}{\partial x_i}(\alpha_w DOV_{wi})=\frac{\partial}{\partial x_i}\left(\alpha_w D_w\frac{\partial DO}{\partial x_i}\right)+G_{gw} \qquad (12)$$

gde je D_w koeficijent disperzije za kiseonik. Postupkom elektrolize oslobađa se kiseonik i vodonik u molarnom odnosu 1:2. Obzirom da tokom eksperimenta vodonik nije odstranjivan, neophodno je rešavati jednačinu (6) za svaki element zasebno.

Odmah po startovanju, formira se struja mehurića povlači vodu sa sobom što uzrokuje vertikalnu cirkulaciju vode, a time i koncentraciju gasovite struje u sredini suda, gledano u osnovi (Slika 4). Vektori brzina u nekoliko horizontalnih ravni, kao i na samoj površini vode su prikazani na slici 5.

Slika 4: Eksperimentalna instalacija za promenu koncentracije kiseonika u vodi u toku procesa elektrolize.
Figure 4: Experimental installation for changing the concentration of oxygen in the water during the process of electrolysis.



plates located on the bottom, used for electrisation (2.3A). Before the start of the experiment, the concentration of dissolved oxygen is chemically reduced, so the initial concentration of dissolved oxygen is close to zero. Beside visual observation of gas phase distribution, changes in the concentration of dissolved oxygen were also measured, in three spots along the vertical axis that passes through the center of the reservoir. Measurement spots were located at 25, 35 and 45 cm from the bottom.

For numerical simulation domain is divided into 5 × 5 cm elements in the base and variable height from 1.2 cm to 5 cm. At 7.5 cm from the bottom, the flux of oxygen and hydrogen bubbles with constant diameter of 0.1 mm was given. Flux of oxygen is calculated based on Faraday's Law:

$$\frac{24.8L/mol\cdot 2.3A}{4\cdot 96485As/mol}=0.148\cdot 10^{-3}L/s, \qquad (11)$$

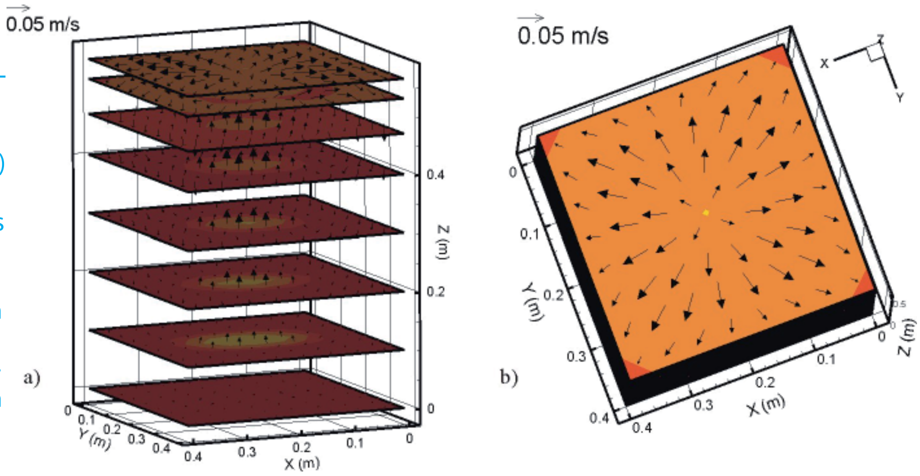
and for pressure of 1.05 atm gives 8.46 × 10^{-3} L/min. It is adopted that the flux of hydrogen is equal to double value for oxygen. For the purpose of the simulation of concentration distribution of oxygen, in addition to the above equations, it is necessary to solve the continuity equation for dissolved oxygen in water:

$$\frac{\partial(\alpha_w DO)}{\partial t}+\frac{\partial}{\partial x_i}(\alpha_w DOV_{wi})=\frac{\partial}{\partial x_i}\left(\alpha_w D_w\frac{\partial DO}{\partial x_i}\right)+G_{gw} \qquad (12)$$

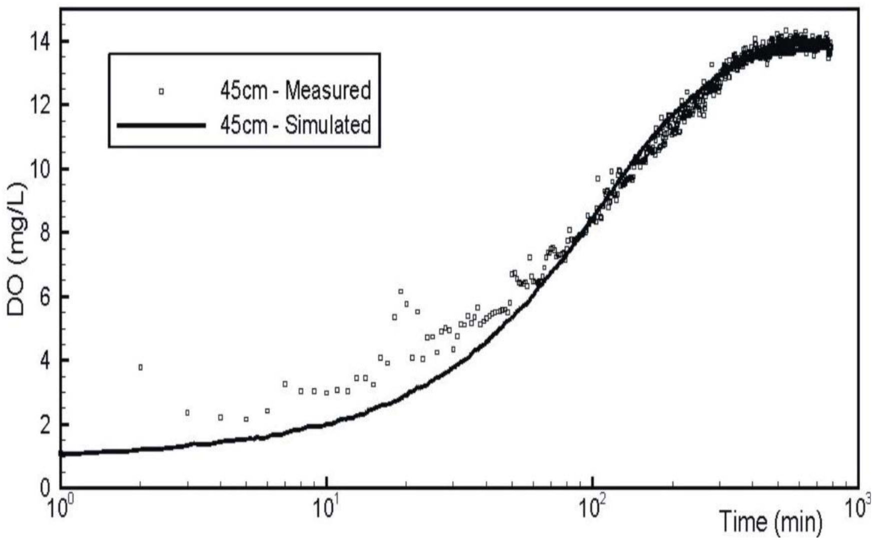
where D_w is dispersion coefficient for oxygen. Electrolysis process releases oxygen and hydrogen in molar ratio 1:2. Considering that during the experiment, hydrogen has not been removed, it is necessary to solve equation (6) for each element separately.

Immediately after startup, the current of bubbles is being formed which drags the water with it, causing vertical circulation of water, and thus the concentration of gas stream in the middle of the reservoir; observing the bottom (Figure 4). Velocity vectors in a horizontal sections and on the surface of water are shown in Figure 5.

Figure 6 shows the comparison of measured and calculated changes in oxygen concentration at 45 cm from the bottom of the reservoir. Initial measured concentrations of oxygen were 0 mg / L at 25 cm, 0.318 mg / L at 35 cm and 0.746 mg / L at 45 cm from the bottom. Immediately after start, the measured values at 35 and 45 cm showed a very rapid increase in concentrations of about 3 to 4 mg / L. As can be seen in the diagram, this rapid increase in the concentration of oxygen in the opening minutes of the experiment, are not well reproduced by the model, while the later matches are



Slika 5. Sračunati profili sadržaja gasovite faze i vektori brzina, 1min. nakon početka eksperimenta: a) vektori brzina u horizontalnim presecima (različite boje označavaju sadržaj gasovite faze), b) vektori brzina na površini vode.
Figure 5 Calculated profiles of gas phase content and velocity vectors, 1min. after the start of the experiment: a) velocity vectors in horizontal sections (different colors indicate the content of gaseous phase), b) velocity vectors on the water surface.



Slika 6. Poređenje izmerenih i simuliranih koncentracija kiseonika na 45 cm od dna suda
Figure 6 Comparison of measured and simulated oxygen concentrations at 45 cm from the vessel bottom

Slika 6 prikazuje poređenje merenih i sračunatim promena koncentracija kiseonika na 45 cm od dna suda. Inicijalne izmerene koncentracije kiseonika su bile 0 mg/L na 25 cm, 0,318 mg/L na 35 cm i 0,746 mg/L na 45 cm od dna. Neposredno po startovanju, izmerene vrednosti na 35 i 45 cm su pokazale vrlo brzo povećanje koncentracije na oko 3 do 4 mg/L. Kao što se može videti na dijagramu, ovo brzo povećanje koncentracije kiseonika u početnim minutima eksperimenta, model nije dobro reprodukovao, dok su kasnije slaganja vrlo dobra. Pretpostavka je da jednačina (5), odnosno vrednost koeficijenta transfera nije odgovarajuća za niske vrednosti koncentracije kiseonika.

4 ZAKLJUČAK

U okviru prikazanih istraživanja razvijen je 3D numerički model za simulaciju dvofaznog strujanja vode i vazduja, koji se može primeniti kod modeliranja različitih hidrauličkih problema. U ovom radu prikazano je nekoliko rezultata procesa validacije modela, kroz simulacije laboratorijskih eksperimenata iz literature, kao i sprovedenih eksperimenata u okviru istraživanja efekata elektrolize na povećanje koncentracija rastvo-renog kiseonika u vodi.

Obzirom na kompleksnost modeliranog procesa, kao i niza usvojenih pretpostavki, može se konstatovati da razvijeni model može da reprodukuje dvofazno tečenje vode i gasovite faze, kao i proces rastvaranja komponenti gasovite faze u vodi.

very good. The assumption is that equation (5), or the value of transfer coefficient is not appropriate for low values of oxygen concentration.

4 CONCLUSION

In the presented study a numerical model has been developed for simulation of bubble flow, which can be applied for modeling of various hydraulic problems. This paper presents several results of a model validation process, through simulations of laboratory experiments from the literature, and experiments conducted in the study of effects of electrolysis for increase of the dissolved oxygen concentration in the water.

Considering the complexity of modelled process, and adoption of a series of assumptions, it can be concluded that the developed model can reproduce two phase flow of water and gas phases, and the dissolving process of the gas components in the water.

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