GEOMETRY, GRAPHICS AND DESIGN IN THE DIGITAL AGE

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Editors

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A METHOD FOR ADJUSTING THE SHAPE OF SEMI-OVAL ARCHES USING HÜGELSCHÄFFER'S CONSTRUCTION

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Abstract

There are numerous ways to construct arches as curved structural elements that span openings in architectural buildings. Arches are present in many historical styles, having the role of an identification mark. Contemporary architecture also uses a resource of inherited styles, combining them into an eclectic blend of modern and classic. However, regardless of the aspirations that investors, architects and future users may have towards some of the complex stylistic forms, the question is whether every contractor will be able to meet the set requirements. Although modern construction technology is apt to perform much more diverse forms than in earlier epochs, there are still situations when some problems, especially geometric ones, need to be solved, since the technology itself has not yet reached the point of its own thinking. One such problem was posed to the authors by a contractor who came up with the question: how to easily and accurately make a template for a semi-oval window arch, with a predefined point (M) through which the arch should pass. Hence, this is not about an elliptical arc for which there are a number of known constructions. The request was to offer a construction of a higher order curve, simple enough to enable quick and easy design and fabrication of templates on the construction site. In semi-oval arcs, the curve deviates from the elliptical one, giving greater curvature at the vertices of the major axis, and lesser at the vertex of the minor semi-axis. To solve this problem, we use the generalization of the Hügelschäffer's egg curve construction. With input data: the major and minor semi-axes *a* and *b,* together with the given position of the point M which defines the deviation of the oval curve from the ellipse, we first determine the displacement of the minor circle of the Hügelschäffer's construction along the y axis. Then, by applying the transformation of hyperbolism, we obtain the points of the semi-oval. The offered construction gives quick and accurate positions of points on the semi-oval arc, moreover, it allows the adjustment of the shape of the semi-oval arch according to needs (aesthetic, stylistic, constructive, functional, etc.). In the digital age, it is no problem to draw a higher order curve, but in order to simplify the construction for the purpose of making templates on the construction site, we can turn to the classic method - the approximation of curves with circles, as with semi-elliptical arcs. We find the centers of circles and use their successively connected arcs to accomplish the task with synthetic tools: compass and ruler.

Keywords: arc, ellipse, oval, Hügelschäffer's construction, architecture.

1 INTRODUCTION

Arches, as elements of architectural constructions, are present in many historical styles, having the role of an identification mark. We meet them as window and door lintels, arched beams and triumphal arches in numerous architectural buildings of historical and cultural importance. Contemporary architecture also uses a resource of inherited styles, combining them into an eclectic blend of modern and classic. However, regardless of the aspirations that investors, architects and future users may have towards some of the complex stylistic forms, the question is whether every contractor will be able to meet the set requirements. Although modern construction technology is apt to perform much more diverse forms than in earlier epochs, there are still situations when some problems, especially geometric ones, need to be solved, since the technology itself has not yet reached the point of its own thinking. One such problem was posed to the authors by a contractor who came up with the question: how to easily and accurately make a template for a semi-oval window arch. Hence, this is not about an elliptical arc for which there are a number of known constructions. The aim was to find a sufficiently simple, and yet sufficiently precise construction of the oval, a curve of a higher (4th) order understandable enough for application in practice.

1.1 Architectural Types of Arches

There are numerous ways to construct arches as curved structural elements that span openings in architectural buildings, just as there are many different shapes of arches, window sills and curved beams in architecture. We will find different systematizations in different professional literature, and we will rely on the sources [1] and [2] that gathered most of the shapes in one place, so among the architectural types of arches we find 30 of them: flat, round, segmental, horseshoe, three-centred arch, four centred arch, triangular, parabolic, inflexed, rampant, keyhole, ogee, oriental, shouldered, draped, cinquefoil, multifoil, complemented with its pointed, rounded and pseudo variants, as shown in the Fig. 1.

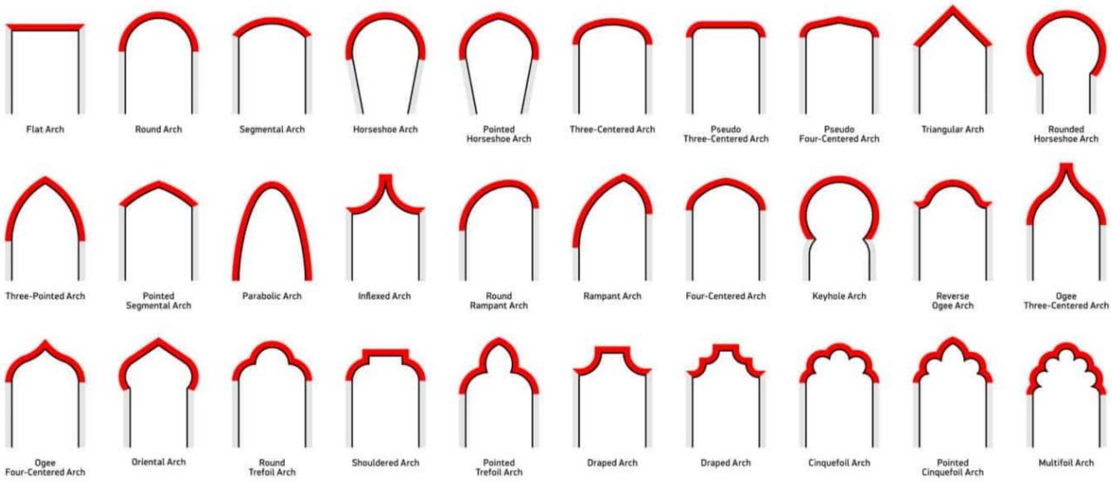


Fig. 1. Architectural types of arches [1]
https://www.homestratosphere.com/types-of-arches/

The elliptical arch, which follows the shape of half of the 2-degree geometric curve (conic) - ellipse, in most sources was not processed from a strictly geometric aspect, so we come across terms such as: semi – arch [3], [4], [5], a three-centred arch [6], or basket handle arch in [2]. From this we see that the geometry of the ellipse itself is often neglected, and we come to practical solutions, which give an approximate, sufficiently smooth and continuous curve, composed of circular segments. However, for a better and more accurate construction, a five-center construction is often used, which gives a semi-oval arc composed of five circular arcs, the centers and radii of which are determined by geometric procedures [7].

It is interesting that, probably due to the archaic nature of the procedure itself, a detailed explanation of this five-center construction is very often completely omitted, so it is left to the reader to reconstruct, if he has enough geometric knowledge. Let us mention several such sources: [7], [8], [3], [4] and [5], where a satisfactory geometrical explanation is not given, but only a procedure for obtaining the desired result. If we now, instead of an ellipse, have an additional requirement that the arch should be adapted to the individual case, e.g. that due to aesthetic, constructive or lighting reasons, this arch should be "broader", i.e. more curved towards the vertices of the major axis, and less curved at the vertex of the minor axis, this geometric problem becomes more complex.

In this paper, a clear distinction between an ellipse and an oval was made, so the constructions concerning the ellipse were left aside. The oval, as part of a higher order (fourth) curve, is treated from the aspect of a geometric construction derived from Hügelschäffer's construction ([9], [10]) although it is not actually the Hügelschäffer's construction itself, but a version resulting from its expansion, i.e. Modifications, as stated in the paper [11].

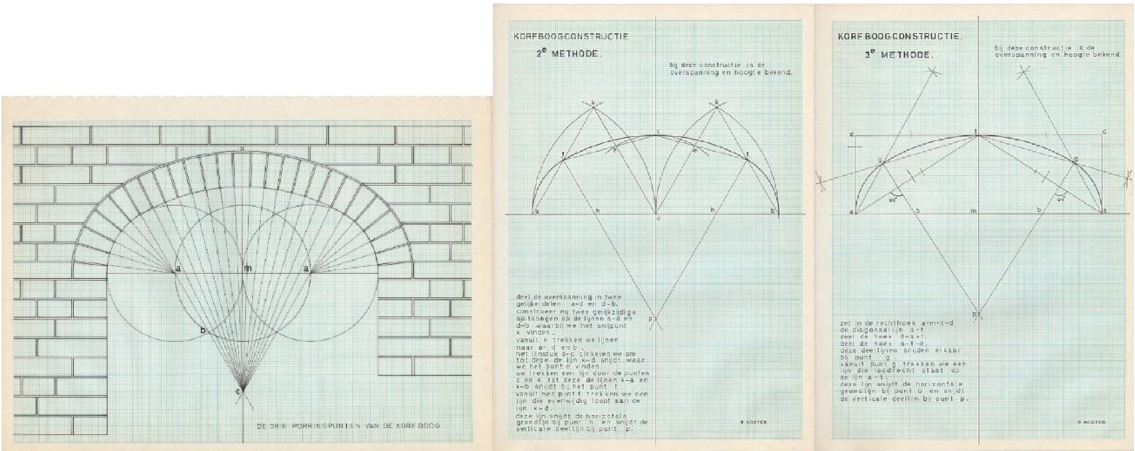
The modification consists in the fact that the center of the hyperbolism is in the center of the chord AB, as for the ellipse, and the center of the minor circle (c_2) is shifted - shifted along the y axis for the value "y".

This construction also has its origins in the ellipse construction, as can be seen in [6], [8], [11] and [12]. A detailed mathematical elaboration of the Hügelschäffer's curve can be seen in [13].

At the request of a construction contractor, we accepted the task of helping him solve this problem and offer a solution that would enable not only easy construction, but also easy creation of templates for work on the construction site.

2 THE GEOMETRICAL REASONING BEHIND THE METHOD

In various sources, e.g. [2], [6], [7], [3], [4] and [5], we find diverse constructive procedures for the construction of a semi-elliptical arc. Many of them are based on approximations of ellipse by circles, i.e. on the superposition of ellipses with circular arches, such as those given in Fig. 2 and Fig 3. Specifically: "korfboog" i.e. "basket handle" oval construction consists of three or five adjacent circular segments, with the smaller circles placed at the end points of the span's segment and the larger circle in the middle. All variants are generally based on pre-defined sizes of span and rise, whereby the resulting arch also has a pre-defined shape, without taking into account possible deviations in curvature in relation to the usual arch shape. Such procedures originate from the late Gothic and Renaissance periods, up to the 17th century, to be used again in the turn of the 19th to the 20th century ([2], [14]). They are mainly based on empirical results of the optimal continuation of circular arches into a semi-oval shape. On the other hand, there are also constructions using hyperbolism: e.g. [6], [8], [11] and [12]. It is based on dividing two concentric circles into a given number of parts by intersecting rays radiating from a common center (Fig. 3b).Thereby the result is an (semi-)ellipse.



a) b) c)

Fig. 2. Basket handle oval constructions (Korfboog constructie oval) a) [4]; b) [5]; c) [3]

In some constructions, especially in those where circles are used, the ellipse itself is less significant in the geometric sense, but a shape that will satisfy the semi-oval, i.e. basket handle arch. Only in few sources [15]15 is the distance clearly stated and it is indicated that it is not an ellipse, but an oval.

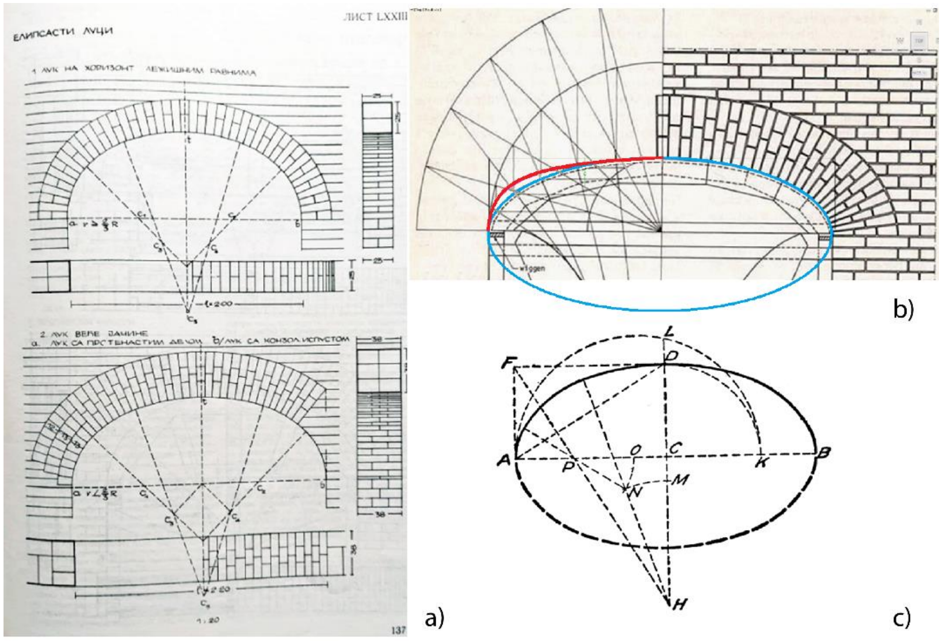


Fig. 3. Several geometric constructions of elliptical arch: a) [7]; b) [8]; c) [15]

Those constructions are certainly useful, but they are mostly defined by the proportions of the span of the opening and the height of the arch, i.e. sagitta or rise of the arch, which define the shape of the ellipse. Thus, the result is known and unambiguous. In some of them, we do not start from the known major diameter or minor radius of the arc, but from the angular values of the diameters of the continuing circles, and only as a result do we get the exact parameters of the ellipse. In some sources, the point through which the arch should pass is already known and assigned, but in other, the point is not substantial, but given “frame”, i.e. diameter of the opening, and the rise of the arch (AB and CD).Hence, we can notice that for the constructors of the earlier centuries, geometric bases of the construction is secondary, or completely neglected. What was required was a quick and easy construction of a template for on-site execution. Thus, among these constructions, the geometric interpretation was often completely omitted. What kind of curve it was, ellipse or an oval, what kind of oval, etc. was not considered important, and especially no solution was given for adapting the oval to the given initial conditions.

In this study, we used an extended (modified) Hügelschäffer’s construction to get an oval, not an ellipse. Generalizing Hügelschäffer’s egg-curve construction, resulted in different types of ovals, fourth order curves and even higher order curves, up to 8th degree [11].

In this paper, we apply the modification of the Hügelschäffer’s construction and give a proposal for a quick and simple construction of an adjustable semi-oval arch, which can have different shapes, i.e. curvature at the vertex points, depending on the given initial parameters.

3 HÜGELSCHÄFFER’S CONSTRUCTION AND THE MODIFIED PROCEDURE

The initial setting of the problem is usually given the diameter **AB** of the wall opening and the radius of the arc **CD**. These two perpendicular lines define the rectangle circumscribed around the arc. We can inscribe a semi-ellipse in this rectangle and it is completely defined by this. However, for various reasons, stylistic, constructive or functional, there is sometimes a need for this arch to deviate from the ellipse and look more like an oval. So, it is necessary for the arch to be "more bent", that its points should be e.g. closer or farther from the vertex of this described rectangle.

We encounter Hügelschäffer’s construction (Fig. 4a) in the sources dealing with egg curve construction, i.e. [16], [17], [18]. The construction itself is based on the transformation of hyperbolism [17], where two circles are used (in the previous sources marked as c_1 and c_2) as in the construction of the ellipse. In contrast, with Hügelschäffer’s construction one circle (c_1, commonly of the minor radius) is shifted along the x axis by some w, and this circle (c_1) is the one in whose center is also set the center of the hyperbolism. As a result of the construction, an oval curve is obtained, more precisely: a part of the curve of the third order, of which the egg-shaped oval is a part [18], [17].

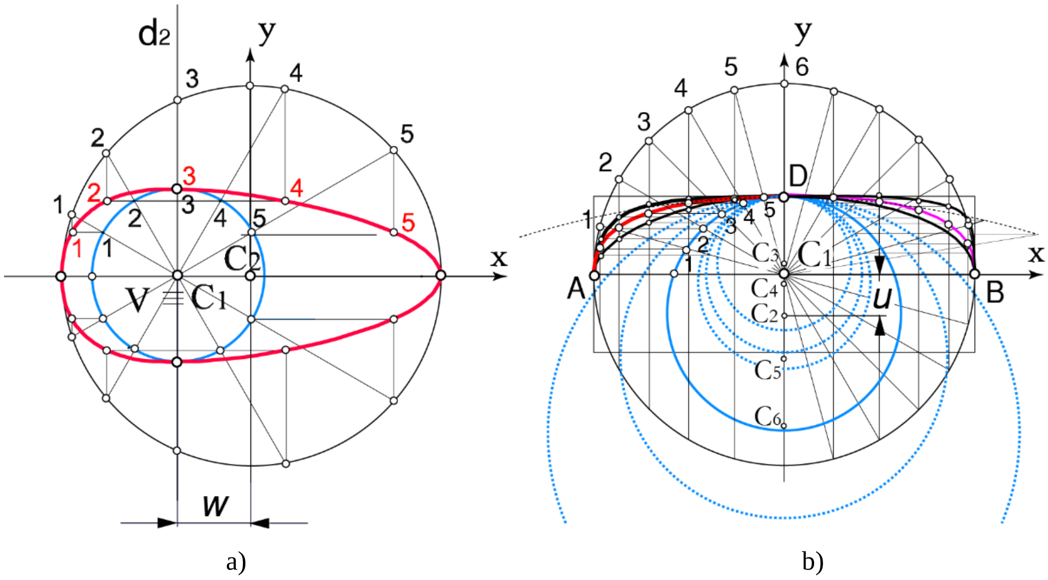


Fig. 4. a) Hügelschäffer's construction of the egg curve; b) modification of the Hügelschäffer's construction applied on a semi-oval arch

By modifying the Hügelschäffer's construction, we can obtain the desired oval arch with a simple construction. In the paper [11], special attention was paid to the variations and modifications of this construction, so diverse cases of displacing the center of the hyperbolism from the center of the circle c_1, in the direction of both the **x** and ***y*** axes were examined. From this study we use the method of shifting the center of the hyperbolism from the center of the "minor" circle (relatively speaking) to the center of the "major" circle. In fact, in this study, the center of the hyperbolism stays in the center $\mathbf{C_1}$ of the circle c_1, having that now "minor" circle is c_2, displaced from the coordinate center at point $\mathbf{O=C_1}$ along the **y** axis by some ***u***, and that its diameter is always $\mathbf{C_2D}$.

In Fig. 4b, we see an example of creating arcs as a product of the described transformation. Along the y axis we will shift the circle with the centre in $\mathbf{C_{2\text{-}n}}$, but so that the circle in each variant passes through the point **D** and has a center on the ***y*** axis We will notice that the larger the diameter of the circle, the more "straightened" the resulting arc will be at the vertex **D**, and more curved in the part that bends towards the corner point **F** (or **G**) of the rectangle - the intersection point of the tangents at the vertices **A** and **D**, or symmetrically: **B** and **D**.

Since both the centers $\mathbf{C_1}$ and $\mathbf{C_2}$ lie on the ***y***-axis, which is also the axis of symmetry for the rectangle **ABFG**, it is clear that the arc (and the whole curve) will be symmetrical with respect to it, so that everything that happens in the left sector of the rectangle, will be repeated symmetrically in the right sector, and vice versa.

If we were to complete the construction and look for a whole curve instead of an arc, as a result we would get a quartic curve, the bean curve [14], [16], similar to the one in [11], Fig. 9]. An example is shown in Fig. 5a**.**

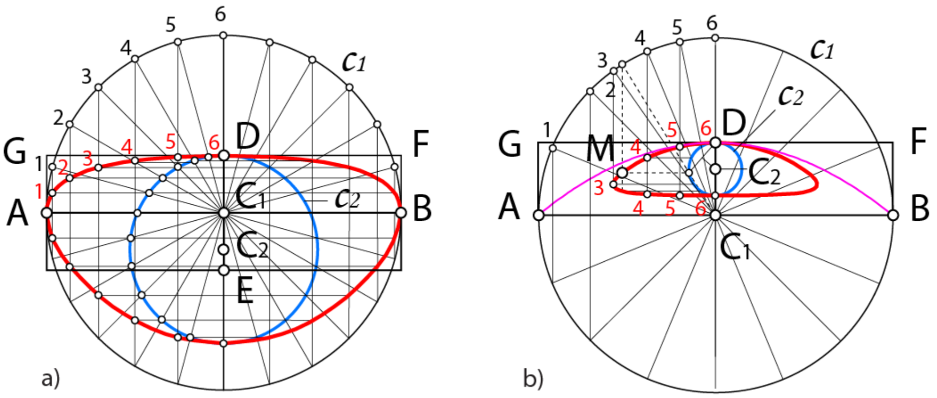


Fig. 5. Finding the center C_2 of the minor circle

Therefore, according to the above and according to Fig. 4b, since it is possible to place more (geometrically: an infinite number) of curves in the same rectangular frame, then it is possible to find the curve that passes through the given point **M**.

However, we must bear in mind that the position of the point **M** is not entirely arbitrary. It is limited by a few conditions:

1) The "minor" circle $\boldsymbol{c_2}$ must not have a diameter smaller than $\mathbf{C_1D}$, because in that case the bean curve of 4 degree is "shrunk" above the $\boldsymbol{x}$-axis and we do not get a curve passing through the points **A** and **B**, which is the initial condition. Therefore, the region in which we can set the point **M** is limited by the parabola $\boldsymbol{p}$ (given in pink in the Fig. 5b), which is obtained by setting the circle $\boldsymbol{c_2}$ so to have the diameter $\boldsymbol{d_2}=\mathbf{C_1D}$, i.e. that $\boldsymbol{c_2}$ is tangent to the $\boldsymbol{x}$ axis.

2) The point **M** should preferably be set inside the area of the "major" circle $\boldsymbol{c_1}$, i.e. not in the area between the rectangular frame and the circle $\boldsymbol{c_1}$, because the resulting curve may be too close to the tangents at the vertices of the arc, which makes the shape of the arc itself barely perceptible close to a rectangle (Fig. 6a). However, the closer the length of the rise of the arc is to the length of $\mathbf{AC_1}$, or be even greater, the more the position of the point **M** outside the circle $\boldsymbol{c_1}$ can be tolerated (Fig. 6b).

Knowing all the aforementioned, we can set up the construction which gets us the necessary elements to generate the part of the curve - the desired oval arc that passes through the assigned point **M**.

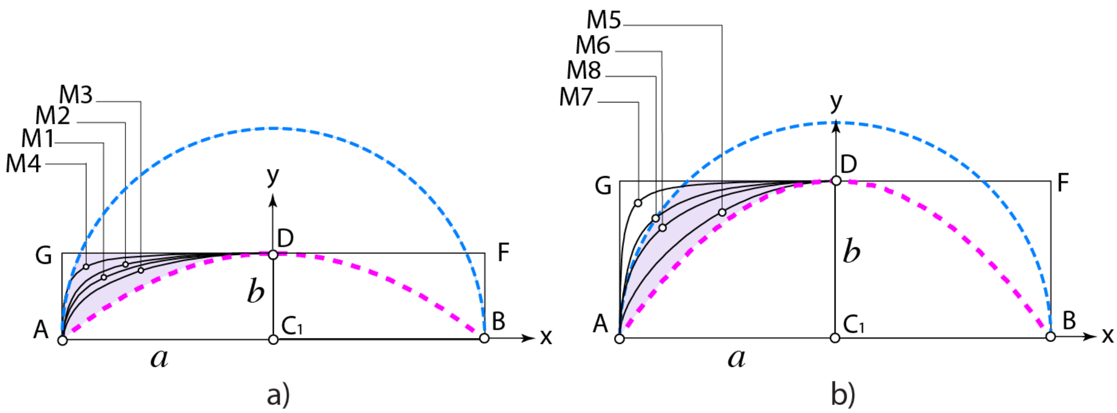


Fig. 6. Borderlines and a preferable position of the given point M

3.1 The Construction: Finding the Circle c_2

Let us assume that there are given: the span of the opening **AB** and the rise of the arch **CD**, where **C** is in the midpoint of the segment **AB**. According to the above limitations, there is also a given point **M**. We need to construct a semi-oval arc which should pass through the points **A, B, D** and **M**. We will use the modified Hügelschäffer's construction and apply its principles according to the considerations above.

The procedure is as follows (see Fig. 7): we draw a rectangle **ABFG**, a tangent parallelogram through the points **A**, **B** and **D**. In the midpoint **C** of the diameter **AB** is the center $\mathbf{C_1}$ of the "major" circle $\boldsymbol{c_1}$ of hyperbolism. Its radius is $\boldsymbol{a}\mathbf{=C_1A=C_1B}$ and we can find it directly.

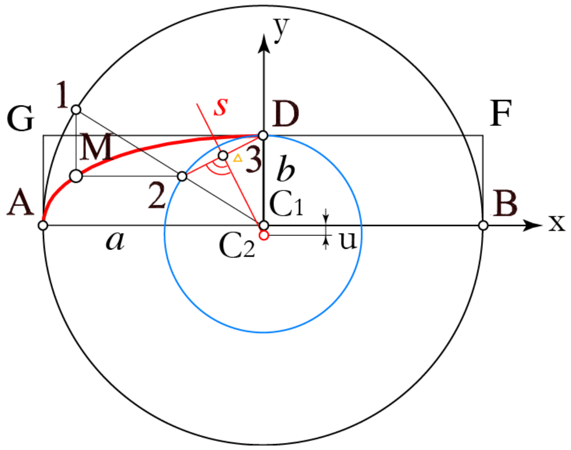


Fig. 7. Finding the center C_2 of the minor circle of the Hügelschäffer's construction based on the given point M

To determine the desired position of the smaller circle with center in $\mathbf{C_2}$, we start from the point **M.** By the inverse procedure we place a vertical line parallel to the ***y*** axis through the point **M** and in its intersection with the circle $\boldsymbol{c_1}$ we find the point 1. The point 1 and the center $\mathbf{C_1}$ define the ray of hyperbolism. In the intersection point of this ray and the horizontal line, parallel to ***x*** axis, from the point **M**, we find point **2** of the "minor" circle $\boldsymbol{c_2}$***.*** To get the center $\mathbf{C_2}$, we connect the points **D** and **2** and find the chord of the smaller circle $\boldsymbol{c_2}$***.*** It is now clear that the center $\mathbf{C_2}$ will be at the intersection point of the bisector of the chord **D-2** and the **y**-axis which passes through the points $\mathbf{C_1}$ and **D**.

Now that we have found the center C_2, we obtain the "minor" circle of the modified Hügelschäffer's construction. The value of the circle radius is:

$$c_2 = \overline{\mathrm{C_2D}} = \overline{\mathrm{C_2 2}}.$$

We then apply hyperbolism, i.e. modified Hügelschäffer's construction. As the result, we obtain two overlapping, bean-shaped, 4th degree curves. We are only interested in the "upper" part of the curve, i.e. the one on the positive side of the ***y***-axis, where point **D** is located.

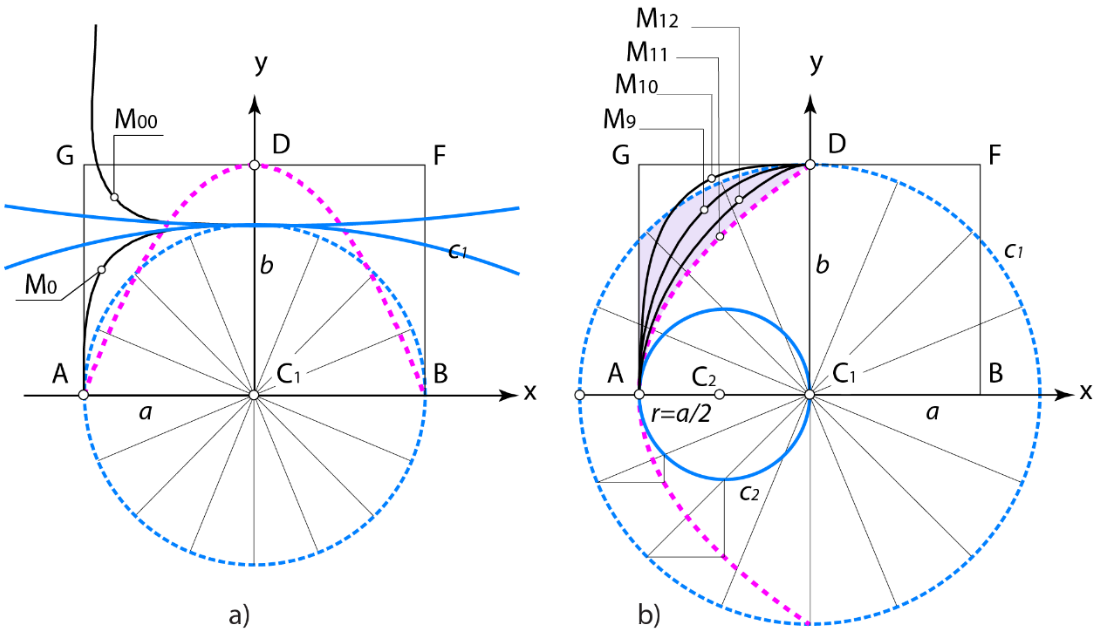


Fig. 8. Finding borderlines for the optimal position of the point M for the case when b>a

Now, however, we can ask the question: what happens if the proportions of the arch span and the rise *a:b*, are such that *b>a*? In that case, as we see in Fig 8a, the radius of the circle c_1 is smaller than the length **C_1D**, so that the vertex of the curve does not reach the vertex **D** of the arc. In addition, in such cases the radius of the circle c_2 is disproportionately larger than the radius of the circle c_1, which makes the construction impractical, and its results unusable. Therefore, we will modify the construction once again. The circle c_1 now has a radius of **$C_1D=b$**, so that one of the vertices of the curve is at the point **D** (Fig. 8b). Meanwhile, given that the same circle now passes outside the rectangle **ABFG**, we will change the direction of the rays of hyperbolism, so the modification of the Hügelschäffer's construction will be somewhat different. Circles c_1 and c_2 change the angles of the hyperbolism rays, so now the circle c_1 is the one from whose end points of intersection the rays of the hyperbolism are horizontal (**II*x***), and the circle c_2 whose center we move along, now, the x axis as with the Hügelschäffer's construction, by the value w. The circle c_2 now is the one from whose points we place vertical rays (**II*y***). The construction for finding the center of the circle c_2 is identical to the previous construction, only rotated by 90 degrees.

In this case, the region in which we search for the optimal position of the point **M** is now limited by the rectangle **ABFG**, the parabola ***p*** whose vertex is at the assigned point of the arc's chord (**A** or **B**, depending on which side of the rectangle we take the starting point **M**), and which passes through the vertex point **D** on one side, and the circle c_1 on the other.

This way, with the application of reflection we get complete arches with a common horizontal tangent at point **D**, as well as some variants with a broken arch, when the point M is exactly on the parabola ***p***.

3.2 Approximation with Circular Arches: Five-Center Construction

In the digital age, generating the curve itself is not a problem. For example, in numerous graphic software, we can perform it quickly, simply and accurately using the spline tool (given by the points it passes through). Then, the drawing can be printed for work on the construction site (in 1:1 scale) or even directly cut the formwork by machines (CNC or laser cutting machines). However, if we wish to return to analogue procedures and prepare the sketch for work with an elementary, low-budget tool, e.g. compasses, we can follow in the footsteps of builders from earlier centuries and translate this construction into a 5-center circle construction, that is, a basket handle construction.

We find the centres (**K_{1-5}**) of the five circles as intersection points of the bisectors of the chords: A-1 and 1-2 for **K_1**, 2-3 and 3-4 for **K_2**, and **4-5** and **y** axis for **K_3**, having that we have used the division of the circle c_1 on 24 segments, i.e. its upper semicircle on 12 segments (Fig. 9). Symmetrically, across the **y** axis, we find the remaining two centers, **K_4** and **K_5**. Using their successively connected arcs A-2, 2-4, 4-8, etc. to perform the task using synthetic tools: compasses and rulers.

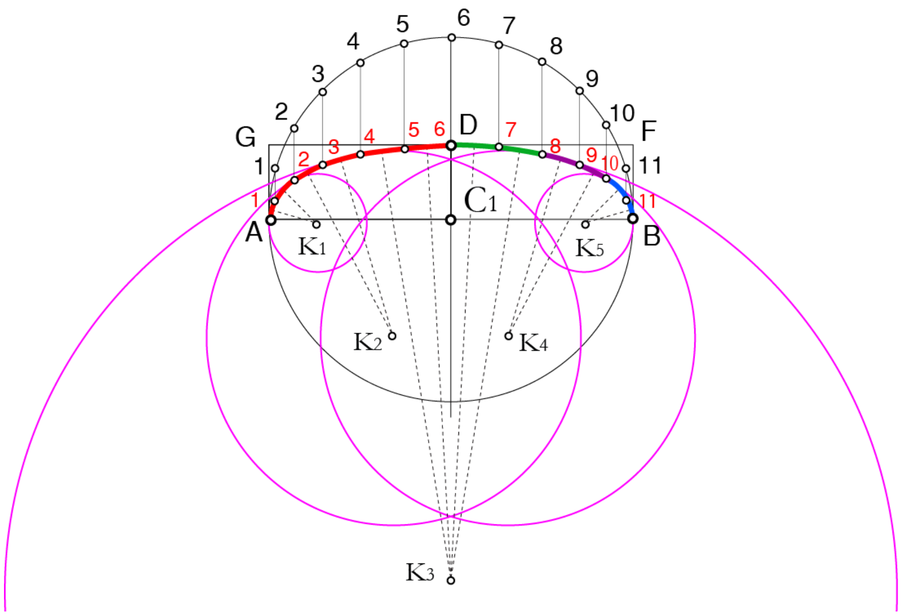


Fig. 9. Finding the centers $C_{3\text{-}5}$ of the approximating circles in order to superpose the semi-oval with circles

4 RESULTS AND DISCUSSION

The kind of arch examined in this study can be quickly and easily drafted with classic accessories and cut as a template that will later be applied on the construction site. The construction is adaptable to any shape of the rectangle and various positions of the point **M** inside it, respecting certain restrictions dictated by the shape of the arch itself, and specifically by the geometric principles of constructing the curve, as explained in section 3. With the described procedure, we can create the most diverse shapes of arches: from semi-circular, semi-elliptical, semi-oval, to pointed arches.

Fig. 10 shows examples of obtaining different shapes of arches depending on the position of point **M** and the rise of the arc $\mathbf{C_1D}$**,** with the given span of the opening: **AB**. We see that the shape of the arc changes by moving the point **M,** for the same initial setting of **AB** (***a***) and $\mathbf{C_1D}$ (***b***). Thus, if the rise is smaller and the arc shallower (Fig 10a), the position of point **M** will affect whether the arc is more curved or straighter in the vertex points.

In Fig. 10b**,** where the rise ***b*** is closer to the length of the semi-axis a, the influence of the position of the point **M** on the shape of the arc is even more noticeable.

The rotated construction is shown in Fig. 10c, where we see that by applying symmetry across the **y** axis, we get arches that completely resemble parts of a continuous oval arch, except when the position of point **M** is exactly on the border parabola ***p***, in which case we get a pointed arc (Fig. 10c-12).

In this way, we get different shapes of arches by applying only one, universal construction. In addition, by applying this procedure we avoid:

a) limitations imposed on us by the previous constructions of semi-oval arches, because they require rigidly defined proportions of the diameter and the sagitta (rise) of the arch;

b) stereotypical arches, without the possibility of correction according to the specific case;

c) (perhaps the most important from the practical side) complicated constructions of oval curves of higher degrees that require greater knowledge of geometry, especially descriptive geometry, and we keep at the construction most similar that of the ellipse;

d) dependence on computer software and technologies (cutting machines) that are not available to every construction company. The whole procedure can be achieved by a synthetic method, with basic tools available to everyone: a compass, a ruler, scissors or some other suitable tool.

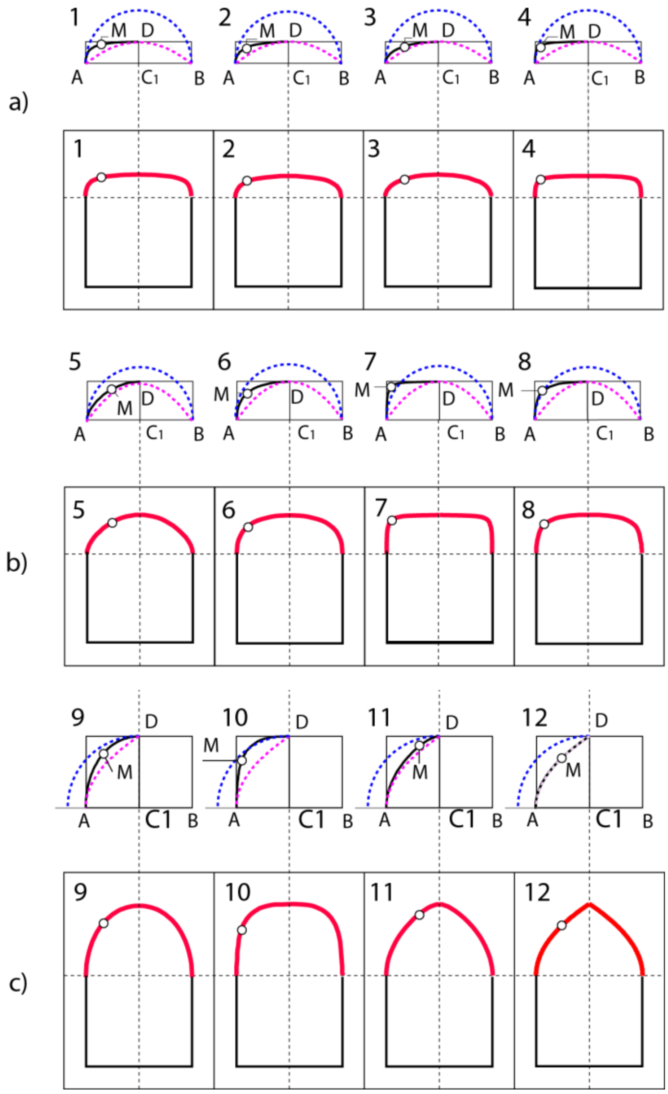


Fig. 10. Examples of different shapes of semi-oval arches depending on the position of the point M and the rise of the arch (C_1D)

CONCLUSIONS

By respecting the geometric regularities present in the architectural elements and understanding them, we are able to maintain a connection with the architectural heritage, which allows us to approach the procedures of preservation, restoration and revitalization of historical buildings more easily.

By applying the modified Hügelschäffer's construction presented in the paper, with the given initial elements: the span of the opening AC_1 (*a*) and the rise of the arc C_1D (*b*) plus the position of one of the points (M) through which the arc should pass, we can find the required semi-oval arc. Furthermore, by moving the point M within the defined region which gives the optimal results for the architectural arch, we can adjust the shape of the arc as desired.

The given construction is suitable for any proportion a:b and we can use it to get almost any arch, except for those derived (composed) from several different arcs, such as, for example: ogee arch, trefoil arch, draped arch or similar.

What should be especially emphasized is that this construction, like the Hügelschäffer's construction itself, can be applied in shaping egg curves. For the same rectangle **ABFG**, two different points $\mathbf{M_L}$ and $\mathbf{M_R}$ can be set from different sides of the **y** axis, and then the two arcs of different curves meet at the vertex **D** with a common horizontal tangent.

Out of the presented construction, it is possible to develop a software application that would generate an arc by changing the position of the point **M** in real time. The authors intend to implement this as the next step of the research.

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