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# NELINEARNA NAPONSKO-DEFORMACIJSKA ANALIZA TERENA PRIMENOM METODE KONAČNIH ELEMENATA***

***Izvod***

*U radu je izvršena naponsko-deformacijska analiza, primenom metode konačnih elemenata, terena u naselju Medaković u Beogradu u toku izgradnje i eksploatacije stambene zgrade od osam spratova. Proračun stabilnosti na klizanje izvršen je korišćenjem podataka dobijenih metodom konačnih elemenata. Primena metode konačnih elemenata, uz korišćenje predloženog inkrementalnog postupka, pokazala je određene prednosti u odnosu na standardne metode analize stabilnosti padina i kosina. Pre svega, na ovaj način moguće je, u svakom trenutku, realno proceniti stanje napona i deformacija u terenu. Na osnovu toga utvrđujemo:*

- *zone lokalnog i/ili opšteg loma tla*
- *kritičnu kliznu površinu*
- *pouzdani faktor sigurnosti*

*Dobijeni rezultati imaju i širi značaj zbog analogije ispitivane padine sa terenima središnjeg dela Beograda koji je poslednjih decenija doživeo intenzivnu urbanizaciju. Posebno naglašavamo da se prikazani naponsko-deformaciski postupak može da primeni kod izgradnje saobraćajnica i deponija i to kako kod građevinskih tako i kod rudarskih radova.*

***Ključne reči:*** *naponsko-deformacijska analiza, metoda konačnih elemenata, koeficijent sigurnosti*

## UVOD

Brojni su primeri u inženjerskoj praksi koji pokazuju da je zbog nedovoljnog poznavanja međusobnog uticaja objekta i terena, u toku izgradnje ili eksploatacije objekta, došlo do narušavanja prirodne ravnoteže. To se ispoljava pomeranjima u terenu koja izazivaju znatna oštećenja postojećih objekata.

Činjenica je da smo često prinuđeni da konstruktivno vrlo složene objekte gradimo

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*** *Ovaj rad proizašao je iz projekta TR 36014 "Geotehnički aspekti istraživanja i razvoja savremenih tehnologija građenja i sanacija deponija komunalnog otpada" koji finansira Ministarstvo prosvete, nauke i tehnološkog razvoja Republike Srbije.*

na uslovno stabilnim terenima – na primer, umirenim klizištima. To nas obavezuje da problemima stabilnosti padina pristupimo sa posebnom odgovornošću, odnosno da primenimo najsavremenije metode istraživanja terena i geostatičkih proračuna.

Ovaj rad predstavlja prilog takvom nastojanju, jer je na osnovu bogatog fonda podataka terenskih i laboratorijskih istraživanja dat savremen postupak proračuna interakcije objekta i terena pomoću metode konačnih elemenata.

U radu je analizirano ponašanje severo-istočne padine Mokroluškog potoka u Beogradu, u području naselja Medaković, pod opterećenjem jednog višespratnog stambenog objekta.

Ovo područje je izabrano stoga što je po svom sastavu i geotehničkim osobinama karakteristično za područje središnjeg dela Beograda u kome je poslednjih decenija izvršena intenzivna urbanizacija i stoga je važno da se utvrdi interakcija već izgrađenih objekata i terena.

## OSNOVNE KARAKTERISTIKE TERENA I OBJEKTA U NASELJU MEDAKOVIĆ

Na levoj, severo-istočnoj padini Mokroluškog potoka, izgrađeno je stambeno naselje Medaković i u okviru njega prateći objekti društvene namene i objekti infrastrukture. U ovom radu analizira se uticaj jednog, višespratnog stambenog objekta na stabilnost padine [1].

### Geotehničke karakteristike terena

Teren u ovome području, kao i u široj okolini (Slika 1), izgrađuju sledeći lito-genetski članovi [2]:

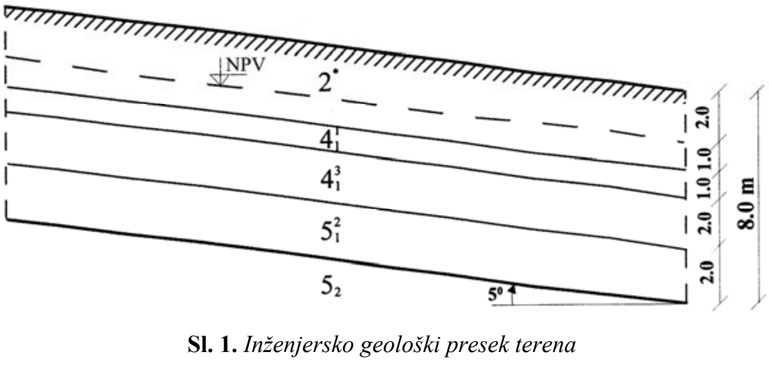
$2^*$ Izmenjeni les prašinasto peskovit sa retkim nakupinama karbonatnog praha, čestim pegama mangana i oolita limonita: žuto smeđe boje; hidrogeološki kolektor-sprovodnik;

$4_1^1$ Laporovita glina, glinoviti lapori, gline i lapori (panon) zona potpunog raspadanja: mrvice (mm i cm dimenzija); bledo žute boje; hidrogeološki kolektor.

$4_1^3$ Glinoviti lapori i laporci (panon) izdeljeni u blokove (dm i m dimenzija) i izmenjeni; svetlo žute boje; hidrogeološki izolator;

$5_1^2$ Glinoviti lapori i gline i peskovite gline sa retkim sočivima peska-uslojeni (sarmat), zona raspadanja: krupno izdeljeni, žuto mrki; hidrogeološki izolator;

$5_2$ Glinoviti lapori i laporci (sarmat) – uslojeni, neizmenjeni, sivi; hidrogeološki izolator. Debljina ove sredine nije tačno utvrđena, ali prelazi 10 m.



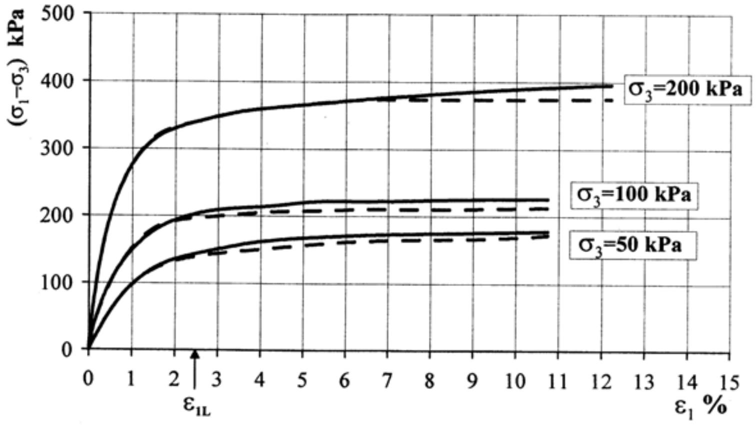

**Sl. 1.** *Inženjersko geološki presek terena*

Navedeni litološki članovi su međusobno približno paralelni i nagnuti niz padinu oko 5°. Maksimalni nivo podzemne vode (NPV) je na 2,0 m ispod površine terena.

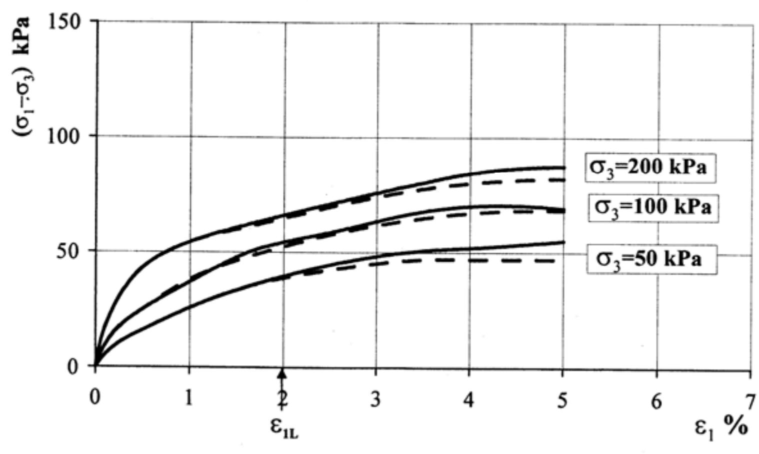
Standardnim laboratorijskim triaksijalnim ispitivanjima reprezentativnih uzoraka merodavnih prirodnih sredina (CD opiti) utvrđeni su karakteristični odnosi napona i deformacija (Slike 2, 3, 4, 5 i 6).

## Analitički modeli naponsko-deformacijskih zavisnosti

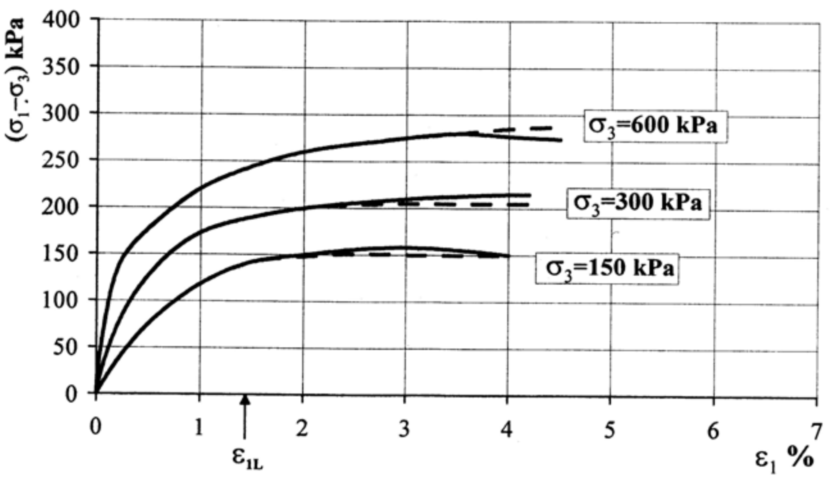
Savremeni pristup rešavanju geotehničkih problema zahteva da se naponsko-deformacijske zavisnosti analitički predstave [3].



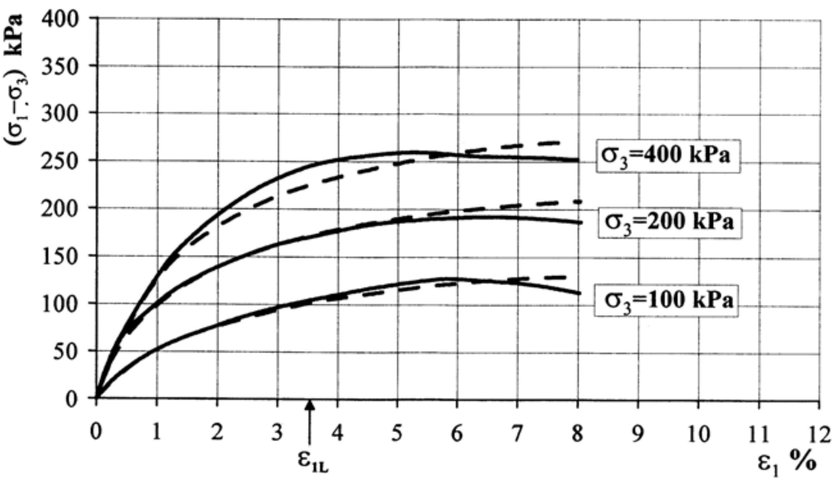

**Sl. 2.** *Naponsko-deformacijske zavisnosti karakterističnih kompleksa za sredinu 2**



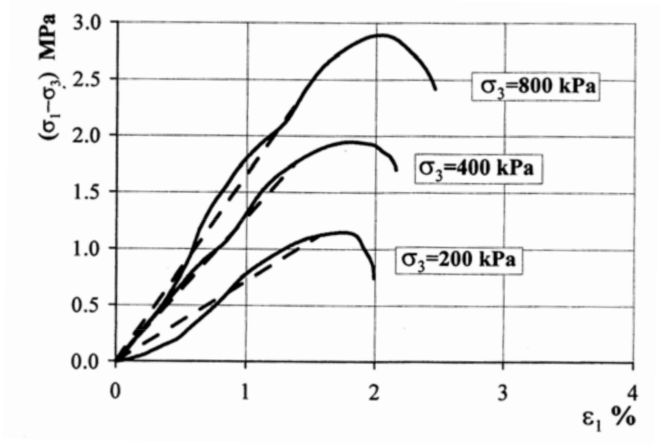

**Sl. 3.** *Naponsko-deformacijske zavisnosti karakterističnih kompleksa za sredinu* $4_1^1$




**Sl. 4.** *Naponsko-deformacijske zavisnosti karakterističnih kompleksa za sredinu* $4_1^3$




**Sl. 5.** *Naponsko-deformacijske zavisnosti karakterističnih kompleksa za sredinu* $5_1^2$




**Sl. 6.** *Naponsko-deformacijske zavisnosti karakterističnih kompleksa za sredinu* $5_2$

S obzirom na prirodu ovih funkcija za sredine $2^*$, $4_1^1$, $4_1^3$, $5_1^2$ usvajamo da je:

$$F = \sigma_1 - \sigma_3 = \frac{A \cdot \varepsilon_1}{B + \varepsilon_1} \tag{1}$$

$$A = \alpha_1 + \beta_1 \cdot \sigma_3 + \gamma_1 \cdot \sigma_3^2 \tag{2}$$

$$B = \alpha_2 + \frac{\beta_2}{\sigma_3} + \frac{\gamma_2}{\sigma_3^2} \tag{3}$$

gde je:

$\sigma_1$, $\sigma_3$ - glavni naponi

$\varepsilon_1$ - glavna dilatacija

U ovim jednačinama su $\alpha_1$, $\alpha_2$, $\beta_1$, $\beta_2$ $\gamma_1$, $\gamma_2$ parametri koji se za svaku sredinu određuju posebno, u zavisnosti od njenih deformacijskih karakteristika.

Činjenica je da se u jednačini (3) naponi $\sigma_3$ pojavljuju u imeniocu, ne umanjuje kvalitet numeričke analize, jer se inkrementalni postupak primenjuje za već poznate primarne napone tj. za $\sigma_3 > 0$.

Predloženi analitički model može vrlo efikasno da se upotrebi u nelinearnim analizama, jer pruža mogućnost da se odredi tangentni modul elastičnosti u proizvoljnoj tački naponsko-deformacijske funkcije. Kada je manji glavni napon konstantan, onda se modul elastičnosti dobija kao

$$E = \frac{d(\sigma_1 - \sigma_3)}{d\varepsilon_1} = \frac{A \cdot B}{(B + \varepsilon_1)^2} \tag{4}$$

U sredini $5_2$ može se, s obzirom na prirodu funkcije $F=F(\sigma_1-\sigma_3, \varepsilon_1)$, usvojiti linearna zavisnost

$$F = \sigma_1 - \beta_3 = E \cdot \varepsilon_1 \tag{5}$$

odnosno, za modul elastičnosti zavisnost

$$E = \alpha_3 + \beta_3 \cdot \sigma_3 + \gamma_3 \cdot \sigma_3^2 \tag{6}$$

gde su $\alpha_3$, $\beta_3$, $\gamma_3$ parametri koji se određuju zavisno od deformacijskih karakteristika sredine $5_2$.

Na Slikama 2, 3, 4, 5 i 6 prikazane su uporedo funkcije dobijene analitički - isprekidane linije i odgovarajuće funkcije F eksperimentalno utvrđene - pune linije [4].

## Tehničke karakteristike objekta

Objekat, čiju interakciju sa terenom analiziramo, izgrađen je od armiranog betona i ima osam spratova. Fundiran je plitko, na betonskoj ploči širine 20 m i dužine 100 m. S obzirom na to, opterećenje od objekta na teren je vertikalno, jednako podeljeno i iznosi 100 kPa.

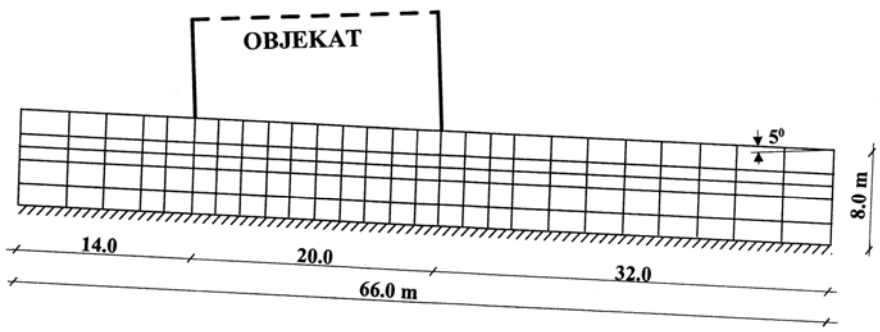
Prilikom naponsko-deformacijske analize usvojeno je da opterećenje od objekta deluje na površini terena što je, s obzirom na način fundiranja, opravdano.

## Formiranje modela terena

Definisanjem geotehničkih karakteristika terena i tehničkih karakteristika objekta, određeni su svi potrebni parametri za formiranje mreže konačnih elemenata. S obzirom da su dimenzije temeljne ploče znatno veće od dimenzija monolita stena (deset, sto i više puta) i da monoliti nisu slobodni nego su zglobno povezani, opravdano je da se teren oko objekta (Slika 1) posmatra kao deo kontinuma. Dalje, zbog položaja objekta u odnosu na pravac pružanja padine, može se smatrati da su zadovoljeni uslovi za ravno stanje deformacije.

Analizom naponsko - deformacijskih funkcija $F=F(\sigma_1-\sigma_3,\ \varepsilon_1)$ zaključujemo da neizmenjeni, sivi lapori (sredina $5_2$), predstavljaju oslonac sredinama iznad njih.

Teren neposredno oko objekta modeliran je mrežom konačnih elemenata i za to su korišćeni ravanski (asolid) konačni elementi (Slika 7).




**Sl. 7.** *Model terena prikazan mrežom konačnih elemenata*

S obzirom na dimenzije temeljne ploče i visinu diskretnog sistema, može se smatrati da se uticaj objekta neće osećati na bočnim stranama modela. Fizičke karakteristike konačnih elemenata, kao što je uobičajno, zadate su preko modula elastičnosti i Poasonovog koeficijenta.

Uslov loma definišemo preko granične deformacije $\varepsilon_{1L}$ koju istraživač uslovljava zavisno od merodavnih karakteristika terena i objekta, odnosno interakcije sistema teren-objekat (Slike 2, 3, 4 i 5).

## ODREĐIVANJE STANJA NAPONA I DEFORMACIJA U TERENU

Određivanje napona i deformacija u terenu vršimo u dva dela, i to za [5]:

(1) - sopstvenu težinu tla i strujni pritisak

(2) - dopunsko opterećenje od objekta

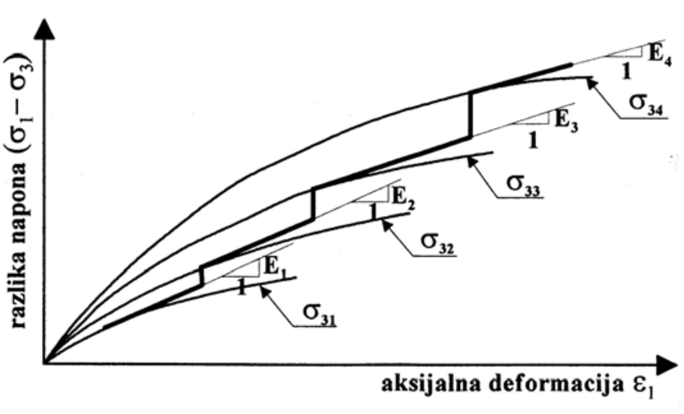
Ovakva podela je opravdana stoga što se za (1) padina može tretirati kao beskonačna sa pravcem filtracije koji je paralelan nagibu padine. U tom slučaju napone i deformacije određujemo neposredno iz uslova ravnoteže i funkcije $F=F(\sigma_1-\sigma_3,\ \varepsilon_1)$.

Proračun napona i deformacija za (2) obavljen je metodom konačnih elemenata [6], korišćenjem programskog paketa SAP 2000.

Početni naponski uslovi su određeni uz predpostavku da se teren pre gradnje nalazi u stanju mirovanja i da je zapreminska težina svih prirodnih sredina $\gamma = 20$ kN/m$^3$.

## Prikaz inkrementalnog postupka

Određivanje napona i deformacija u terenu vrši se inkrementalnim postupkom, u četiri koraka. U prvom koraku određuju se uticaji od sopstvene težine tla i strujnog pritiska. Opterećenje od objekta nanosi se postupno, u preostala tri inkrementa i to: prvo 20 kPa, zatim 30 kPa i na kraju 50 kPa. Na ovaj način simuliran je proces izgradnje objekta i uticaj izgradnje na naponsko-deformacijsko stanje u terenu. Shematski prikaz inkrementalnog postupka dat je na Slici 8.




**Sl. 8.** *Shema inkrementalnog postupka*

U skladu sa predloženim inkrementalnim postupkom, fizičko-mehaničke karakteristike prirodnih sredina izražavamo Poasonovim koeficijentom i tangentnim modulom elastičnosti. S obzirom na izvedene CD opite i činjenicu da je uticaj Poasonovog koeficijenta na stanje napona i deformacija u terenu znatno manji nego što je uticaj modula elastičnosti, usvojeno je da je u svim sredinama $\nu$ konstantno, i to za sredine $2^*$, $4_1^1$ usvojeno je $\nu = 0.45$, a za sredine $4_1^3$, $5_1^2$ usvojeno je $\nu = 0.35$. Što se tiče modula elastičnosti $E_i$ (i=1,...4) oni se određuju iz jednačine (4). Napominjemo da predloženi inkrementalni postupak uvažava kako nelinearnost napon sko-deformacijske funkcije, tako i uticaj manjeg glavnog napona $\sigma_3$.

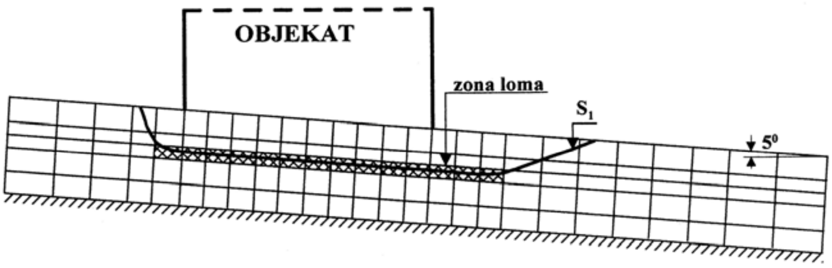
## Ocena sračunatih rezultata

Obradom izračunatih podataka, a u skladu sa predloženim inkrementalnim postupkom, određeno je stanje napona i deformacija u terenu i to kako u toku građenja objekta tako i po završenoj izgradnji.

Utvrđeno je da:

- maksimalno pomeranje u pravcu upravno na nagib padine iznosi 10.17 cm;
- maksimalno pomeranje u pravcu nagiba padine iznosi 8,86 cm;
- deformacije $\varepsilon_1$ najveće su u sredini $4_1^1$ (laporovita glina, zona mrvica); maksimalna deformacija je $\varepsilon_1 = 4,52$ %;

- promena modula elastičnosti je najveća u sredini $4_1^1$, a maksimalna promena je E4/E1= 0.083; u sredini 2* (izmenjeni les) nastaje povećanje modula elastičnosti, a najveće povećanje je $E_4/E_1$= 1.91;
- zona loma prolazi kroz sredinu $4_1^1$ u delu terena koji je ispod objekta (Slika 9).




**Sl. 9.** *Model terena sa zonom loma*

Intenzitet promena napona i deformacija opada relativno brzo sa udaljavanjem od objekta, tako da bokovi diskretnog sistema praktično ne osećaju njegovo dejstvo. Ovo potvrđuje ispravnost usvojenih dimenzija modela.

S obzirom na sličnost ispitivane padine Mokroluškog potoka sa terenima središnjeg dela Beograda, ovi rezultati imaju širi praktičan značaj.

## ANALIZA STABILNOSTI PADINE

S obzirom na utvrđenu zona loma u sloju $4_1^1$, predpostavljena je klizna površina $S_1$ (Slika 9). Njenu stabilnost određujemo za dva granična slučaja:

- pre početka izgradnje objekta i
- po završenoj izgradnji objekta

Metodom konačnih elemenata, korišćenjem programskog paketa SAP 2000, ne može direktno da se odredi koeficijenat sigurnosti padine u odnosu na lom tla klizanjem. S obzirom da je taj podatak bitan za inženjera geotehničara, predlažemo da se rezultati dobijeni metodom konačnih elemenata iskoriste kako bi se odredio koeficijent sigurnosti. S tim u vezi, poznato je da su deformacije u terenu od primarnog inženjerskog značaja, jer njihova veličina uslovljava početak loma. Stoga, u daljem tekstu, predlažemo određivanje koeficijenta sigurnosti koje vodi računa o deformacijama u terenu. U skladu sa tim, koeficijent sigurnosti definišemo kao količnik čvrstoće na smicanje duž klizne površine za graničnu deformaciju ($\varepsilon_1 = \varepsilon_{1L}$) i odgovarajućeg smičućeg napona za ostvarenu deformaciju ($\varepsilon_1 = \varepsilon_{1S}$). Na taj način sračunati koeficijenti sigurnosti za kliznu površinu $S_1$ iznose:

- pre početka izgradnje objekta
  FS = 3.76
- po završenoj izgradnji objekta
  FS = 1.69

Prikazanom numeričkom analizom utvrđeno je da izgradnja objekta smanjuje koeficijent sigurnosti. Međutim njegova vrednost je i dalje zadovoljavajuća, tako da bezbednost objekta nije dovodena u pitanje.

U vezi sa dobijenim rezultatima treba naglasiti da opterećenje od objekta ne izaziva zone loma koje bi mogle da dovedu u pitanje nosivost temeljnog tla. Naime, poznato je da se ove zone prvo pojavljuju ispod ivica temelja i da se, sa povećanjem opterećenja, proširuju sve dok ne dođe do njihovog spajanja [7]. To je i bio razlog zašto nismo posebno analizirali nosivost temeljnog tla. Međutim, interesantno je da opterećenje od objekta izaziva zonu loma, po širini temelja u sredini $4_1^1$ (laporovita glina, raspadnuta), što može da izazove klizanje tog dela padine i objekta na njoj. Zbog toga smo stabilnost na klizanje padine i objekta posebno analizirali.

## ZAKLJUČCI

Proračun stabilnosti padine Mokroluškog potoka u naselju Medaković ukazao je na određene mogućnosti koje u ovoj oblasti Geotehnike obezbeđuje metoda konačnih elemenata, tj.:

- predloženi postupak proračuna omogućava realnu procenu stanja napona i deformacija koja uvažava stvarne fizičko-mehaničke osobine prirodnih sredina. Ovde naročito treba istaći mogućnost dobijanja realnih deformacija i pomeranja jer su one često od izuzetnog značaja za budući objekat, a jedino se mogu dobiti metodom konačnih elemenata;
- inkrementalni postupak pruža mogućnost da se prati promena naponskog i deformacijskog stanja u terenu, zavisno od toka izgradnje objekta. Na taj način stabilnost terena posmatramo kao aktivan proces koji je uslovljen faktorima na koje možemo, po potrebi, uticati;
- dobijeni rezultati omogućavaju da se odrede zone lokalnog i/ili opšteg loma tla;
- poznavanje zona loma, i uopšte deformacija u terenu, obezbeđuje određivanje potencijalnih kliznih površina;
- brojni naponsko-deformacijski podaci koje daje metoda konačnih elemenata omogućavaju određivanje pouzdanijeg koeficijenta sigurnosti nego što je to slučaj kod ostalih metoda analize stabilnosti terena i kosina. U ovom radu je predloženo određivanje koeficijenta sigurnosti u zavisnosti od veličine deformacija u terenu – vodeći istovremeno računa o sigurnosti objekta.

Sve ovo omogućava da se realno sagleda sadejstvo terena i objekta i da se problem stabilnosti padine, u celini, reši na optimalan način. Pri tome, na ovaj način određena pomeranja i faktori sigurnosti su u dozvoljenim granicama za višespratne armirano betonske objekte što omogućava njihovo bezbedno korišćenje [8]. Osim toga, ovako dobijene vrednosti su dragocene prilikom odlučivanja o nadgradnji objekata.

Na kraju želimo da istaknemo da se navedeni postupak naponsko-deformacijske analize može u potpunosti da primeni kod izgradnje saobraćajnica i deponija i to kako kod građevinskih tako i kod rudarskih radova [9], [10].

## LITERATURA


[1] S. Ćorić, Primena metode konačnih elemenata kod rešavanja problema stabilnosti terena u naselju Medaković, Beograd, Magistarski rad, Građevinski fakultet Univerziteta u Beogradu, 1980.

[2] D. Božinović, Geotehničke osobine terena u području Beograda koje izgrađuju koherentni gornjomiocenski sedimenti, Izgradnja 1979, Beograd, str. 245.

[3] C. Liu, L. Zhang, B. Bai, J. Chen, J. Wang, Nonliner Analysis of Stress and Strain for a Clay Core Rock-fill Dam with FEM, International Conference on Advances in Computational

Modeling and Simulation, Procedia Engineering 31, 2012., pp. 497-501.

[4] Z. Wang, Y. Li, R. F. Shen, Correction of Parameters in Calculation of Embankment Settlement Using a BP Network Back-analysis Model, Engineering Geology 91, 2007., pp. 168-177.

[5] R. N. Chowdhury., Slope Analasis, Elsevier Publishing Company, Amsterdam 1978. pp. 422.

[6] C. S. Desai and J. T. Christian, Numerical Methods in Geotechnical Engineering, Mc Graw Hill, New York 1977. pp. 783.

[7] S Ćorić, Geostatički proračuni, Rudarsko-geološki fakultet & Izgradnja 2008., Beograd. str. 460.

[8] Canadian Foundation Engineering Manual, Canadian Geotechnical Society, Richmond., 2006.

[9] S. Ćorić, L. Čaki, D. Rakić, S. Jotić, B. Ubiparip, M. Ljubojev, Three-dimensional Aspects of the Slope Stability Analysis, Rudarski radovi, 3 (2011), pp. 51-60.

[10] D. Rakić, Konstitutivne zavisnosti komunalnog otpada sa deponija u Srbiji, Doktorska disertacija (u izradi), 2012.

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# NONLINEAR STRESS-STRAIN ANALYSIS OF TERRAIN USING THE FINITE ELEMENT METHOD***

***Abstract***

*This paper presents the stress-strain analysis of terrain using the finite element method in the settlement Medaković in Belgrade during the construction and occupation of the eighth floor residential building. The slope stability analysis was performed using the finite element method. Application the finite element method, with the use of proposed incremental procedure has shown some advantages compared to the classical methods of stability analysis. First of all, using this method, it is possible to assess at any time the stress and strain values in terrain. Based on these values, it is possible to determine: zones of failure (local and/or general), critical sliding surface and reliable safety factor*

*The obtained results are of great significance for the most part of the Belgrade terrain which was built up in a similar way and in the last few decades has been intensively urbanized.*

*It has to be emphasized that the proposed stress-strain procedure has its application in construction of highways and landfills, both in civil engineering and mining engineering.*

**Keywords:** *stress-strain analysis, finite element method, safety factor*

## INTRODUCTION

There are many examples in the engineering practice, showing that the lack of knowledge the interactions between the object and terrain, during the construction or occupation of object, there was a disruption of the natural balance. That reflects in shifts of the ground, which cause considerable damages to the existing structures.

The fact is that we are often forced to constructively build very complex structures on the conditionally stable grounds -

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for example, soothed landslides. It obliges us to approach the problems of stability the slopes with a special responsibility, and to apply the latest methods of field research and geostatic calculations.

This paper is a contribution to such efforts, as based on a rich fund of data from the field and laboratory research, it gives a modern procedure for calculation the interaction of objects and terrain using the finite element method.

This paper analyzes the behavior of the north-eastern slopes of the Mokroluški stream in Belgrade, in the area of the settlement Medaković under the load of a multi-storey residential building.

This area was chosen because it is, by its composition and geotechnical properties, characteristic for the area of the central part of Belgrade, where an extensive urbanization has been made in the last decades, and it is therefore important to determine the interaction of already constructed buildings and terrains.

## BASIC CHARACTERISTICS OF TERRAIN AND OBJECTS IN THE SETTLEMENT MEDAKOVIĆ

On the left, north-eastern slope of the Mokroluški stream, a residential settlement Medaković was built with its social purpose outbuildings and infrastructure facilities. In this paper, impact of a single, multi-storey residential building is analyzed on the slope stability [1].

### Geotechnical characteristics of the terrain

Terrain in this area, as well as in the wider area (Figure 1), is built by the following lithogenetical members [2]:

$2^*$ Altered silty sandy loess with rare clusters of carbonate powder, frequent spots of manganese and limonite oolite limonite: yellow-brown color; hydrogeological collector;

$4_1^1$ Marly clay, clayey marl, clay and marl (Pannonian) zone of complete decomposition crumbs (mm and cm dimensions); pale yellow; hydrogeological collector;

$4_1^3$ Clayey marls and marls (Pannonian), divided into blocks (dm and m dimensions) and alteres; bright yellow; hydrogeological insulator;

$5_1^2$ Clayey marl and clay and sandy clay with rare lenses of sand-stratified (Sarmatian), zone of decomposition: coarse divided, yellow brown; hydrogeological insulator;

$5_2$ Clayey marls and marls (Sarmatian) - stratified, nonaltered, gray; hydrogeological insulator. Thickness of this region is not precisely defined, but more than 10 m.

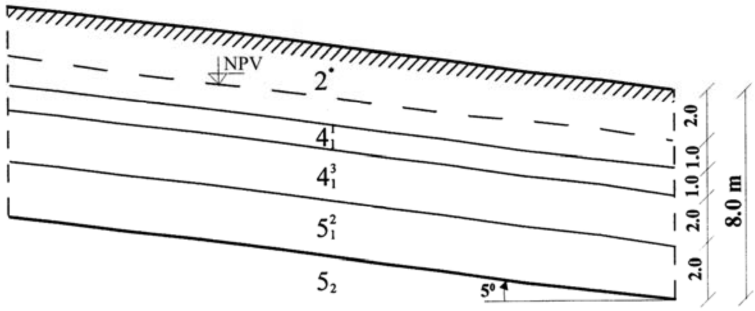


**Fig. l.** *Engineering-geological section of the terrain*

The aforementioned lithologic members are approximately parallel to each other and inclined down the slope about 5°. Maximum groundwater level (NPV) is 2.0 m below the ground surface.

Standard laboratory triaxial testing the representative samples of competent natural environments (CD experiments) have determined the characteristic relations of stress and strain (Figures 2, 3, 4, 5 and 6).

## Analytical models of stress-strain dependence

Modern approach to solving the geotechnical problems requires the analytical presentation of the stress-strain dependence [3].

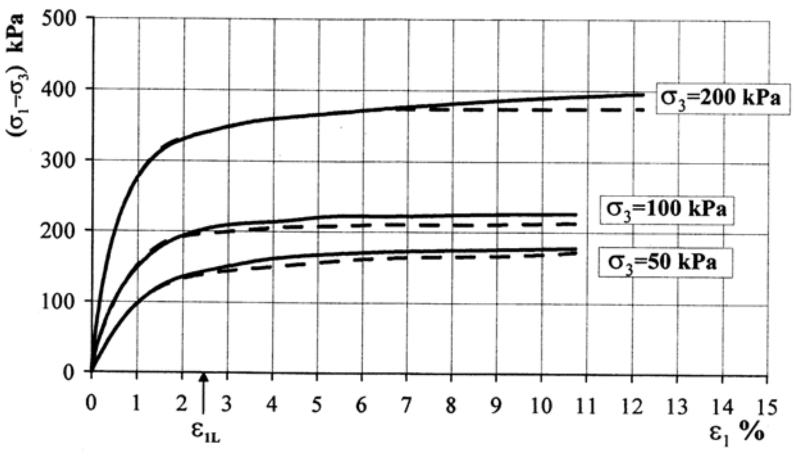


**Fig. 2.** *Stress-strain dependences of characteristic complexes for area 2*[*]

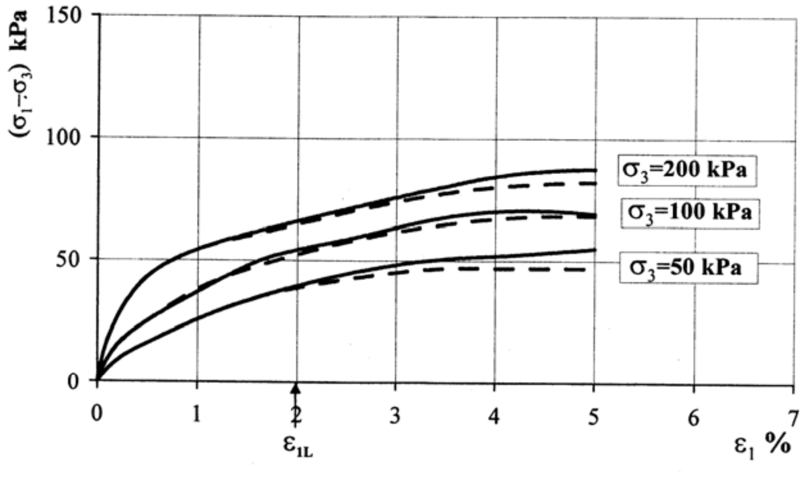


**Fig. 3.** *Stress-strain dependences of characteristic complexes for area* $4_1^1$

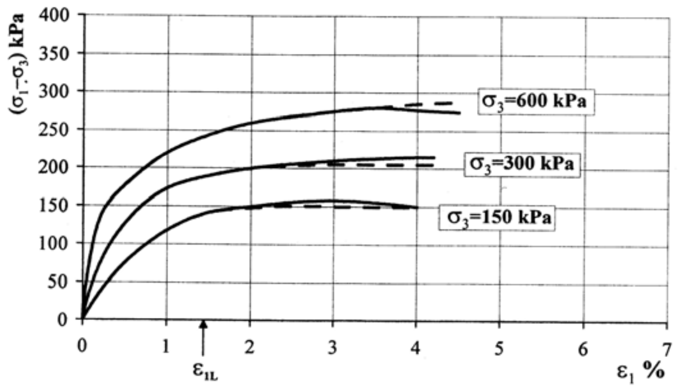


**Fig. 4.** *Stress-strain dependences of characteristic complexes for area* $4_1^3$

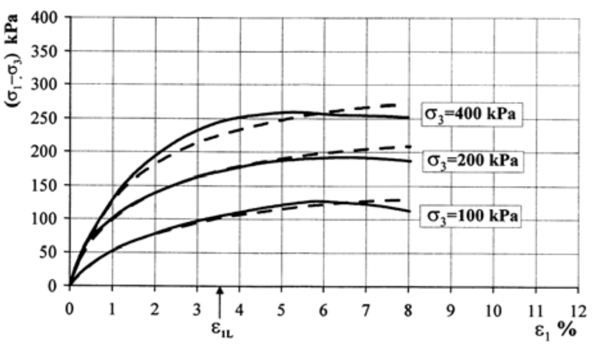


**Fig. 5.** *Stress-strain dependences of characteristic complexes for area* $5_1^2$

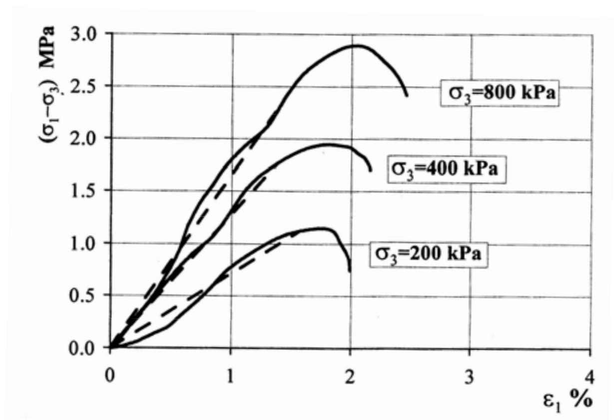


**Fig. 6.** *Stress-strain dependences of characteristic complexes for area* $5_2$

Considering the nature of these functions for areas $2^*$, $4_1^1$, $4_1^3$, $5_1^2$, it is adopted that:

$$F = \sigma_1 - \sigma_3 = \frac{A \cdot \varepsilon_1}{B + \varepsilon_1} \quad (1)$$

$$A = \alpha_1 + \beta_1 \cdot \sigma_3 + \gamma_1 \cdot \sigma_3^2 \quad (2)$$

$$B = \alpha_2 + \frac{\beta_2}{\sigma_3} + \frac{\gamma_2}{\sigma_3^2} \quad (3)$$

where:

$\sigma_1$, $\sigma_3$ – principal stresses

$\varepsilon_1$ - principal dilatation

The parameters $\alpha_1$, $\alpha_2$, $\beta_1$, $\beta_2$ $\gamma_1$, $\gamma_2$ are in these equations that are determined for each area particularly, depending on its deformation characteristics.

The fact that in equation (3) stresses $\sigma_{3,}$ appear in denominator, do not diminish the quality of numerical analysis, because an incremental process is used for primary voltages already known primary stresses, i.e. for $\sigma_3 > 0$.

The proposed analytical model can be effectively used in nonlinear analyses, as it provides the ability to determine the tangent modulus of elasticity at an arbitrary point of the stress-strain function. When lower principal stress is constant, then the modulus of elasticity is obtained as

$$E = \frac{d(\sigma_1 - \sigma_3)}{d\varepsilon_1} = \frac{A \cdot B}{(B + \varepsilon_1)^2} \quad (4)$$

In the area $5_{2,}$ due to the nature of function $F=F(\sigma_1-\sigma_3, \varepsilon_1)$, the linear dependence can be adopted

$$F = \sigma_1 - \sigma_3 = E \cdot \varepsilon_1 \quad (5)$$

That is, dependence for the elasticity modulus

$$E = \alpha_3 + \beta_3 \cdot \sigma_3 + \gamma_3 \cdot \sigma_3^2 \quad (6)$$

where $\alpha_3$, $\beta_3$, $\gamma_3$ are parameters that have to be determined depending on the deformation characteristics of area $5_2$.

Figures 2, 3, 4, 5 and 6 show in parallel the functions obtained analytically - dashed lines and corresponding experimentally determined functions F – full lines [4].

**Technical characteristics of object**

The analyzed object with terrain was built of reinforced concrete and has eight floors. It is shallow founded on a concrete slab, width of 20 m and length of 100 m. Since, the load of the building on the ground is vertically, equally divided and it is 100 kPa.

During the stress-strain analysis, it was assumed that the load of building acts on terrain surface which is, due to the way of funding, justified.

**Formation of terrain model**

Defining the geotechnical characteristics of terrain and technical characteristics of building, all necessary parameters were determined for formation the network of finite elements. Since the given dimensions of the base plate are much higher than dimensions of monolith rocks (ten, a hundred or more times) and that the monoliths are not free but are hingedly connected, it is reasonable that the area around the building (Figure 1) is seen as part of the continuum. Furthermore, due to the position of object relative to the direction of slope, it can be considered that the requirements for plane state of strain are met.

Analyzing the stress-strain functions $F=F(\sigma_1-\sigma_3, \varepsilon_1)$, it is concluded that non-altered gray marl (area $5_2$), are the mainstay of areas above them.

Terrain immediately around the building was modeled using the network of finite elements and the plane (asolid) finite elements were used for this (Figure 7).

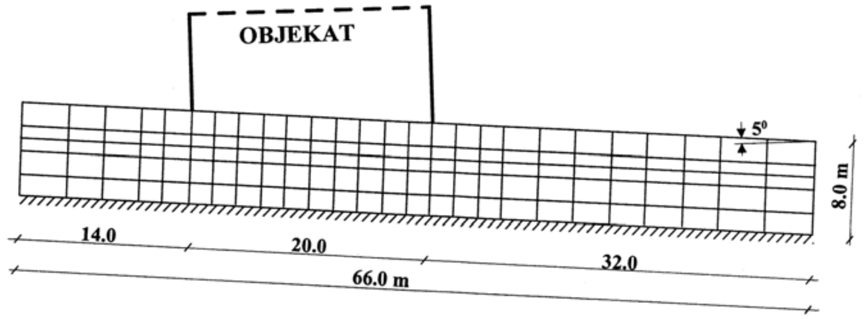


**Fig. 7.** *A terrain model is shown by the network of finite elements*

Considering the base plate dimensions and discrete system height, it can be considered that the impact of object will not be on the lateral sides of model. Physical characteristics of finite elements, as it is common, are given through the elasticity modulus and the Poisson coefficient.

The condition of fracture is defined through the fracture limit strain $\varepsilon_{1L}$ which is conditioned by researcher depending on the characteristics of terrain and object, i.e. the interaction of terrain-object system (Figures 2, 3, 4 and 5).

## DETERMINATION THE STRESS-STRAIN CONDITIONS IN THE GROUND

Determination the stress-strain conditions in the ground is carried out in two parts, as well as for [5]:

(1) – own weight of ground and current pressure

(2) – additional stress of the object

Such division is justified because it can be treated for (1) slope as an infinite with direction of filtration that is parallel to the slope angle. In this case, the stresses and strains are directly determined from the equilibrium condition and function $F=F(\sigma_1-\sigma_3,\ \varepsilon_1)$. Calculation of stress and strains for (2) was performed by the finite element method [6] using the software package SAP 2000.

The initial stress conditions are determined assuming that the area, prior to the construction, is in the idle mode and that the bulk density of all natural environment is $\gamma = 20\ kN/m^3$.

### Preview of incremental method

Determination of stresses and strains in the ground is carried out using the incremental method, in four steps. The first step determines the effects of the soil own weight and current pressure. Load of the building is applied gradually, and the other three increments are: the first 20 kPa, then 30 kPa, and at the end 50 kPa. In this way, the simulated process of object construction and the impact of construction on the stress-strain condition in the ground. Schematic representation of the incremental method is given in Figure 8.

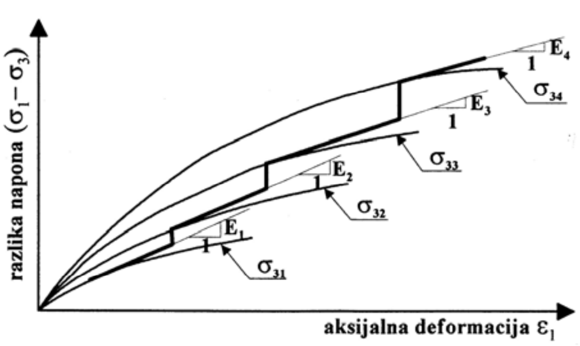


**Fig. 8.** *Diagram of incremental method*

In accordance with the proposed incremental method, the physical-mechanical properties of the natural environment are expressed by the Poisson coefficient and tangent modulus of elasticity. Since the performed CD experiments and the fact that the influence of the Poisson coefficient of the stress and strain condition in the ground are much smaller than the impact of elasticity, it is assumed that in all areas ν is constant as well as for the area * 2, adopted ν = 0.45, and the areas $4_1^3$, $5_1^2$, adopted ν = 0.35. As for the elastic modulus $E_i$ (i = 1,...4), they are determined from the equation (4). We note that the proposed incremental method take into account both the non-linearity of the stress-strain function, and the impact of a smaller principal stress $\sigma_3$.

## Evaluation of calculated results

Processing the calculated data in accordance with the proposed incremental method has determined the stress and strain condition in the ground both during construction and after completion of construction.

It was found that:

- maximum displacement in a perpendicular direction to the slope inclination is 10.17 cm;
- maximum displacement in a direction of inclination of the slope is 8.86 cm;
- strains $\varepsilon_1$ are the largest in the area $4_1^1$ (marly clay, zone of crumbs); maximum deformation $\varepsilon_1$ = 4.52%;
- change in the elasticity modulus is the largest in the area $4_1^1$, and maximum change is $E_4/E_1$= 0.083; in the area 2* (altered loess) is the result of increase the elasticity modulus, and the largest increase is $E_4/E_1$= 1.91;
- fracture zone runs through the area $4_1^1$ in the ground part which is below the object (Figure 9).

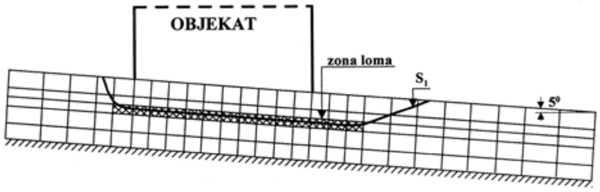


**Fig. 9.** *A terrain model with the fracture zone*

The intensity of change the stress and strain decreases relatively quickly with distance from the object, so that the sides of a discrete system do not practically feel its effect. It is confirmed by the validity of adopted model dimensions.

Considering the similarity of tested slope of the Mokroluški stream with the terrains of the central part of Belgrade, these results have broader practical significance.

## ANALYSIS OF THE SLOPE STABILITY

Considering the established fracture zone in the layer $4_1^1$, the sliding surface $S_1$ is presumed (Figure 9). Its stability is determined for two boundary cases:

- before the start of construction the object, and
- after completion the construction of object

By the finite element method, using the software package SAP 2000, the coefficient of slope safety cannot be determined compared to the fracture of soil sliding. Since that this data is essential for the engineers of geotechnics, we suggest that the results obtained by the finite element method would be used to determine the factor of safety. In this regard, it is known that the strains in the ground have the primary engineering importance, because their size is caused by the fracture initiation. Therefore, in further text, we suggest determining the safety coefficient which takes into account the strains of the ground. In accordance to this, the safety factor is defined as the quotient of shear strength along the sliding surface for boundary strain ($\varepsilon_1 = \varepsilon_{1L}$) and the appropriate shearing stress for achieved strain ($\varepsilon_1 = \varepsilon_{1S}$). In this way, the calculated safety factors for sliding surface $S_1$ are as follows:

- before the start of construction the object, $F_S = 3.76$
- after completion the construction of object, $F_S = 1.69$

The present numerical analysis has shown that the construction of object reduces the safety coefficient. However, its value is still satisfactory, so that the safety of object is not brought into question.

In connection with the obtained results, it should be noted that the load of object does not cause a fracture zone that could compromise the capacity of foundation soil. It is known that these zones first appear under the edges of foundation, and with increasing load, expanding until their connection [7]. That was the reason why we did not specifically analyze the capacity of the foundation soil. However, it is interesting that the load of object causing the fracture zone, along the width of foundation in the area $4_1^1$ (marly clay, decomposed), which can cause sliding of that part of the slope and the buildings on it. Therefore, the stability of slope sliding was specifically analyzed.

## CONCLUSIONS

Calculation of the slope stability of the Mokroluški stream in the settlement Medaković pointed to the specific possibilities in this area of geotechnics, provided by the finite element method, that is:

- the proposed method of calculation provides a realistic assessment of the stress and strain condition that takes into account the actual physical-mechanical properties of natural environments. Here, it should be noted in particular the possibility of obtaining the real strains and displacements because they are often of great importance for the future object, and they can be only obtained by the finite element method;
- the incremental method provides the possibility to monitor the changes in stress and strain condition in the ground, depending on the course of construction. In this way, the stability of the ground, viewed as an active process, which is determined by factors that can, if necessary, be affected;
- the obtained results allow to determine the zones of local and/ or general soil failure;
- knowing the fracture zones, and general strains in the ground, provides determination of the potential sliding surfaces;
- a number of stress-strain data provided by the finite element method allows determination of the coefficient of reliable safety than it is the case for other methods of analysis the stability of terrain and slopes. This work proposes determination the safety coefficient depending on the size of strains in the ground - at the same time taking into account the safety of object.

All this makes it possible to realistically look at the terrain and synergy the facility and that the problem of the stability of the slope, in general, solved in an optimal way. In fact, in this way the certain shifting and safety factors are within acceptable limits for multi-story reinforced concrete buildings allowing their safe use [8]. In addition, the values obtained in this way are valuable when deciding on superstructure of building.

At the end we want to emphasize that given procedure of the stress-strain analysis can be completely used in construction of highways and landfills, both in civil engineering and mining engineering [9], [10].

## REFERENCES


[1] S. Ćorić, Application the Finite Element Method in Solving the Problem of Terrain Stabillity in the Settlement Medaković, Belgrade, Mater Thesis, Faculty of Civil Engineering, University of Belgrade, 1980 (in Serbian);

[2] D. Božinović, Geotechnical Properties of the Terrain in the Area of Belgrade gornjomiocenski that Build the Coherent Upper Miocene Sediments, "Izgradnja" 1979, Belgrade, pp. 245 (in Serbian);

[3] C. Liu, L. Zhang, B. Bai, J. Chen, J. Wang, Nonliner Analysis of Stress and Strain for a Clay Core Rock-fill Dam with FEM, International Conference on Advances in Computational Modeling and Simulation, Procedia Engineering 31, 2012, pp. 497-501;

[4] Z. Wang, Y. Li, R. F. Shen, Correction of Parameters in Calculation of Embankment Settlement Using a BP Network Back-analysis Model,

Engineering Geology 91, 2007, pp. 168-177;
[5] R. N. Chowdhury, Slope Analysis, Elsevier Publishing Company, Amsterdam 1978, pp. 422;
[6] C. S. Desai and J. T. Christian., Numerical Methods in Geotechnical Engineering, Mc Graw Hill, New York 1977, pp. 783,
[7] S. Ćorić, Geostatic Calculations, Faculty of Mining and Geology & Izgradnja 2008, Belgrade, pp. 460 (in Serbian);
[8] Canadian Foundation Engineering Manual, Canadian Geotechnical Society, Richmond, 2006;
[9] S. Ćorić, L. Čaki, D. Rakić, S. Jotić, B. Ubiparip, M. Ljubojev, Three-dimensional Aspects of the Slope Stability Analysis, Mining Enginnering, 3 (2011), pp. 51-60;
[10] D. Rakić, Constitutive Dependences of the Municipal Waste from the Landfills in Serbia, Ph.D. Thesis (in preparation), 2012 (in Serbian).