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TERMOELASTIČNOST OŠTEĆENIH ELASTOMERA - EFEKTIVNE KARAKTERISTIKE MATERIJALA

THERMOELASTICITY OF DAMAGED ELASTOMERS - EFFECTIVE MATERIAL PROPERTIES

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- efektivni kontinuum

Keywords
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Izvod

U radu se razmatra elastomer kao deformabilno telo sa slučajnom 3D-raspodelom paralelnih elipsoidnih šupljina. Korišćenjem Levinovih formula numerički se dobija efektivna krutost kao i tenzor koeficijenata termičkog širenja. Slučaj male neuredjenosti se malo razlikuje od paralelnih šupljina. Za takvu neuređenost su nađene ove efektivne karakteristike. Zatim se brzina oslobađanja energije koristi kao uslov otvaranja i zatvaranja postojećih prslina. Dobijeni rezultati daju mogućnost primene kod rešavanja kinetičkih jednačina.

Abstract

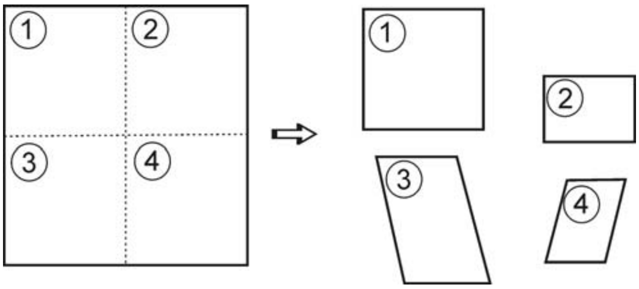
The paper deals with an elastomer body having a random 3D-distribution of ellipsoidal mutually parallel voids. By making use of Levin's formulae effective stiffness as well as thermal expansion tensors are found numerically. Slight disorder case differs unsignificantly from parallel voids. For such a disorder these effective material features are found. Subsequently, the energy release rate is used as a triggering tool for opening and closing of existing cracks. The accomplished results make possible a further application to kinetic equations.

UVOD

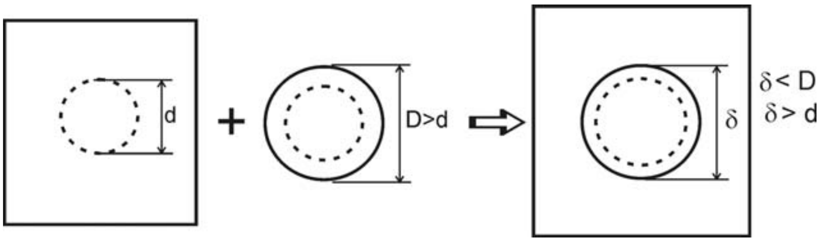
U klasičnim radovima posvećenim neprekidnoj teoriji dislokacija kao glavni uzrok zaostalih napona se pojavljuje inkompatibilnost plastičnih ili kvaziplastičnih deformacija (termičkih ili nekih drugih). Na sl. 1. je predstavljena elementarna vizuelizacija sopstvenih deformacija zbog takve nekompatibilnosti. Ključna činjenica ovde je da ako se zapreminski elementi u Kondovom prostoru prirodnog stanja /1/ deformišu slobodno, tada oni ne mogu da spoje bez zaostalih napona. Iako je takav pristup, koji je prvi uveo Krener /2/, izgledao veoma obećavajući u plastičnosti zasnovanoj na neprekidnim dislokacijama, nedavni radovi koriste mahom jedan alternativni pristup usađivanja, predložen od strane Ešelbija /3/. Veoma uprošćena interpretacija takvog pristupa je prikazana na sl. 2.

INTRODUCTION

In classical texts devoted to continuum theory of dislocations as the principal source of residual stresses incompatibility either of plastic strains or quasi-plastic strains (thermal and some others) is considered. An elementary visualization of eigenstrains caused by such an incompatibility is presented in Fig. 1. The key point here is that if volume elements in natural state space of Kondo /1/ deform freely then they cannot be connected without residual stresses. While such an approach, promoted originally by Kroener /2/, looked very promising in plasticity based on continuum dislocations, the recent papers use mainly an alternative approach of implantations as proposed by Eshelby /3/. One very simplified picture of such an approach is depicted in Fig. 2.



Slika 1. Nekompatibilnost deformacija u prostoru prirodnog stanja Konda /9/
Figure 1. Incompatibility of strains in natural state space of Kondo /9/



Slika 2. Ešelbijeva usadna deformacija za isti materijal uključka i matrice /7/
Figure 2. Eshelby's implanting eigenstrain by the same material of inclusion and matrix /7/.

EŠELBIJEV POSTUPAK ZA UKLJUČKE-MATRICA I UKLJUČENJE SU OD ISTOG MATERIJALA

Ešelbijev tensor za isti materijal matrice i uključka

Prema Muri /4/ ograničene usadne deformacije izazvane slobodnim deformacijama nazivaju se "eigen-strains" (sopstvene deformacije). Ograničene i slobodne deformacije su povezane poznatom Ešelbijevom formulom:

$$\mathbf{E}^{constr} = \mathbb{S} : \mathbf{E}^{free} \qquad (1)$$

izvedenom u /7/. U ovoj formuli slobodna deformacija $\mathbf{E}^{free}$ (deformacija u sredini sl. 2) je povezana sa usadnom sopstvenom deformacijom $\mathbf{E}^{constr}$ (deformacija na desnoj strani sl. 2) tenzorom cetvrtog ranga $\mathbb{S}$. U specijalnom slučaju za isti izotropni materijal matrice i uključka Ešelbijev tensor 4 ranga glasi:

$$\frac{8\pi(1-\nu)}{3} S_{mnkl} = \left(I_{MK} a_K^2 - \frac{1-2\nu}{3} I_M \right) \delta_{mn}\delta_{kl} + \left(\frac{a_M^2 + a_N^2}{2} I_{MN} + \frac{1-2\nu}{3} \frac{I_M + I_N}{2} \right) \left(\delta_{mk}\delta_{nl} + \delta_{nk}\delta_{ml} \right)$$

gde su

$$I_M = 2\pi a_1 a_2 a_3 \, lim_{\xi\to 0} \int_{\xi}^{\infty} \frac{d\xi}{\left(a_M^2 + \xi\right) \Delta}$$

$$I_{MN} = \frac{2}{3} \pi a_1 a_2 a_3 \, lim_{\xi\to 0} \int_{\xi}^{\infty} \frac{d\xi}{\left(a_M^2 + \xi\right)\left(a_N^2 + \xi\right) \Delta}$$

$$\Delta^2 \equiv \left(a_1^2 + \xi\right)\left(a_2^2 + \xi\right)\left(a_3^2 + \xi\right)$$

S druge strane, u slučaju najopštijeg anizotropnog materijala, numerička procena je jedini način za proračun $\mathbb{S}$. Formula Kunina i Sosnina /5/ na način kako su objasnili Yaguchi i Busso /6/ je primenjena u ovom radu.

ESHELBIAN APPROACH TO EIGENSTRAINS-MATRIX AND INCLUSION OF THE SAM MATERIAL

Eshelby's tensor for same materials of matrix and inclusion

According to Mura /4/ constrained implanting strains induced by free strains are termed as "eigen-strains". Constrained and free strains are connected by the known Eshelby formula:

$$\mathbf{E}^{constr} = \mathbb{S} : \mathbf{E}^{free} \qquad (1)$$

derived in /7/. In the above formula the unconstrained strain $\mathbf{E}^{free}$ (strain in the middle of Fig. 2) is related to implanting "eigen-strain" $\mathbf{E}^{constr}$ (strain on the right hand side of Fig. 2) by the fourth rank tensor $\mathbb{S}$. In the special case of the same isotropic materials of matrix and inclusion Eshelby's 4-tensor reads:

$$\frac{8\pi(1-\nu)}{3} S_{mnkl} = \left(I_{MK} a_K^2 - \frac{1-2\nu}{3} I_M \right) \delta_{mn}\delta_{kl} + \left(\frac{a_M^2 + a_N^2}{2} I_{MN} + \frac{1-2\nu}{3} \frac{I_M + I_N}{2} \right) \left(\delta_{mk}\delta_{nl} + \delta_{nk}\delta_{ml} \right)$$

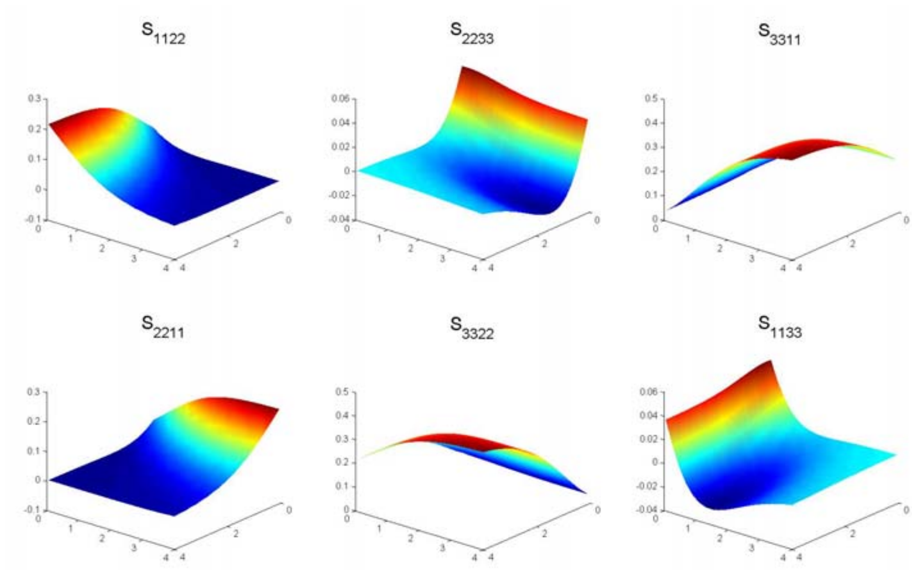
where

$$I_M = 2\pi a_1 a_2 a_3 \, lim_{\xi\to 0} \int_{\xi}^{\infty} \frac{d\xi}{\left(a_M^2 + \xi\right) \Delta}$$

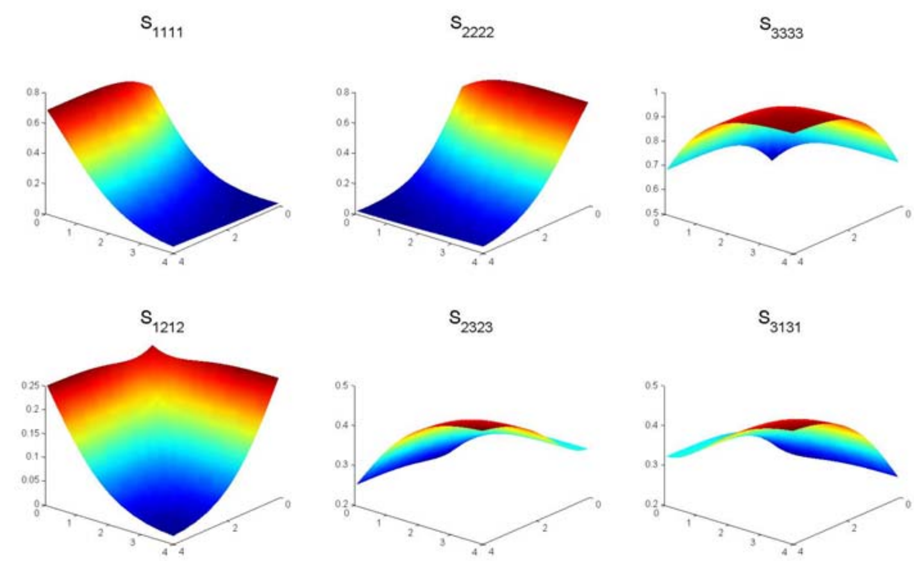
$$I_{MN} = \frac{2}{3} \pi a_1 a_2 a_3 \, lim_{\xi\to 0} \int_{\xi}^{\infty} \frac{d\xi}{\left(a_M^2 + \xi\right)\left(a_N^2 + \xi\right) \Delta}$$

$$\Delta^2 \equiv \left(a_1^2 + \xi\right)\left(a_2^2 + \xi\right)\left(a_3^2 + \xi\right)$$

On the other hand, in the case of general anisotropic material a numerical estimation is the only way to calculate $\mathbb{S}$. Formula of Kunin and Sosnina /5/ in the way explained by Yaguchi and Busso /6/ is applied in this paper.



Slika 3. Nedijagonalne komponene Ešelbijevog tenzora za razne logaritme odnosa poluosa elipsoida i izotropne materijale
Figure 3. Pair off-diagonal components of Eshelby tensor for logarithms of diverse principal axes ratios and isotropic materials.



Slika 4. Dijagonalne i zavojne komponente Ešelbijevog tenzora za razne logaritme odnosa glavnih osa elipsoida i izotropne materijale
Figure 4. Diagonal and pair-screw components of Eshelby tensor for logarithms of diverse principal axes ratios for isotropic materials

OSNOVNE JEDNAČINE ZA TERMOELASTIČNE NAPONE

Odgovarajuće nelokalne integralne jednačine za jedan uključak date su u radu (Kanaun i Kudrjavceva, /7/):

$$\boldsymbol{\Sigma}(x)-\int_V \mathbb{M}(x-x')\delta\left(\mathbb{D}^{-1}(x')\right)\boldsymbol{\Sigma}(x')dx' = \int_V \mathbb{M}(x-x')\delta\left(\boldsymbol{\alpha}(x')\right)\theta dx' \quad (2)$$

Ovde je jezgro $\mathbb{M}(x)\equiv\mathbb{D}(x)\mathbb{K}(x)\mathbb{D}(x)-\mathbb{D}(x)\delta(x)$ formirano izvodima Grinove funkcije kao tenzora 4. ranga $(\mathbb{K})_{abcd}=(\partial_a\partial_d G_{bc})_{(ab)}$ i skokom krutosti između uključka i matrice $\delta\mathbb{D}\equiv\mathbb{D}-\mathbb{D}_M$.

GOVERNING EQUATIONS FOR THERMOELASTIC STRESSES

Governing non-local integral equation for the single inclusion problem (Kanaun & Kudrjavceva, /7/):

$$\boldsymbol{\Sigma}(x)-\int_V \mathbb{M}(x-x')\delta\left(\mathbb{D}^{-1}(x')\right)\boldsymbol{\Sigma}(x')dx' = \int_V \mathbb{M}(x-x')\delta\left(\boldsymbol{\alpha}(x')\right)\theta dx' \quad (2)$$

Here the kernel $\mathbb{M}(x)\equiv\mathbb{D}(x)\mathbb{K}(x)\mathbb{D}(x)-\mathbb{D}(x)\delta(x)$ is formed by means of $(\mathbb{K})_{abcd}=(\partial_a\partial_d G_{bc})_{(ab)}$ being 4-tensor of Green function derivatives and stiffness jump between inclusion and matrix $\delta\mathbb{D}\equiv\mathbb{D}-\mathbb{D}_M$.

EFEKTIVNI TENZORI

Efektivna krutost

Tenzor efektivne krutosti prema postupku koji je dao Levin u /8/ se izvodi na sledeci nacin. Mikroformulacija Hukovog napona ima oblik

$\langle \boldsymbol{\Sigma}_\Lambda \rangle = \mathbb{D}^{\text{eff}} : \langle \mathbf{E}_{\Lambda e} \rangle$ dovodi do $\mathbf{S} = \mathbb{D}^{\text{eff}} : \mathbf{E}_e$

gde je

$\mathbf{E}_{\Lambda e} = \mathbf{E}_{\Lambda E} - \boldsymbol{\alpha}_\Lambda \theta$.

Levinov izraz za efektivni tenzor krutosti glasi:

$$\mathbb{D}_\Lambda^{eff} = \mathbb{D}_M + \delta\mathbb{D}_\Lambda \left(\mathbb{I} - \langle \mathbb{A}\mathbb{S} \rangle_\omega \mathbb{D}_M \delta\mathbb{D}_\Lambda \right)^{-1} \langle \mathbb{A} \rangle_\omega \quad (3)$$

Ovde za uključak Λ, $(\Lambda \in \{1,...,N\})$ i za matricu (skelet) označenu sa M imamo:

$$\mathbb{D}_M = \langle \mathbb{D} \rangle_\omega, \ \delta\mathbb{D}_\Lambda \equiv \mathbb{D}_\Lambda - \mathbb{D}_M,$$

$$\mathbb{A}_\Lambda = \left(\mathbb{I} + \mathbb{S}_\Lambda \mathbb{D}_M^{-1} \delta\mathbb{D}_\Lambda \right)^{-1},$$

$$\mathbb{S}_\Lambda \equiv -\mathbb{D}_M \int_{\Delta V_\Lambda} \mathbb{K}(x - x')\, dV'.$$

Ovde je jezgro 4-tenzor formiran drugim izvodima Grinove funkcijom, t.j. $(\mathbb{K})_{abcd} = (\partial_a \partial_d G_{bc})_{(ab)} \cdot \mathfrak{A}$. Za blago neuređenu /9/ raspodelu sa malim skokom krutosti sa relativnom krutošću uključka $\mathfrak{D}_\Lambda \equiv \mathbb{D}_M^{-1} \delta\mathbb{D}_\Lambda$ ovde se koristi izraz izveden u radu (Mićunović, /10/):

$$\mathbb{D}_\Lambda^{eff} = \mathbb{D}_M \left\{ \mathbb{S} + \mathfrak{D}_\Lambda \left(\mathbb{I} + \langle \mathbb{S} \rangle_\omega \mathfrak{D}_\Lambda - \langle \mathbb{S}\, \mathfrak{D} \rangle_\omega \right) \right\} \quad (4)$$

Efektivni tenzor termičkog koeficijenta širenja

Prema /1/, osnovne diferencijalne jednačine problema su:

$$div\mathbf{S}(x) = 0, \quad curl\ curl\left(\mathbb{D}^{-1}(x)\mathbf{S} + \boldsymbol{\alpha}(x)\theta \right) = \mathbf{0}$$

pa se za tenzor efektivnog termickog širenja dobija /1/:

$$\boldsymbol{\alpha}_\Lambda^* = \boldsymbol{\alpha}_M - \delta\boldsymbol{\alpha}_\Lambda \left(\mathbb{I} - \langle \mathbb{B}\mathbb{Q} \rangle_\omega \delta\left(\mathbb{D}_\Lambda^{-1} \right) \right) \langle \mathbb{B} \rangle_\omega \quad (5)$$

gde za uključak Λ, $(\Lambda \in \{1,...,N\})$ i matricu (skelet) označen sa M dobijamo:

$$\mathbb{B}_\Lambda = \left(\mathbb{I} + \mathbb{Q}_\Lambda \delta\left(\mathbb{D}_\Lambda^{-1} \right) \right)^{-1}, \qquad \mathbb{Q}_\Lambda \equiv \mathbb{D}_M \left(\mathbb{I} + \mathbb{P}_\Lambda \mathbb{D}_M \right)$$

Uvodeći ponovo $\mathfrak{D}_\Lambda \equiv \mathbb{D}_M^{-1} \delta\mathbb{D}_\Lambda$ i razvijanjem tenzora 4. reda u red dobijamo:

$$\boldsymbol{\alpha}_\Lambda^* = \boldsymbol{\alpha}_M + \delta\boldsymbol{\alpha}_\Lambda \left(\mathbb{I} + \mathbb{D}_M \left(\langle \mathbb{S}\mathfrak{D} \rangle - \mathfrak{D}_\Lambda \right) \mathbb{D}_M^{-1} \right) \quad (6)$$

Ovo je novi rezultat za blagu neuređenost – dat na osnovu aproksimacije drugog reda po $\mathfrak{D}$ i $\delta\boldsymbol{\alpha}_\Lambda$.

KINETIČKE JEDNAČINE ZA RAST ŠUPLJINA I NJIHOVO ZATVARANJE

U ovom odeljku razmatra se korišćenje kinetičkih jednačina uvedenih od strane Krajčinovića i Šumaraca u radu /11/. One su uvedene da opišu porast veličine i promenu

EFFECTIVE TENSORS

Effective stiffness

Effective stiffness tensor is derived by the procedure originally proposed by Levin /8/ as follows. Micro formulation of Hooke's law has the form

$\langle \boldsymbol{\Sigma}_\Lambda \rangle = \mathbb{D}^{\text{eff}} : \langle \mathbf{E}_{\Lambda e} \rangle$ leads to $\mathbf{S} = \mathbb{D}^{\text{eff}} : \mathbf{E}_e$

where

$\mathbf{E}_{\Lambda e} = \mathbf{E}_{\Lambda E} - \boldsymbol{\alpha}_\Lambda \theta$.

Levin's expression for effective stiffness reads:

$$\mathbb{D}_\Lambda^{eff} = \mathbb{D}_M + \delta\mathbb{D}_\Lambda \left(\mathbb{I} - \langle \mathbb{A}\mathbb{S} \rangle_\omega \mathbb{D}_M \delta\mathbb{D}_\Lambda \right)^{-1} \langle \mathbb{A} \rangle_\omega \quad (3)$$

Here for inclusion Λ, $(\Lambda \in \{1,...,N\})$ and matrix (skeleton) denoted by M we have:

$$\mathbb{D}_M = \langle \mathbb{D} \rangle_\omega, \ \delta\mathbb{D}_\Lambda \equiv \mathbb{D}_\Lambda - \mathbb{D}_M,$$

$$\mathbb{A}_\Lambda = \left(\mathbb{I} + \mathbb{S}_\Lambda \mathbb{D}_M^{-1} \delta\mathbb{D}_\Lambda \right)^{-1},$$

$$\mathbb{S}_\Lambda \equiv -\mathbb{D}_M \int_{\Delta V_\Lambda} \mathbb{K}(x - x')\, dV'.$$

Here the kernel is a 4-tensor formed by second derivatives of Green function i.e. $(\mathbb{K})_{abcd} = (\partial_a \partial_d G_{bc})_{(ab)} \cdot \mathfrak{A}$. For a slight disorder /9/ with small jump of stiffness by relative inclusion stiffness $\mathfrak{D}_\Lambda \equiv \mathbb{D}_M^{-1} \delta\mathbb{D}_\Lambda$ the expression derived in (Micunovic, /10/) is used here:

$$\mathbb{D}_\Lambda^{eff} = \mathbb{D}_M \left\{ \mathbb{S} + \mathfrak{D}_\Lambda \left(\mathbb{I} + \langle \mathbb{S} \rangle_\omega \mathfrak{D}_\Lambda - \langle \mathbb{S}\, \mathfrak{D} \rangle_\omega \right) \right\} \quad (4)$$

Effective tensor of thermal expansion coefficients

According to /1/, governing differential equations are:

$$div\mathbf{S}(x) = 0, \quad curl\ curl\left(\mathbb{D}^{-1}(x)\mathbf{S} + \boldsymbol{\alpha}(x)\theta \right) = \mathbf{0}$$

which leads to effective thermal expansion tensor /1/:

$$\boldsymbol{\alpha}_\Lambda^* = \boldsymbol{\alpha}_M - \delta\boldsymbol{\alpha}_\Lambda \left(\mathbb{I} - \langle \mathbb{B}\mathbb{Q} \rangle_\omega \delta\left(\mathbb{D}_\Lambda^{-1} \right) \right) \langle \mathbb{B} \rangle_\omega \quad (5)$$

where for inclusion Λ, $(\Lambda \in \{1,...,N\})$ and matrix (skeleton) denoted by M we have:

$$\mathbb{B}_\Lambda = \left(\mathbb{I} + \mathbb{Q}_\Lambda \delta\left(\mathbb{D}_\Lambda^{-1} \right) \right)^{-1}, \qquad \mathbb{Q}_\Lambda \equiv \mathbb{D}_M \left(\mathbb{I} + \mathbb{P}_\Lambda \mathbb{D}_M \right)$$

Introducing again $\mathfrak{D}_\Lambda \equiv \mathbb{D}_M^{-1} \delta\mathbb{D}_\Lambda$ and developing 4-tensors into power series we arrive at:

$$\boldsymbol{\alpha}_\Lambda^* = \boldsymbol{\alpha}_M + \delta\boldsymbol{\alpha}_\Lambda \left(\mathbb{I} + \mathbb{D}_M \left(\langle \mathbb{S}\mathfrak{D} \rangle - \mathfrak{D}_\Lambda \right) \mathbb{D}_M^{-1} \right) \quad (6)$$

This new result for slight disorder is acquired by making use of second order approximation in $\mathfrak{D}$ and $\delta\boldsymbol{\alpha}_\Lambda$

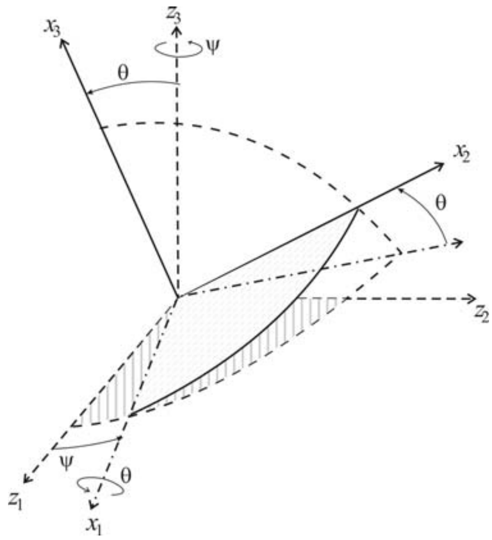
KINETIC RELATIONS FOR VOID GROWTH AND THEIR CLOSURE

In this section use of kinetic equations, introduced by Krajcinovic and Sumarac in /11/, is considered. They are introduced to describe the increase of size and orientation orijentacije šupljina. Tačan opis ponašanja čitavog skupa šupljina bi doveo do eksplicitnih vremenski zavisnih efektivnih materijalnih konstanti zavisnih od oštećenja. To je bio konačni cilj.

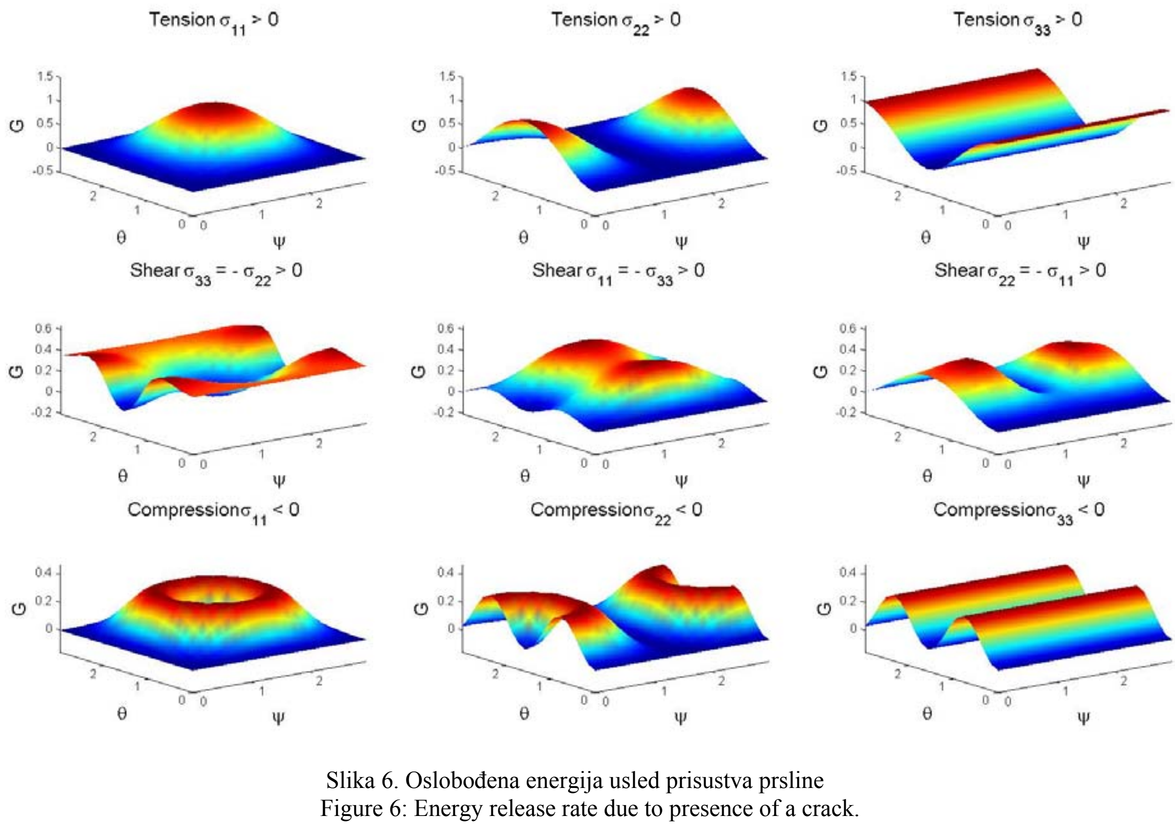
Međutim, s obzirom na složenost postavljenog problema zasad ograničavamo našu paznju na efekt naponskog stanja na mnogostrukost paralelnih proizvoljno orijentisanih (ali slučajno raspoređenih) prslina u obliku diska. Jedna takva prslina je specijalni slučaj degenerisane kružne elipsoidne šupljine kada upravna poluosa teži nuli. Četvrtina takve prsline je pokazana za proizvoljne Ojlerove uglove na sl. 5.

change of pores. A proper description of behaviour of the whole time dependent ensemble of pores would lead to explicit time dependent effective material properties dependent on damage. This was the final goal.

However, due to complexity of the problem at this stage of research we confine our investigation to show how stress state acts on a manifold of arbitrarily oriented but parallel (and stochastically distributed) penny shaped cracks. Such a crack is a special case of degenerated circular ellipsoidal void when perpendicular semi-axis tends to zero. A quarter of such a crack is shown for arbitrary Euler angles in Fig. 5.



Slika 5. Četvrtina prsline u obliku diska
Figure 5. Quarter of penny shaped crack.



Slika 6. Oslobođena energija usled prisustva prsline
Figure 6: Energy release rate due to presence of a crack.

Izraz za oslobođenu energiju zbog prisustva prsline glasi:

$$G = \{K_I \ K_{II} \ K_{III}\} \, diag\mathbf{C} \, \{K_I \ K_{II} \ K_{III}\}^T \qquad (7)$$

Svi faktori intenziteta napona $\{K_I K_{II} K_{III}\}$ zavise od normalnog napona i dve komponente smičućeg napona (normalno-radijalne, a takođe normalno-tangencijalne, redom) u lokalnom koordinatnom sistemu $\{x_1,...,x_3\}$.

Tenzor drugog ranga $\mathbf{C}$ u istom koordinatnom sistemu je dijagonalan za izotropni materijal i dat je u radu /5/ na sledeci nacin:

$$\mathbf{C} = \frac{1+\nu}{E}\begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1-\nu & 0 \\ 0 & 0 & 1-\nu \end{Bmatrix} \qquad (8)$$

gde su ν, E - Puasonov koeficijent i Jangov modul elasticnosti. Najlakši način za simuliranje bilo zatezanja, pritiska ili smicanja je da neke od dijagonalnih komponenata tenzora napona budu različite od nule, a sve ostale jednake nuli, u globalnom koordinatnom sistemu referencije $\{z_1,...,z_3\}$.

Rezultati računanja za neka, iako najprostija, pa ipak ilustrativna naponska stanja su prikazani na sledećih devet dijagrama (sl. 6). Brzina oslobođene energije je normalizovana deljenjem sa $G(\sigma_{11} > 0, \psi = 0, \theta = 0) \equiv G_0$. Rezultati proračuna će biti korišćeni u budućim formulacijama zatvaranja i porasta individualnih prslina i odgovarajućim posledicama na evoluciju oštećenja preko efektivnih krutosti i tenzora koeficijenata termičkog širenja.

The expression for energy release rate due to existing crack is:

$$G = \{K_I \ K_{II} \ K_{III}\} \, diag\mathbf{C} \, \{K_I \ K_{II} \ K_{III}\}^T \qquad (7)$$

All stress intensity factors $\{K_I K_{II} K_{III}\}$ depend on perpendicular stress component and two shear stresses (perpendicular and radial, perpendicular and circumferential, respectively) in the local coordinate system $\{x_1,...,x_3\}$.

The second rank tensor $\mathbf{C}$ in the same coordinate system is diagonal for isotropic materials and given in /5/ as follows:

$$\mathbf{C} = \frac{1+\nu}{E}\begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1-\nu & 0 \\ 0 & 0 & 1-\nu \end{Bmatrix} \qquad (8)$$

where ν, E - are Poisson coefficient and Young modulus, respectively. The easiest way to simulate either tension, compression or shear is to take some of normal stresses different from zero and all the others equal to zero in global coordinate system of reference $\{z_1,...,z_3\}$.

The result of calculation for some simplest, yet illustrative, stress states is shown on next nine diagrams (Fig. 6). Energy release rate is normalized by its division with $G(\sigma_{11} > 0, \psi = 0, \theta = 0) \equiv G_0$. The results of calculation are going to be used in forthcoming simulation of closing or opening individual crack and consequences to change of effective elastic moduli and thermal expansion tensor due to corresponding damage evolution.

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