POREĐENJE PONAŠANJA TANKIH CILINDRIČNIH I KONUSNIH LJUSKI OD UGLJENIČNOG I NERĐAJUĆEG ČELIKA

BEHAVIOUR OF THIN-WALLED CYLINDRICAL AND CONICAL SHELLS - CARBON vs. STAINLESS STEEL

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1 UVOD

Tanke cilindrične ljuske su konstruktivni elementi koji su našli široku primenu u praksi. Bilo da je reč o limenci piva ili o delu za rakete, neophodno je temeljno poznavanje svih njihovih karakteristika. Posebno treba naglasiti analizu stabilnosti i raspored (formu) izbočine koja se formira u postkritičnoj oblasti − dijamantsku formu, ili u literaturi još poznatu kao Yoshimura šablon [19].

Stvarne ljuske su elementi koji imaju određene imperfekcije. Moguće je klasifikovati imperfekcije na tri osnovna tipa: geometrijske, strukturne i imperfekcije u opterećenju. U analizi, najčešće se koriste geometrijske imperfekcije zbog jednostavnosti njihovog definisanja, a pokazale su se i kao adekvatne za opisivanje bilo koje vrste imperfekcija. Dobro je poznato da najveći uticaj na stabilnost cilindra imaju geometrijske imperfekcije zadate u obliku koji odgovara sopstvenim oblicima izbočavanja (jednom imperfekcijom ili kombinacijom više njih). Ovakva pretpostavka značajno odstupa od realne slike, ali je u građevinarstvu našla široku primenu. Jedan od razloga za to jeste mali broj dostupnih merenja imperfekcija na

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1 INTRODUCTION

Thin-walled cylindrical shells are structural elements that are widely used in practice. No matter if it is about a beer can or a rocket section, it is necessary to thoroughly know all their characteristics. The analysis of stability and buckling pattern formed in the post-buckling area should be especially emphasized. Buckling pattern can be a diamante pattern or the Yoshimura pattern, as it alternative known in literature [19].

The actual shell structures are elements which have certain imperfections. It is possible to classify these imperfections to three basic types: geometrical, structural and loading imperfections. The most often used are geometrical imperfections due to the simplicity in their definition. Moreover, it is proved that geometrical imperfections are adequate for describing any kind of imperfections. It is well known that geometrical imperfections have the highest influence on the cylinder stability. The geometrical imperfections are set in the form which corresponds to the eigenmodes (one or several of them combined). Such assumption considerably deviates from

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izvedenim konstrukcijama. Pored velikih ekonomskih izdataka, koji su neophodni da bi se jedno ovakvo merenje izvršilo, treba napomenuti i to da su imperfekcije funkcija s velikim brojem promenljivih. Znatan uticaj na pojavu imperfekcija imaju različite faze gradnje, način eksploatacije, klimatski uslovi (temperatura), kao i mnogi drugi uslovi.

U građevinarstvu, izbočavanje cilindrične ljuske merodavan je kriterijum za dimenzionisanje dimnjaka, stubova vetrogeneratora, silosa i sličnih objekata. Za praktičnu primenu, potrebno je pronaći ravnotežu između sigurnosti i ekonomičnosti, zadržati se u finansijski opravdanim granicama, primenom manje konzervativnih rešenja, ali ostati na strani sigurnosti u pogledu nosivosti i stabilnosti. U svemu navedenom leži i cilj istraživanja ovog rada, a to jeste uticaj različitih vrednosti amplituda početnih geometrijskih imperfekcija, koje su zadate u obliku prvog sopstvenog oblika izbočavanja, na ponašanje ljuski, uz analizu izbora materijala od kog su ljuske izrađene.

U radu su prikazani rezultati numeričke analize stabilnosti cilindričnih i konusnih ljuski, primenom programa Abaqus. Analiziran je uticaj materijalne i geometrijske nelinearnosti na kritičan napon i nosivost na izbočavanje. Materijalna nelineranost uzeta je u proračun primenom eksperimentalnih krivih napon–dilatacija ugljeničnog i nerđajućeg čelika, dok je geometrijska nelineranost obuhvaćena različitim vrednostima početnih imperfekcija. Rezultati sprovedene numeričke analize poređeni su s preporukama datim u EN 1993-1-6 [9] i definisane su vrednosti redukcionog faktora kao odnosa kritičnog napona i nosivosti na izbočavanje. Vrednosti redukcionog faktora poređene su s podacima dostupnim u referentnoj literaturi.

2 AKSIJALNO OPTEREĆENE KRUŽNE CILINDRIČNE I KONUSNE LJUSKE

Tokom poslednjih decenija, veliki broj istraživanja bio je usmeren na analizu problema stabilnosti ljuski. Naročito se izdvojio slučaj aksijalno napregnutih cilindričnih ljuski izloženih dejstvu pritiska, kao karakterističan za ovaj tip konstruktivnih elemenata, koji je u samom radu i analiziran. Razlog za to jeste značajno drugačije ponašanje cilindričnih ljuski pri aksijalnom opterećenju u poređenju s ponašanjem ploča i stubova [3]. Navedeno je ilustrativno prikazano na slici 1 za različite konstruktivne elemenate izložene dejstvu aksijalnog pritiska. Prikazane su krive sila–deformacija dobijene analizom ponašanja za inicijalnu fazu, graničnu fazu pri kojoj dolazi do gubitka stabilnosti i postkritičnu fazu. Za slučaj elemenata bez početnih imperfekcija, na slici 1 uočava se da se nakon dostizanja Eulerovog kritičnog opterećenja u slučaju ploča opterećenje (*P*) povećava s povećanjem aksijalne deformacije (Δ). Kod stubova ono se ne povećava, ali se i ne smanjuje, dok kod cilindričnih ljuski s daljim porastom deformacija (Δ), opterećenje (*P*) opada nakon dostizanja Eulerovog opterećenja. Kada se uzme u obzir uticaj početnih imperfekcija, kod stubova se može uočiti da je Eulerovo opterećenje maksimalna nosivost za sve aksijalno pritisnute elemente, s početnim imperfekcijama i bez njih. Kod ploča se uočava porast deformacije (Δ) u slučaju elementa s početnom imperfekcijom, ali oba slučaja mogu da prihvate opterećenje

the actual condition, but it has been widely used in civil engineering. One of the reasons is the small number of available imperfection measurements on the structures. In addition to the considerable financial cost that is necessary to perform such a measurement, it should be highlighted that imperfections are the function with a large number of variables. Different phases of construction, and the way of operation, climate conditions (temperature) and many other factors have a great impact on imperfections.

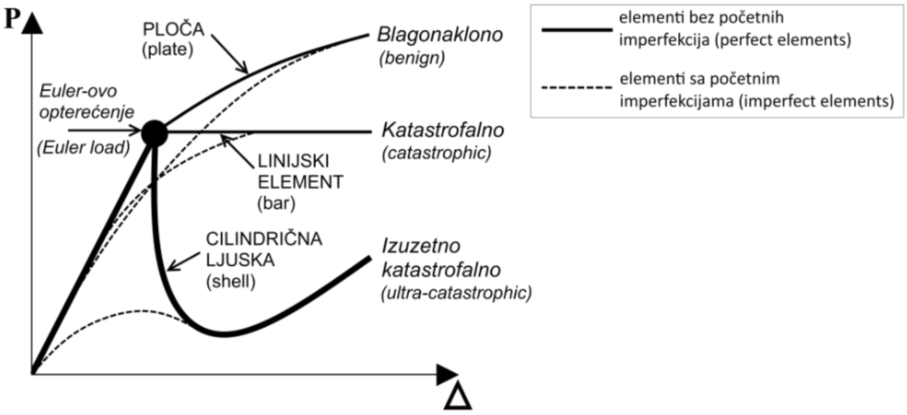
In civil engineering, buckling of cylindrical shell is a relevant criterion for design of chimneys, wind turbine towers, silos and similar structures. It is necessary to find a balance between safety and cost-effectiveness, but to remain in the financially justified limits at the same time, using the less conservative design procedures that will retain safety in terms of design resistance and stability. This is the aim of the paper, i.e. the influence of different values of amplitudes of initial geometrical imperfections, set in the form of the first eigenmode, on shell behaviour with the analysis of the choice of materials from which the shells are made.

The results of the numerical analysis of the stability of cylindrical and conical shells using software package Abaqus are presented in the paper. The influence of material and geometric nonlinearity on critical buckling stress and buckling resistance is analysed. Material nonlinearity has been taken into calculation using the experimental stress-strain curves of carbon and stainless steel, while the geometric nonlinearity has been included through various levels of initial imperfections. The results of the conducted numerical analysis have been compared with the recommendations given in EN 1993-1-6 [9] and the values of the reduction factor, calculated as ratio between critical buckling stress and buckling resistance have been defined. The values of the reduction factor are compared with the data available in the reference literature.

2 AXIALLY COMPRESSED CYLINDRICAL AND CONICAL SHELLS

During the recent decades, a large number of researches have been focused on the analysis of shell stability problem. The case of axially loaded cylindrical shells exposed to compression stands out as characteristic for this type of structural elements, and it is analysed in this paper. The reason lies in considerably different behaviour of cylindrical shells under axial load than of the plates and columns [3]. This is graphically presented in figure 1 for different structural elements exposed to axial compression. The force-strain curves obtained by the analysis for the initial phase, ultimate phase – where the loss of stability occurs and post-buckling phase are given. In figure 1, for perfect plates the load *(P)* can actually increase above the Euler buckling load with increasing axial deformation (Δ). For perfect bars the load (*P*) neither increases nor decreases, while for perfect shells with further increase of deformation (Δ) the load (*P*) decreases beyond the Euler buckling load. When considering the influence of initial imperfections, the Euler load for bars is the maximum load for all axially compressed elements, with and without initial imperfections. The plate shows an increase in deformation (Δ) in the case of an element koje je veće od Eulerovog opterećenja. Cilindrična ljuska s početnim imperfekcijama gubi stabilnost pri opterećenju znatno manjem od Eulerovog opterećenja, čime je ponašanje ovog elementa okarakterisano kao izuzetno katastrofalno [10].

with initial imperfections, but in both cases they can accept the load that is greater than Euler's. A cylindrical shell with initial imperfections loses stability at a load significantly smaller than Euler's which has been characterized as extremely catastrophic behaviour [10].



Slika 1. Krive sila–deformacija za različite pritisnute konstruktivne elemente [10]
Figure 1. The force-strain curves for different axially compressed structural elements [10]

2.1 Linearno-elastična teorija stabilnosti – kritično opterećenje

Ponašanje kružnih cilindričnih ljuski moguće je opisati jedinstvenim setom jednačina, ali zbog svoje kompleksnosti, bez praktične su primene. Iz navedenih razloga, uvode se određena uprošćenja, zanemarujući veličine koje imaju mali uticaj na razmatrani fenomen. Jedan takav set jednačina za opisivanje ponašanja ljuski predložio je Donnell [6]. U slučaju aksijalno opterećenih ljuski, set jednačina moguće je svesti na jednu jednačinu osmog reda, koja je poznata kao Donnell-ova jednačina i može se koristiti za određivanje kritičnog opterećenja – kako usled aksijalnog pritiska, tako i usled torzije i unutrašnjeg pritiska. Kritičnu vrednost opterećenja moguće je dobiti rešavanjem Donnell-ove jednačine i jedno od rešenja dao je Batdorf [2] za slobodno oslonjen cilindar na oba kraja. Dobijena vrednost kritičnog napona, σ_{cr}, u literaturi još nazivana i klasično rešenje, prikazana je jednačinom (1):

2.1 Linear-elastic theory – critical load

The behaviour of cylindrical shells can be described by a single set of equations, but due to its complexity they are impracticable. Therefore, certain simplifications are made by ignoring the parameters that have a small influence on the considered phenomenon. One such set of equations for description of shell behaviour was proposed by Donnell [6]. In the case of axially loaded shells, the set of equations can be reduced to one equation of the eighth order, which is known as Donnell's equation and which can be used for determining the critical load, both under the axial compression and to the torsion and internal pressure. The critical load value can be obtained by solving the Donnell's equation and such a solution was provided by Batdorf [2] for a cylinder pinned on both ends (pinned). The obtained solution, alternatively called the classical solution in the literature, is presented by the eq. (1):

$$\sigma_{cr} = \frac{1}{\sqrt{3(1-\mu^2)}}\frac{Et}{R} \tag{1}$$

gde je:

- E – modul elastičnosti;
- t – debljina ljuske;
- μ – Poisson-ov koeficijent;
- R – poluprečnik krivine cilindra.

Jednačina (1) primenjiva je na cilindre srednje dužine, koji su najzastupljeniji u praksi, pa shodno tome i veoma važni.

U slučaju konusnih ljuski, teorijske analize [13] pokazale su da se kritično opterećenje, P_{cr}, može izraziti na sledeći način:

where:

- E – elastic modulus;
- t – shell thickness;
- μ – Poisson's ratio in elastic range;
- R – radius of curvature.

The eq. (1) is applicable for cylinders of medium length which are the most common in practice, and therefore very important.

In the case of the conical shells, theoretical analyses [13] demonstrated that the critical load can be expressed in the following way:

$$P_{cr} = \frac{2\pi E t^2 \cos^2\alpha}{\sqrt{3(1-\mu^2)}} \tag{2}$$

gde je:

- E – Young-ov modul;
- t – debljina ljuske;
- μ – Poisson-ov koeficijent;
- α – nagib izvodnice konusa u odnosu na vertikalnu osu.

Jednačina (2) primenjiva je za vrednost ugla α između 10° i 75°. Za uglove manje od 10° može se uzeti vrednost koja odgovara cilindričnim ljuskama iste visine i debljine, a poluprečnika osrednjene vrednosti poluprečnika konusa.

2.2 Lom i postkritično ponašanje

Eksperimentalna ispitivanja ljuski pokazala su da je kritičan napon – pri kome dolazi do loma – često znatno manji od teorijski dobijenog primenom linearno elastične teorije. Stoga, može se izvesti zaključak da je ona neadekvatna za opisivanje ponašanja aksijalno pritisnutih cilindara i da se opisivanje relanog ponašanja može postići uvođenjem materijalne nelinearnosti u teoriju velikih deformacija.

Značajan napredak u razumevanju ponašanja cilindričnih ljuski napravio je Donnell 1934. godine [7]. On je uvideo važnost primene nelinearne teorije, odnosno značaj iznalaženja ne samo opterećenja pod kojim dolazi do bifurkacione stabilnosti cilindrične ljuske, već i postkritičnog ponašanja ljuski. Eulerova kritična sila na slici 1 predstavlja tačku bifurkacione stabilnosti, nakon čijeg dostizanja se funkcija ravnoteže menja. Zbog velikih uprošćenja, dobijeni rezultati nisu bili primenljivi. Dalja istraživanja bila su usmerena ka detaljnijem opisivanju ponašanja cilindra u postkritičnoj oblasti, kao i ispitivanjima uloge početnih imperfekcija u ponašanju cilindra. Izdvojiće se rad Donnell-a i Wan-a iz 1950. godine [8]. Oni su definisali diferencijalne jednačine kojima se opisuje ponašanje cilindričnih ljuski s početnim imperfekcijama. Grafički prikaz rešenja dat je na slici 2a. Sa A_0/t na slici definisan je odnos amplitude početne imperfekcije prema debljini ljuske t. Iako su dalja eksperimentalna ispitivanja pokazala da su dobijeni rezultati manje tačni, izvedeni su sledeći zaključci:

where:

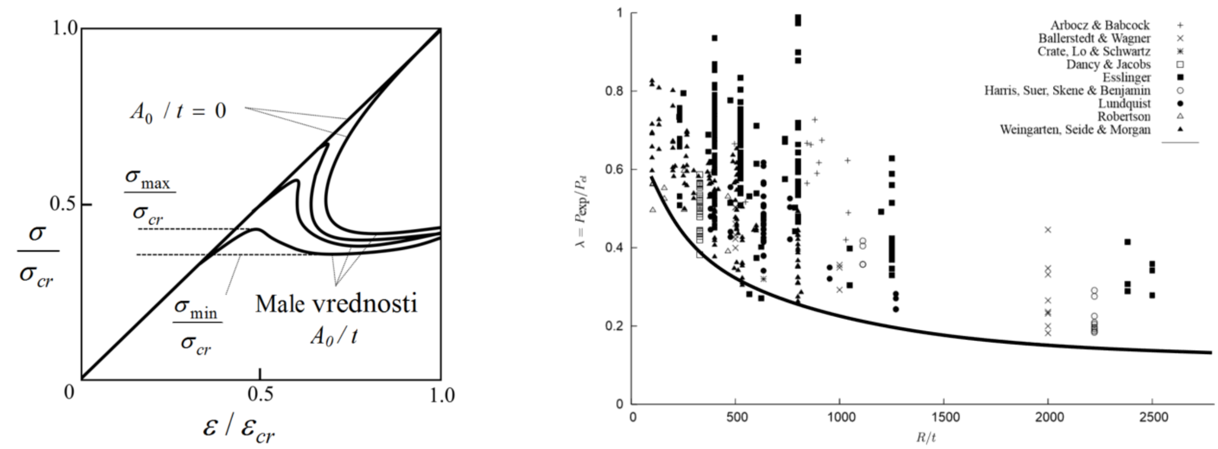
- E – elastic modulus;
- t – shell thickness;
- μ – Poisson's ratio in elastic range;
- α – semi-vertex angle of cone.

The eq. (2) is applicable for the value of angle α between 10° and 75°. For the angles smaller than 10° one can assume a value corresponding to the cylindrical shells of the same height and thickness, whose radius is an averaged value of the cone radius.

2.2 Failure and post-buckling behaviour

Experimental research showed that the critical buckling stress at which failure occurs is often considerably lower than the theoretically calculated one obtained using the linear-elastic theory. Therefore, it can be concluded that the linear-elastic theory is inadequate for description of behaviour of axially compressed cylinders, and that describing actual behaviour can be accomplished using the nonlinear large deformation theory.

A considerable advance in understanding cylindrical shell behaviour was made by Donnell in 1934 [7]. He grasped the importance of implementation of nonlinear theory, i.e., the importance of not only finding the load under which the bifurcation stability of a cylindrical shell occurs, but also learning about the post-buckling behaviour of the shells. Euler's critical force in figure 1 represents the point of bifurcation stability after which the equilibrium function changes. Due to the large simplifications, the obtained results were not applicable. Further research was directed towards a more detailed description of the behaviour of the cylinder in the post-critical field, as well as the role of initial imperfections in its behaviour. The work of Donnell and Wan of 1950 will be highlighted [8]. They defined differential equations describing behaviour of cylindrical shells with initial imperfections. The graphical presentation of the solution is provided in figure 2a. With A_0/t, the relationship between the amplitude of the initial imperfections and the shell thickness t is defined in the figure. Although the



a) b)

Slika 2. a) Efekti imperfekcija na postkritično ponašanje cilindara [8]; b) Rezultati ispitivanja aksijalno pritisnutih izotropnih cilindričnih ljuski [4]

Figure 2. a) Effect of imperfections on post-buckling behaviour of cylinders [8], b) Test data for axially compressed isotropic shells [4]

– i najmanja imperfekcija vodi do znatnog smanjenja vrednosti kritičnog napona;

– minimalna vrednost napona (σ_{min}) nije pod velikim uticajem veličine imperfekcije, pa bi usvajanje ove vrednosti, kao kritične, dovelo do konzervativnog rešenja za projektante.

Do rešenja koje je veoma doprinelo rešavanju problema u praksi došao je Koiter [11], koristeći uprošćenu teoriju velikih deformacija na primeru aksisimetričnih početnih imperfekcija. Na slici 2b prikazani su rezultati eksperimentalnih ispitivanja cilindričnih ljuski, koje su objavili različiti autori. Na apscisi je označen odnos prečnika (R) i debljine ljuske (t), dok je na ordinati definisana normalizovana vrednost kritičnog opterećenja (λ), koja predstavlja odnos kritičnog opterećenja dobijenog eksperimentalnim putem (P_{exp}) i opterećenja dobijenog primenom klasičnog rešenja (P_{cl}). Punom linijom na grafiku obeleženo je rešenje koje je dobijeno množenjem opterećenja dobijenog klasičnom teorijom s korekcionim faktorom definisanim u proračunskim preporukama NASA-e [14]. Utemeljeno je objašnjenje da glavni razlog za veliku disperziju rezultata eksperimentalnih ispitivanja u odnosu na teoriju predstavljaju početne imperfekcije.

2.3 Proračunske preporuke

Pri projektovanju tankih cilindričnih ljuski, u inženjerskoj praksi koriste se različiti standardi i priručnici. Svi oni u osnovi primenjuju klasično rešenje dobijeno linearno-elastičnom teorijom (jednačina (1)), a zatim ga množe odgovarajućim redukcionim faktorom (*knockdown factor*), kako bi dobili vrednost opterećenja, koju cilindar može da prenese. Ovakva vrednost opterećenja određuje se na osnovu pune zakrivljene linije kojom je obeležena donja granica rezultata ispitivanja na slici 2b.

Postoje različite analitičke formulacije redukcionog faktora za izotropne cilindrične ljuske, aksijalno opterećene. Neke od prvih preporuka potiču s početka 20. veka, kao što su NASA SP-8007 [14]. Prema njima, kritičan napon dobija se iz sledeće formule:

further research proved that the obtained results are less accurate, the following conclusions were drawn:

– even the slightest imperfection causes a considerable lower critical stress value;

– the minimal value is not significantly impacted by the size of the imperfection, so adopting this value as a critical one would lead to a conservative solution for the designers.

The solution which greatly contributed to solving the problems in practice was discovered by Koiter [11] using a simplified large deformation theory on the example of axisymmetric initial imperfections. The obtained result has a good agreement with the lower values obtained by experimental research. In figure 2b the results of the experimental investigation of cylindrical shells published by various authors are presented. The ratio of the diameter (R) and the shell thickness (t) is indicated on the abscissa, while the normalized critical load (λ) is defined on the ordinate, which represents the ratio of the critical load obtained by the experiments (P_{exp}) and the load obtained using the classical solution (P_{cl}). The full line on the graphic is a solution obtained by multiplying the load obtained by the classical theory with the correction (knockdown) factor defined in NASA's design recommendations [14]. An explanation for a large dispersion of the results of experimental tests, in comparison with the theory was established and initial imperfections were allocated as the main reason.

2.3 Design recommendations

Different standards and manuals used in engineering practice give design recommendations for cylindrical and conical shells. All of them basically implement a classical solution obtained using the linear theory, eq. (1), and then multiply it by the corresponding knockdown factor to obtain the load which can be transferred by the cylinder. This factor is determined based on the full curvature marking the lower boundary of test results in figure 2b.

There are different analytical formulations of the knockdown factor for axially compressed isotropic cylindrical shells. Some of the first recommendations date back to the beginning of 20th century, such as NASA SP-8007 [14]. According to them, the critical stress is obtained from the following formula:

$$\sigma_{cr} = \gamma \sigma_{cl} \tag{3}$$

$$\gamma = 1 - 0.901(1 - e^{-\phi}), \quad \phi = \frac{1}{16}\sqrt{\frac{R}{t}} \tag{4}$$

gde je:

σ_{cl} – kritični napon definisan jednačinom (1);

γ – redukcioni (knockdown) faktor definisan jednačinom (4);

t – debljina ljuske;

R – poluprečnik krivine cilindra.

Da bi se dobile formule (3) i (4), rezultati eksperimentalnih ispitivanja su grubo uzeti u obzir, a da se pritom nije vodilo računa o načinu ispitivanja ili proizvodnje elemenata. Primena je ograničena samo za slobodno oslonjene cilindre na oba kraja. Dodatno, treba biti oprezan s korišćenjem ovih formula za opsege $L/R>5$, jer ne postoje eksperimentalni podaci za ovu

where:

σ_{cl} – critical stress defined by eq. (1);

γ – knockdown factor defined by eq. (4);

R – radius of curvature;

t – shell thickness.

In order to obtain eq. (3) and (4), the results of the experimental tests are roughly taken into account, without considering the method of testing or manufacturing the elements. Application is limited only to shells pinned at both ends. Additionally, one should be cautious about using these formulas for range $L/R>5$ conditions, because there are no experimental data. Taking the influence of imperfection into account, the oblast. Imajući uticaj imperfekcija u vidu, dobijeni kritični napon i dalje je na strani sigurnosti.

U Evropskom standardu [9], koristi se isti izraz za kritičan napon, jednačina (3), ali je redukcioni faktor definisan putem naredne dve jednačine:

obtained critical stress remains on the safety side.

In the European standard [9], the same expression for critical stress, eq. (3), is used, but the reduction factor is defined by the following two equations:

$$\gamma = \frac{0.82}{\sqrt{1+0.01R/t}}, \quad R/t < 212 \tag{5}$$

$$\gamma = \frac{0.70}{\sqrt{0.1+0.01R/t}}, \quad R/t > 212 \tag{6}$$

gde je:

γ – redukcioni (knockdown) faktor;
R – poluprečnik krivine cilindra;
t – debljina ljuske.

Kako bi se isključila mogućnost sveukupnog Eulerovog izvijanja stuba, formule (3), (5) i (6) važe isključivo za cilindre koji ispunjavaju uslov:

where:

γ – knockdown factor;
R – radius of curvature;
t – shell thickness.

In order to exclude the possibility of the overall Euler's buckling of the column, eq. (3), (5) and (6) should be applied exclusively to the cylinders that fulfil the condition:

$$\frac{L}{R} \leq 0.95\sqrt{\frac{R}{t}} \tag{7}$$

gde je:

L – dužina cilindra;
R – poluprečnik krivine cilindra.
t – debljina ljuske;

Imperfekcije se u [9] uzimaju u obzir – u skladu sa slikom 3. Dužina poremećaja definisana je izrazom:

where:

L – length of shell;
R – radius of curvature;
t – shell thickness.

Imperfections are taken into account in [9] in accordance with figure 3. The length of the disorder is defined by the expression:

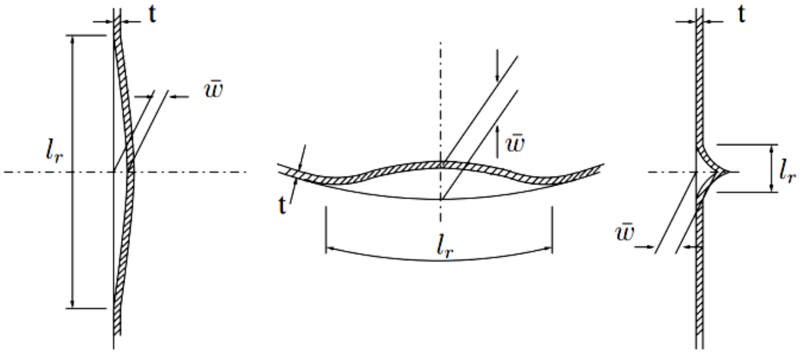
$$l_r = 4\sqrt{Rt} \tag{8}$$

gde je:

R – poluprečnik krivine cilindra.
t – debljina ljuske;

where:

R – radius of curvature;
t – shell thickness.



Slika 3. Merenje dubine $\bar{w}$ početnih imperfekcija [9]
Figure 3. Measurement of depths $\bar{w}$ of initial imperfections [9]

Kada je odnos najveće amplitude $\bar{w}$ u odnosu na odgovarajuće l_r manji od 0,01, treba primeniti formule (5) i (6) za γ; kada je jednak 0,02 vrednosti γ treba uzeti u iznosu od 50%, a između raditi interpolaciju. Kada je ovaj odnos veći od 0,02, ne postoje nikakve preporuke, sugerišući time da bi takve elemente trebalo izbaciti iz primene.

Oba rešenja i preporuke u EN 1993-1-6 [9], kao i preporuke koje je dala NASA-a u SP8007 [14], u osnovi su potekle iz rada Weingarten-a [17]. Osnovna razlika

When the ratio of the largest amplitude $\bar{w}$ relative to the corresponding l_r is less than 0,01, eq. (5) and (6) should be applied to γ; when equal 0,02 values of γ should be taken in the amount of 50%, and between the interpolations should be done. When this ratio is greater than 0,02, there are no recommendations, suggesting that such elements should be excluded from the application.

Both solutions, the recommendations in EN 1993-1-6 [9] and the recommendations provided by NASA in između njih jeste to što je u Evrokodu uzet u obzir kvalitet proizvodnje kroz proizvodne klase (A, B ili C). Druga velika razlika ogleda se u preporukama Evrokoda da se primeni dodatni faktor sigurnosti od 4/3 za aksijalno pritisnute cilindre.

Kod konusnih ljuski, vrednost redukcionog faktora za opseg $10° \leq \alpha < 75°$ iznosi 0,33, dok za opseg $\alpha \geq 75°$ on treba da bude proveren eksperimentalnim putem, kako je preporučeno u [15]. Navedeni redukcioni faktor, kao i u slučaju cilindričnih ljuski, množi se s klasičnim rešenjem datim u jednačini (2).

3 NUMERIČKA ANALIZA

S ciljem kvantifikacije uticajnih parametara koji su značajni za nosivost cilindričnih i konusnih ljuski, sprovedena je numerička parametarska analiza metodom konačnih elemenata. Detaljnom komparativnom analizom, izvršena je procena usklađenosti numeričkih rezultata s rezultatima proračunskih modela koji su dati u evropskom standardu EN 1993-1-6 [9]. Numerička analiza cilindričnih i konusnih tankih ljuski urađena je korišćenjem programa Abaqus, verzija 6.12-3 [1]. U numeričkoj analizi problema stabilnosti, korišćene su dve metode:

– analiza sopstvenih oblika izbočavanja (*LBA – Linear Buckling Analysis*);

– analiza odgovora nakon gubitka stabilnosti ili analiza loma (Postbuckling analysis) primenom GMNIA analize (GMNIA – Geometrically and materially nonlinear analysis with imperfections included).

U inicijalnoj fazi proračuna, koja daje predviđanje sopstvenih oblika izbočavanja, korišćena je linearno-elastična analiza. Analiza stabilnosti elementa primenom GMNIA analize zasniva se na rešavanju nelinearne jednačine ravnoteže, primenom odgovarajuće numeričke metode. U radu je primenjena metoda konstantnog sfernog luka, poznata i kao Riksova metoda [12].

Numerička analiza prikazana u ovom radu obuhvatila je analizu cilindrične ljuske dužine 10 m i poluprečnika 2,5 m, debljine zida ljuske od 6,0 mm do 30,0 mm, koja pripada opsegu cilindričnih ljuski srednje dužine, kako je definisano u EN 1993-1-6, Aneks D [9]. Takođe, numerička analiza obuhvatila je i konusnu ljusku iste dužine i promenjivog poluprečnika od 1,25 m do 2,5 m, debljine zida ljuske 10,0 mm. Modeli cilindričnih i konusnih obostrano zglobno oslonjenih ljuski formirani su u Abaqus-u [1], pomoću površinskih S4R konačnih elemenata. Mreža za sve analizirane modele formirana je od konačnih elemenata S4R aproksimativne veličine 200 x 200 mm, za koju je utvrđeno da rezultati počinju značajnije da konvergiraju (razlika u kritičnom naponu izbočavanja koji odgovara prvom obliku izbočavanja manja je od 0,20%).

Mehanička svojstva materijala definisana su nelinearnom vezom napona i dilatacija dobijenih ispitivanjem pri zatezanju epruveta izrađenih od vruće valjanog profila od ugljeničnog čelika kvaliteta S275 [16] i epruveta od hladno valjanog nerđajućeg čelika austenitne mikrostrukture sa oznakom 1.4301 [5] (kako je prikazano na slici 4a). Zglobno oslanjanje ostvareno je defini-

SP8007 [14] basically originate from the work of Weingarten [17]. The basic difference between them is that Eurocode takes into account the quality of production, through fabrication tolerance quality classes (A, B or C). The other large difference between the European standard [9] and SP8007 [14] is reflected in the Eurocode recommendations to implement an additional safety factor of 4/3 for axially compressed cylinders.

In the case of conical shells, the value of the knock-down factor for the range $10° \leq \alpha < 75°$ amounts to 0,33, while for the range $\alpha \geq 75°$ it must be verified experimentally, as recommended in [15]. The mentioned knock-down factor, as in the case of cylindrical shells, is multiplied with the classical solution provided in the eq. (2).

3 NUMERICAL ANALYSIS

In order to quantify the influence of main parameters on the resistance of cylindrical and conical shells, a numerical parametric analysis was performed using the finite element method. A detail comparative analysis was conducted in order to quantify the compliance of numerical analysis results with design recommendations given in European standard EN 1993-1-6 [9]. Numerical analysis of cylindrical and conical thin-walled shells was performed using the Abaqus software, version 6.12-3 [1]. In the numerical analysis of the stability problem, two methods were used:

– analysis of the linear bifurcation eigenvalue (*LBA - Linear Buckling Analysis*);

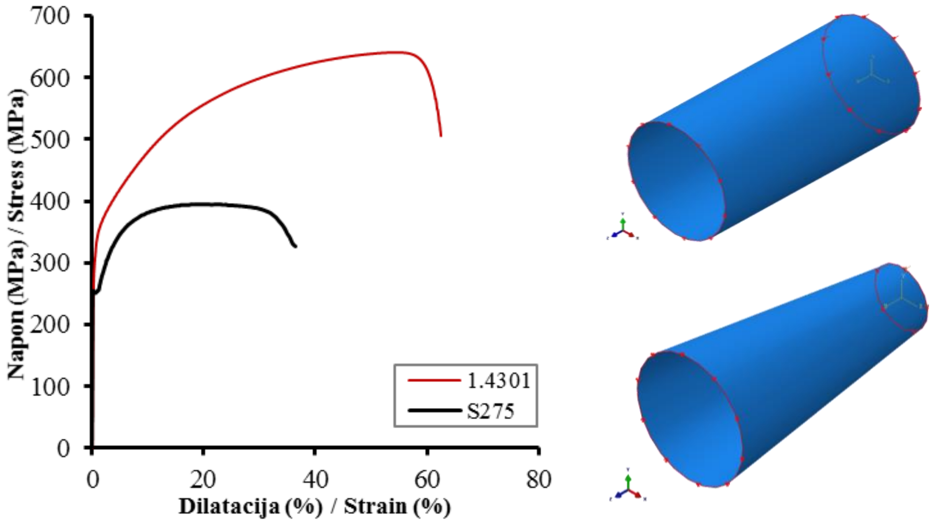
– analysis of response after stability loss or nonlinear analysis (Postbuckling analysis) with GMNIA analysis (GMNIA - Geometrically and materially nonlinear analysis with imperfections included).

A linear elastic analysis was used in the initial phase of the calculation which provides prediction of eigenmodes. The element stability analysis with GMNIA is based on solving the nonlinear equilibrium equation using an appropriate numerical method. An arc length method, also known as the Riks method [12] is implemented in the paper.

The numerical analysis presented in this paper included the cylindrical shell 10 m long with 2,5 m radius, with the wall thickness ranging between 6,0 mm and 30,0 mm, which belongs to the medium long cylindrical shells as defined in EN 1993-1-6, Annex D [9]. In addition, the numerical analysis included the conical shell of variable radius ranging between 1,25 m and 2,5 m, with 10,0 mm wall thickness. The models of cylindrical and conical shells pinned on both ends are defined in Abaqus [1] using shell S4R finite elements. The mesh for all analysed models has been formed using S4R finite elements of the approximate size of 200 x 200 mm for which the results start to converge more considerably (the critical buckling stress which corresponds to the first eigenmode is lower than 0,20%).

Mechanical properties of the material are defined by a nonlinear stress-strain obtained by tensile tests of coupons made of hot-rolled section of carbon steel grade S275 [15] and coupons made of cold-formed stainless steel of austenitic microstructure designated as 1.4301 [5], as given in figure 4a. Pinned supports are realized by defining the boundary conditions along the sanjem graničnih uslova po obimu ljuske, i to na jednom kraju sprečavanjem pomeranja u pravcu sve tri glavne ose U1=U2=U3=0, a na suprotnom kraju elementa na kojem se nanosi opterećenje dozvoljeno je pomeranje u pravcu globalne Z-ose, što odgovara podužnoj X-osi ljuske (U1=U2=0), kako je prikazano na slici 4b. Opterećenje u obliku aksijalnog pritiska nanosi se na jednom kraju ljuske, kao raspodeljeno opterećenje svuda po obimu.

shell circumferences. On one end of the shell restraining displacement in the direction of all three main axes U1=U2=U3=0 is adopted. On the opposite end on which the load is applied, the displacement in the direction of the global Z-axis is free, which corresponds to the longitudinal X-axis of the shell (U1=U2=0), as displayed in figure 4b. The axial compression loading is applied to one end of the shell as a load distributed along the entire circumference.



a) b)

Slika 4. a) Dijagram napon-dilatacija za ugljenični čelik S275 i nerđajući čelik 1.4301; b) model konusne i cilindrične ljuske i uslovi oslanjanja

Figure 4. a) stress-strain curves for carbon steel S275 and stainless steel 1.4301 b) model of conical and cylindrical shell and boundary conditions

4 REZULTATI NUMERIČKE ANALIZE I POREĐENJE S PREPORUKAMA DEFINISANIM U EN 1993-1-6

Vrednosti elastičnog kritičnog napona izbočavanja za cilindrične ljuske debljine od 6,0 do 30,0 mm i konusne ljuske debljine 10,0 mm definisane su primenom LBA metode u Abaqusu [1] i izrazu definisanom u EN 1993-1-6 [9], što je ujedno i klasično rešenje prikazano jednačinom (1). Materijalna nelinearnost uvedena je u numeričke primere putem stvarne veze napona i dilatacije za dva analizirana materijala, a početna geometrijska imperfekcija zadata je kao pomeranje određene amplitude koje odgovara prvom sopstvenom obliku izbočavanja (LBA analiza u Abaqus-u [1]).

Uticaj različitih vrednosti imperfekcija na nosivost na izbočavanje analiziran je na primeru cilindrične i konusne ljuske debljine 10,0 mm, primenom metode materijalne i geometrijske nelinearne analize sa imperfekcijama (*GMNIA*).

4 NUMERICAL ANALYSIS RESULTS AND COMPARISON WITH RECOMMENDATIONS GIVEN IN EN1993-1-6

The values of the elastic critical buckling stress for cylindrical shells with thickness from 6,0 mm to 30,0 mm and conical shell with 10,0 mm thickness are defined using the LBA analysis in Abaqus [1] and according to design recommendations given in EN 1993-1-6 [9] which is at the same time the classical solution presented in the eq. (1). Material nonlinearity is introduced in the numerical models through the real stress-strain relation for two analysed materials, and the geometrical imperfection is set as displacement of a certain amplitude, which corresponds to the first eigenmode (LBA analysis in Abaqus [1]).

The impact of different values of imperfections on the critical buckling stress is analysed on the example of cylindrical and conical shells 10,0 mm thick, with the geometrically and materially nonlinear analysis with imperfections included (*GMNIA*).

4.1 Rezultati numeričke analize – cilindrične ljuske debljine 10 mm

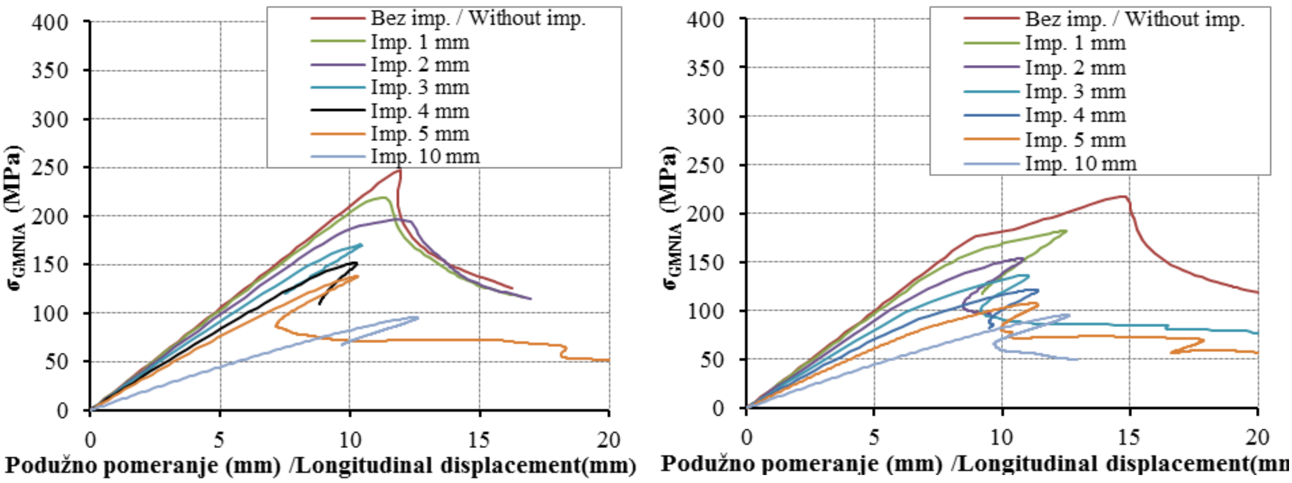
Na slici 5 prikazana je nosivost na izbočavanje cilindrične ljuske od ugljeničnog čelika S275 i nerđajućeg čelika 1.4301.

Cilindrična ljuska od nerđajućeg čelika ima 12,5% manju nosivost na izbočavanje u poređenju sa istim cilindrom od ugljeničnog čelika, za model bez početnih imperfekcija (MNA – *materially nonlinear analysis*). Kritičan napon izbočavanja cilindrične ljuske debljine 10,0 mm od ugljeničnog i nerđajućeg čelika, dobijen primenom LBA metode u Abaqusu [1], iznosi 509,3 MPa i 485,0 MPa, respektivno.

4.1 Results of numerical analysis - cylindrical shells with thickness of 10 mm

Buckling resistance of the cylindrical shell of S275 carbon steel and 1.4301 stainless steel is given in figure 5.

The cylindrical shell of stainless steel has 12,5% lower buckling resistance in comparison with the same cylinder made of carbon steel, regarding the numerical model without initial imperfections (*MNA - materially nonlinear analysis*). Critical buckling stress obtained from LBA analysis in Abaqus [1], for cylindrical shell made from carbon and stainless steel with 10,0 mm thickness of 509,3 MPa and 485,0 MPa, respectively.



a) b)

Slika 5. Nosivost na izbočavanje cilindrične ljuske t=10,0 mm za različite vrednosti početne imperfekcije: a) ugljenični čelik S275; b) nerđajući čelik 1.4301

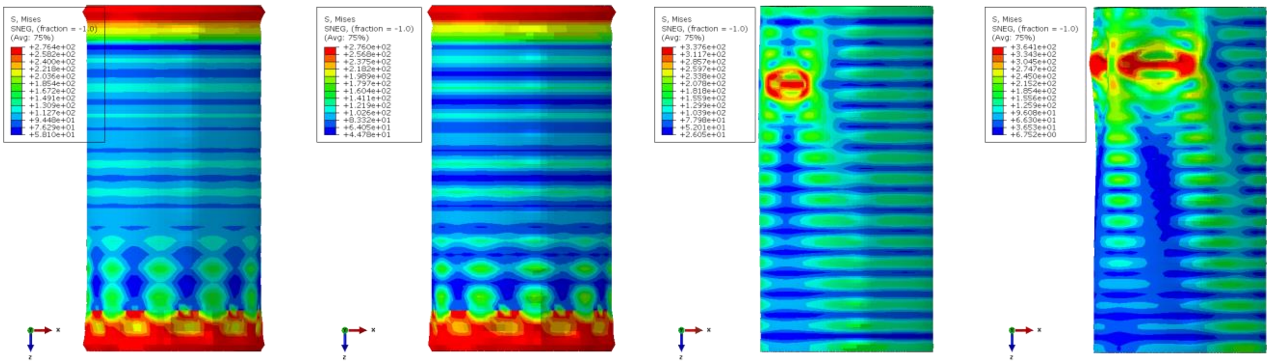
Figure 5. Buckling resistance of cylindrical shell t=10,0 mm for different initial imperfektions: a) carbon steel S275, b) stainless steel 1.4301

Kod cilindričnih ljuski od ugljeničnog čelika, imperfekcije od 1,0 i 2,0 mm izazivaju aksijalno simetrično izbočavanje ljuske u zoni neposredno uz oslonce, odnosno neposredno ispod zone unošenja opterećenja, kao na slici 6a i 6b. Oblik izbočavanja cilindra od nerđajućeg čelika, pri manjim vrednostima imperfekcija, u obliku je nesimetričnog izbočavanja (*dimple buckling*) u zoni unošenja opterećenja, koje je prikazano na slici 6c i 6d, a koje se – kao karakterističan oblik izbočavanja – zadržava i pri povećanju imperfekcija do 10,0 mm.

Takođe, kod nerđajućeg čelika, u oblasti napona između napona proporcionalnosti f_p i konvencionalne granice razvlačenja f_{02} javlja se progresivni pad tangentnog modula elastičnosti E_t, što utiče i na znatno smanjenje krutosti koja je uočljiva na slici 5b, kod cilindričnih ljuski od nerđajućeg čelika.

In the case of the cylindrical shells of carbon steel, the imperfections of 1,0 and 2,0 mm cause axially symmetrical buckling in the zone immediately next to supports, i.e. immediately below the zone where the load is applied, as given in figures 6a and 6b. The response of the stainless steel cylindrical shell at lower values of imperfections has the form of dimple buckling in the zone where the load is applied, as displayed in figures 6c and 6d, which is a characteristic response of the structure retained even when the imperfections are increased up to 10,0 mm.

In addition, in the case of stainless steel, in the stress range between the proportionality limit stress f_p and the 0.2% proof stress f_{02} there is a progressive decline of tangent elastic modulus E_t, which causes the considerable decrease of stiffness as observable in figure 5b, in cylindrical shells of stainless steel.



a) b) c) d)

Slika 6. Izbočavanje cilindrične ljuske debljine od 10,0 mm za različite vrednosti imperfekcija: a) 1 mm - S275; b) 2 mm - S275; c) 1 mm - 1.4301; d) 2 mm - 1.4301

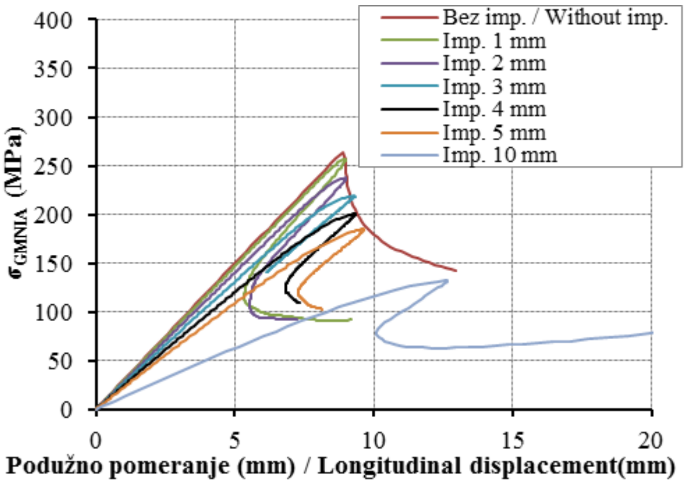
Figure 6.Buckling of cylindrical shell 10,0 mm thick for different values of imperfections - a) 1 mm – S275, b) 2 mm – S275, c) 1 mm – 1.4301, d) 2 mm - 1.4301

4.2 Rezultati numeričke analize – konusne ljuske debljine 10 mm

Uticaj različitih vrednosti početnih geometrijskih imperfekcija zadatih u obliku prvog sopstvenog oblika izbočavanja na nosivost na izbočavanje konusne ljuske od ugljeničnog čelika S275 prikazan je na slici 7.

4.2 Results of numerical analysis - conical shells with thickness of 10mm

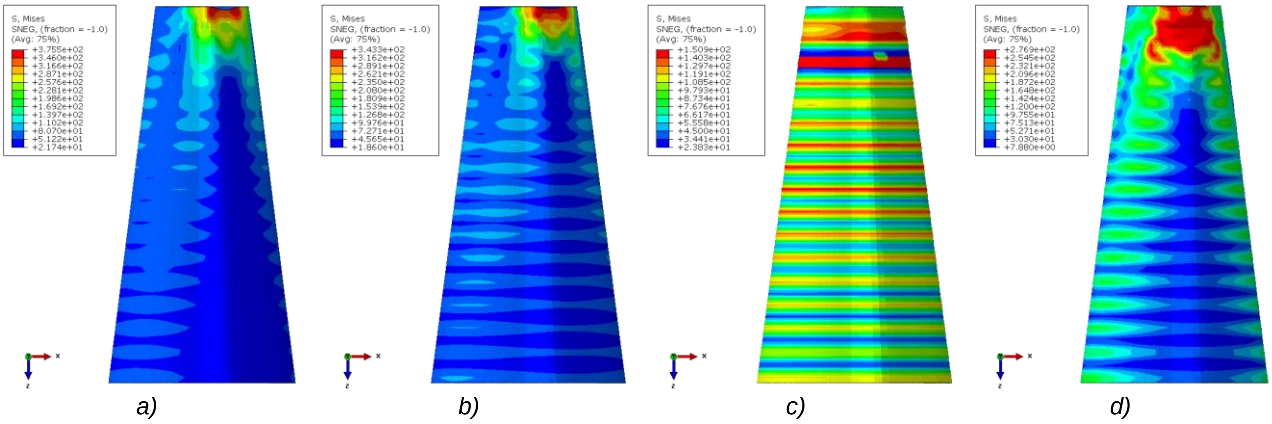
The impact of different values of initial geometrical imperfections on the buckling resistance of the conical shell made of S275 carbon steel is given in figure 7.



Slika 7. Nosivost na izbočavanje konusne ljuske t=10,0 mm za različite vrednosti početne imperfekcije
Figure 7. Buckling resistance of conical shell t=10,0 mm for different initial imperfektions

Kritičan napon izbočavanja konusne ljuske od ugljeničnog čelika debljine 10,0 mm, dobijen primenom LBA metode u Abaqusu [1], iznosi 504,3 MPa. Odgovor analizirane konusne ljuske jeste u obliku nesimetričnog izbočavanja (*dimple buckling*) u zoni unošenja opterećenja, koje se kao karakterističan odgovor konstrukcije zadržava i pri povećanju imperfekcija do 10 mm (kako je prikazano na slici 8).

Critical buckling stress obtained from LBA analysis in Abaqus [1] is 504,3 MPa for conical shell made of carbon steel with 10,0 mm thickness. The response of a conical shell made of carbon steel is in a form of dimple buckling in the zone where the load is applied which is a characteristic response of the structure retained even when the imperfections are increased up to 10,0 mm, as displayed in figure 8.



Slika 8. Izbočavanje konusne ljuske debljine od 10,0 mm od ugljeničnog čelika S275 za različite vrednosti imperfekcija: a) 1 mm; b) 2 mm; c) 3 mm; d) 4 mm

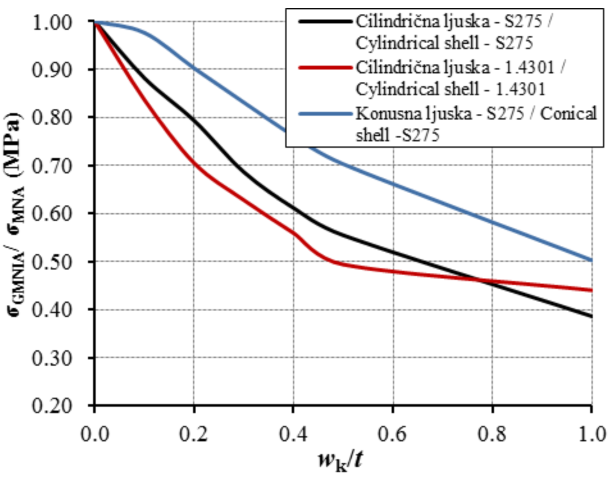
Figure 8. Buckling of cylindrical shell 10,0 mm thick made of S275 carbon steel for different values of imperfections: a) 1 mm, b) 2 mm, c) 3 mm, d) 4 mm

4.3 Poređenje dobijenih rezultata s proračunskim preporukama i literaturom

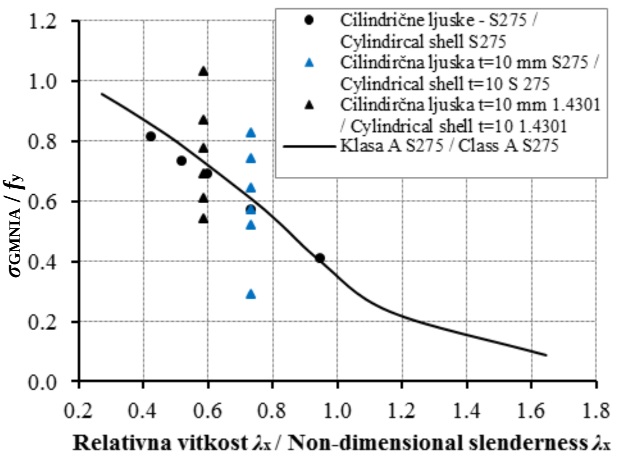
Redukcija nosivosti cilindrične i konusne ljuske od dva analizirana materijala debljine 10,0 mm, usled povećanja geometrijskih imperfekcija, prikazana je na slici 9a. Na horizontalnoj osi prikazan je odnos zadate početne geometrijske imperfekcije i debljine ljuske (w_k/t). Na vertikalnoj osi prikazan je odnos nosivosti ljuske na izbočavanje usled materijalne nelinearnosti i zadatih početnih geometrijskih imperfekcija (σ_{GMNIA}) i bez početnih geometrijskih imperfekcija (σ_{MNA}). Imperfekcija od 1,0 mm izaziva redukciju nosivosti od 12% odnosno 17%, za ugljenični čelik i nerđajući čelik kod cilindričnih ljuski, respektivno. Međutim, ova redukcija nosivosti kod konusnih ljuski od ugljeničnog čelika za istu vrednost imperfekcije znatno je manja i iznosi 2%. Takođe, povećanjem početnih geometrijskih imperfekcija do 5,0 mm, redukcija nosivosti dostiže i do 45% kod ugljeničnih čelika, odnosno 50% kod nerđajućeg čelika u slučaju cilindričnih ljuski i 30% u slučaju konusnih ljuski od ugljeničnih čelika. Cilindrične i konusne ljuske od ugljeničnog čelika pokazuju ujednačen pad nosivosti pri povećanju imperfekcija, ali sa značajnom kvantitativnom razlikom u pogledu vrednosti redukcije nosivosti. S druge strane, cilindrična ljuska od nerđajućeg čelika pri povećanju imperfekcija od 5,0 mm do 10,0 mm pokazuje manji pad nosivosti, od 51% do 57% (kako je prikazano na slici 9a), dok je kod konusne ljuske od ugljeničnog čelika povećanje geometrijskih imperfekcija za istu vrednost rezultovalo padom nosivosti od 20%. Konusne cilindrične ljuske od ugljeničnog čelika pokazuju manji pad vrednosti napona izbočavanja u poređenju s cilindričnim ljuskama, u proseku 10% za istu vrednost početnih geometrijskih imperfekcija.

4.3 Comparison of the obtained results with design recommendations and literature

The reduction of buckling resistance of cylindrical and conical shell made of two analysed materials with 10,0 mm thickness caused by the increase of geometrical imperfections is presented in figure 9a. The horizontal axis represent the initial geometrical imperfections amplitude vs. shell thickness ratio (w_k/t). The vertical axis represent the ratio between buckling resistance shell due to geometrically and materially nonlinear analysis with imperfections included (σ_{GMNIA}) and buckling resistance without imperfections (σ_{MNA}). An imperfection of 1,0 mm causes the buckling resistance reduction of 12%, i.e. 17%, for carbon steel and stainless steel in cylindrical shells, respectively. However, this reduction of buckling resistance of conical shells of carbon steel for the same value of imperfection is considerably lower and amounts to 2%. In addition, by increasing the initial geometrical imperfections up to 5,0 mm, the reduction of buckling resistance is 45% for carbon steel, i.e. about 50% for stainless steel in the case of cylindrical shells and 30% in the case of conical shells of carbon steel. Cylindrical and conical shells made of carbon steel demonstrate a uniform decline of buckling resistance as the imperfections increase, but with a considerable quantitative difference in terms of the values of buckling resistance reduction. On the other hand, a cylindrical shell made of stainless steel, when the imperfections are increased from 5,0 mm to 10,0 mm exhibits a small decline of buckling resistance, from 51% to 57%, as presented in figure 9a. In the case of a conical shell made of carbon steel, the increase of geometrical imperfections for the same value resulted in the buckling resistance drop of 20%. Conical cylindrical shells made of carbon steel show a lower drop of buckling stress value in respect to the cylindrical shells, which is 10% in average for the same value of initial geometrical imperfections.



a) redukcija napona izbočavanja za ljuske debljine 10 mm
a) reduction of bucking stress of 10 mm thick shells



b) rezultati numeričke analize
b) comparison of the results of the numerical analysis with design recommendations

Slika 9. Poređenje rezultata numeričke analize s preporukama za proračun
Figure 9. Impact of geometrical imperfections

Poređenje rezultata numeričke analize za cilindrične ljuske od ugljeničnog i nerđajućeg čelika s proračunskim preporukama definisanim u EN 1993-1-6 [9], za krivu izvijanja definisanu za klasu A proizvodnih tolerancija, prikazano je na slici 9b. Analizirane su clindrične ljuske od čelika S275 debljine od 6,0 mm do 30,0 mm i od nerđajućeg čelika 1.4301 debljine 10,0 mm. Prema EN 1993-1-6 [9], početna geometrijska imperfekcija w_k određuje se u funkciji poluprečnika ljuske i njene debljine, kako je definisano jednačinom (8), kao i parametra lokalne izbočine $U_{0,max}$=0,006 (*dimple tolerance parameter*). Vrednosti zadatih početnih geometrijskih imperfekcija prikazane su u Tabeli (1). Ostvareno je dobro poklapanje rezultata numeričke analize za analizirane debljine cilindrične ljuske i proračunskih preporuka za klasu A proizvodnih tolerancija. Takođe, na slici 9b prikazani su rezultati numeričke analize za cilindrične ljuske od ugljeničnog i nerđajućeg čelika debljine 10,0 mm i početnih geometrijskih imperfekcija u opsegu od 1,0 do 10,0 mm.

Za cilindričnu ljusku debljine 10,0 mm od ugljeničnog čelika S275 i geometrijske imperfekcije do 4,0 mm rezultati numeričke analize zadovoljavaju empirijski definisane preporuke u EN 1993-1-6 [9], kako je prikazano na slici 9b. Za imperfekcije veće od 4,0 mm, što su ujedno i vrednosti imperfekcije koju preporučuje Evrokod za analiziranu cilindričnu ljusku i klasu A proizvodnih tolerancija (Tabela [1]), rezultati numeričke analize ne zadovoljavaju proračunske preporuke. Iako proračunske preporuke za ljuske od nerđajućeg čelika još uvek nisu definisane, analiza ponašanja ljuski od ovoga materijala bila je posebno značajna zbog specifičnosti njegovih mehaničkih svojstava. Kada se mehanička svojstva nerđajućeg čelika primene u proračunskim preporukama za ljuske, datim u EN 1993-1-6 [9], uočava se takođe da se najbolja poklapanja rezultata numeričke analize, sa ovako definisanim proračunskim preporukama, dobija za imperfekcije do 4,0 mm (slika 9b). Svakako, definisanje jasnih proračunskih preporuka u ovoj oblasti zahteva opsežna numerička i eksperimentalna ispitivanja.

Comparison of the results of the numerical analysis for cylindrical shells from carbon and stainless steel and the design recommendations given in EN 1993-1-6 [9] for the buckling curve defined for class A of fabrication tolerances is given in figure 9b. The analysis included cylindrical shells with thicknesses in the range from 6,0 mm to 30,0 mm made from carbon steel S275 and shell made from stainless steel 1.4301 with 10,0 mm thickness. According to EN 1993-1-6 [9] initial geometrical imperfection w_k should be determined as a function of shell radius, thickness and dimple tolerance parameter $U_{0,max}$=0,006, as given in eq. (8). The values of initial geometrical imperfections are presented in Table (1). The suitable prediction of results is achieved for analysed cylindrical shells and design recommendations according to class A of fabrication tolerances. Besides, the results of numerical analysis for cylindrical shells with 10,0 mm thickness made form carbon and stainless steel with initial geometrical imperfections in the range from 1,0 mm to 10,0 mm are presented in figure 9b.

The results of numerical analysis satisfy empirically defined recommendations in EN 1993-1-6 [9] as shown in figure 9b in the case of cylindrical shell with 10,0 mm thickness made of S275 carbon steel with geometrical imperfections up to 4,0 mm. The results of numerical analysis fail to satisfy the design recommendations in the case of imperfections larger than 4,0 mm which is at the same time the value of imperfection recommended by the Eurocode for the analysed cylindrical shell and class A of fabrication tolerances (Table 1). Even though the design recommendations for the stainless steel shells have not been defined yet, the analysis of behaviour of shells made from this material was particularly significant due to the specific mechanical properties of this material. When the mechanical properties of stainless steel are implemented in the design recommendations for shells provided in EN 1993-1-6 [9], one may also observe that the best agreement of the numerical analysis results with such defined design recommendations are obtained for imperfections of up to 4,0 mm (figure 9b). Certainly, defining clear design recommendations in this field requires extensive numerical and experimental research.

Tabela 1. Rezultati parametarske analize – cilindrične ljuske – S275
Table 1. Parametric analysis results – cylindrical shells– S275

t (mm)	Kritičan napon [MPa] / *Elastic critical buckling stress [MPa]*		Nosivost elementa [MPa] / *Buckling resistance [MPa]*				KDF
	EN 1993-1-6 [9] $\sigma_{x,Rcr}=0{,}605EC_xt/r$	Abaqus [1] σ_{LBA}	EN 1993-1-6 [9] w_k (mm)	EN 1993-1-6 [9] $\sigma_{x,Rk}=\chi_x f_y$	Abaqus [1] σ_{MNA}	Abaqus [1] σ_{GMNIA}	
6	304,9	305,8	3	109,6	230,7	108,2	0,35
8	406,6	407,4	3,4	145,0	243,2	137,7	0,36
10	508,2	509,3	3,8	166,7	247,9	151,6	0,30
12	609,8	610,8	4,2	182,1	250,6	163,8	0,30
15	762,3	762,7	4,6	198,3	253,4	183,4	0,24
20	1016,4	1016,4	5,4	215,6	256,9	193,9	0,19
30	1524,6	1522,2	6,6	234,9	260,0	215,9	0,14

Komparativna analiza rezultata numeričke analize, metodom konačnih elemenata i proračunskih preporuka datih u EN 1993-1-6 [9] za cilindrične ljuske od ugljeničnog čelika S275, debljine od 6,0 do 30,0 mm, prikazana je u Tabeli (1). Geometrijske imperfekcije zadate su prema preporuci za definisanje geometrijske imperfekcije w_k date u [9] za klasu A proizvodnih tolerancija. Iste proračunske preporuke u pogledu zadavanja početnih geometrijskih imperfekcija w_k primenjene su i na konusnu ljusku od ugljeničnog čelika S275, s debljinom zida ljuske od 10,0 mm, kako je prikazano u Tabeli (2).

Takođe, u tabelama (1) i (2), prikazan je odnos nosivosti cilindrične i konusne ljuske, dobijen primenom GMNIA analize u Abaqus-u [1] i kritičnog napona izbočavanja, koji je definisan u EN 1993-1-6 [9], što je definisano kao redukcioni faktor (*KDF - knockdown factor*). Napon izbočavanja, dobijen kao rezultat geometrijske i materijalne nelinearnosti (GMNIA), dobijen u Abaqus-u [1] i primenom proračunskih preporuka u EN 1993-1-6 [9], ima približno iste vrednosti. Razlika u ovako dobijenim rezultatima u opsegu je do 10% za cilindričnu ljusku i 17% za konusnu ljusku od konstrukcionog čelika S275. Takođe, ukoliko se uporede rezultati numeričke MNA analize s rezultatima GMNIA analize u Abaqus-u [1], može se uočiti da se s porastom debljine zida cilindrične ljuske razlika u nosivosti elementa, dobijena na ova dva načina, smanjuje. Može se zaključiti i to da se za kompaktnije preseke, veće debljine zida ljuske, smanjuje uticaj početnih geometrijskih imperfekcija na nosivost elementa, a povećava uticaj materijalne nelinearnosti.

Comparative analysis of results obtained from numerical analysis with design recommendations given in EN 1993-1-6 [9] for cylindrical shells made of S275 carbon steel 6,0 to 30,0 mm thick, is given in Table (1). Comparison of the results is performed in order to define the compliance of numerical analysis with recommendations given in European standard. Geometrical imperfections are defined according to the recommendations for geometrical imperfection w_k provided in EN 1993-1-6, Annex D [9] for class A of fabrication tolerances. The same design recommendations considering definition of geometrical imperfections w_k are also implemented on the conical shell made of S275 carbon steel, with the shell wall thickness of 10,0 mm, as given in Table (2).

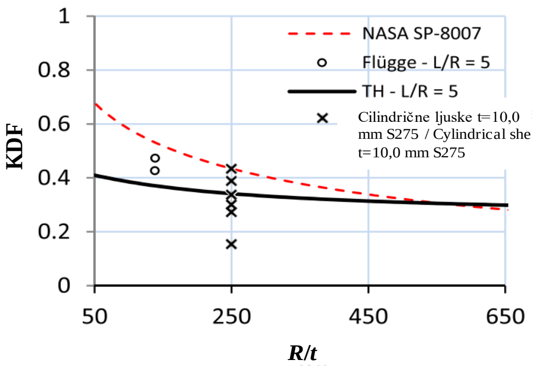
In addition, the relation between buckling resistance of cylindrical and conical shells obtained using the GMNIA analysis in Abaqus [1] and the critical buckling stress which is defined in EN 1993-1-6 [9], defined as knockdown factor - KDF, is given in Table (1) and (2). The buckling resistance obtained as a result of geometrical and material nonlinearity (*GMNIA*) in Abaqus [1] and that obtained using the design recommendations in EN 1993-1-6 [9] have approximately identical values. The difference between obtained results is in the range of 10% for cylindrical shell and 17% for conical shell made form carbon steel S275. When comparing the results of numerical analysis for material nonlinearity (*MNA analysis*) without initial imperfections and the results of an analysis including geometrical and material nonlinearities (*GMNIA*) in Abaqus [1], it can be observed that with the increase of the cylindrical shell thickness, the difference in the buckling resistance obtained by these two methods is reducing. It can be concluded that for the more compact cross-sections, having a higher shell wall thickness, the impact of initial geometrical imperfections on the buckling resistance is decreasing, and the impact of material nonlinearity is increasing.

Tabela 2. Rezultati parametarske analize – konusne ljuske – S275
Table 2. Parametric analysis results – conical shells – S275

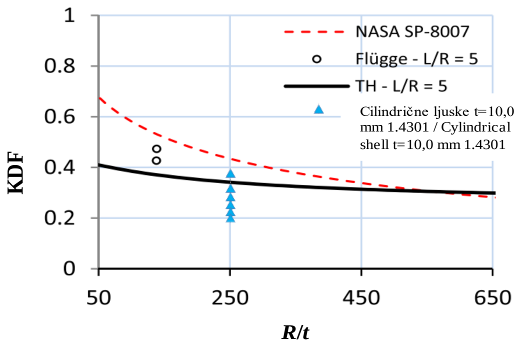
t (mm)	Kritičan napon [MPa] / *Elastic critical buckling stress [MPa]*		Nosivost elementa [MPa] / *Buckling resistance [MPa]*				KDF
	EN 1993-1-6 [9] $\sigma_{x,Rcr}=0{,}605EC_x t/r$	Abaqus [1] σ_{LBA}	EN 1993-1-6 [9] w_k (mm)	EN 1993-1-6 [9] $\sigma_{x,Rk}=\chi_x f_y$	Abaqus [1] σ_{MNA}	Abaqus [1] σ_{GMNIA}	
10	504,3	504,8	3,8	166,0	263,3	200,8	0,40

Na slici 10 prikazana su poređenja rezultata numeričke analize za cilindričnu ljusku debljine 10,0 mm, od ugljeničnog i nerđajućeg čelika, čiji rezultati su prikazani na slici 5 (sa geometrijskim imperfekcijama od 1,0 mm do 10,0 mm), s trenutno razvijenim empirijskim preporukama za redukcioni faktor (KDF). Definisane preporuke date su u formi krivih koje pokazuju zavisnost redukcionog faktora i odnosa poluprečnika ljuske i debljine (*R*/*t*). Na slici 10 prikazane su preporuke za redukcioni faktor prema NASA SP-8007 [14], modifikovane krive u zavisnosti od odnosa dužine i poluprečnika ljuske TH - *L*/*R*=5 prema Wagner-u [18], eksperimentalni rezultati prema Flügge-u i rezultati sopstvene numeričke analize. Modifikovana kriva TH - *L*/*R*=5 prema Wagner-u [18] obuhvatila je zajednički uticaj imperfekcija u opterećenju i geometrijskih imperfekcija (*SBPA – single boundary perturbation approach*), i za odnos dužine i poluprečnika ljuske *L*/*R*=5 definiše značajno niže vrednosti redukcionog faktora (KDF) od preporuka datih u NASA SP-8007 [14].

Figure 10 shows comparisons of numerical analysis results for a cylindrical shell of 10,0 mm thick of carbon and stainless steel given in Figure 5 (with geometric imperfections from 1,0 mm to 10,0 mm), with the currently developed empirical recommendations for the knockdown factor (KDF). The defined recommendations are provided in the form of the curves showing dependence of the knockdown factor and ratio of the shell radius and thickness (*R*/*t*). Figure 10 shows recommendations for the reduction factor according to NASA SP-8007 [14], modified curves depending on the ratio of the shell length and radius *L*/*R* according to Wagner [18], experimental results by Flügge and the results of the presented numerical analysis. The modified curve TH - *L*/*R*=5 according to Wagner [18] includes the mutual impact of imperfection in load and geometric imperfection (SBPA - single boundary perturbation approach), and for the ratio of length and radius of shell *L*/*R*=5 defines significantly lower value of the knockdown factor (KDF) from the recommendations given in NASA SP-8007 [14].



a)



b)

Slika 10. Redukcioni faktor za cilindrične ljuske: a) ugljenični čelik S275; b) nerđajući čelik 1.4301 [18]
Figure 10. Knockdown factor for cylindrical shells - a) carbon steel S275, b) stainless steel 1.4301 [18]

Cilindrične ljuske od ugljeničnog čelika S275 pokazuju relativno dobro slaganje sa modifikovanom krivom prema Wagner-u [18] za geometrijske imperfekcije definisane u skladu s preporukama datim u EN 1993-1-6 [9] za analiziranu dužinu i prečnik ljuske. Za ljusku debljine 10,0 mm od ugljeničnog čelika S275 i geometrijske imperfekcije od 1,0 do 3,0 mm rezultati numeričke analize zadovoljavaju empirijski definisane preporuke prema Wagner-u [18]. Za imperfekcije od 4,0 mm, što su ujedno i vrednosti imperfekcije koju preporučuje Evrokod za debljinu zida ljuske od 10,0 mm i klasu A proizvodne tolerancije, rezultati su nešto konzervativniji. Isto ponašanje uočeno je za veće vrednosti početnih imperfekcija. Rezlutati numeričke

The cylindrical shells made of S275 carbon steel exhibit relatively good agreement with modified curves by Wagner [18] for geometrical imperfections defined in accordance with the recommendations provided in EN 1993-1-6 [9] for the analysed shell length and radius. In the case of the shell 10,0 mm thick made of S275 carbon steel and the geometrical imperfections from 1,0 to 3,0 mm, the results of the numerical analysis satisfy empirically defined recommendations according to Wagner [18]. For imperfections of 4,0 mm, which are simultaneously the imperfection values recommended by the Eurocode for the shell wall thickness of 10,0 mm and class A of fabrication tolerance, the results are slightly more conservative. The same behaviour is observed for analize za cilindričnu ljusku od nerđajućeg čelika takođe su konzervativni u poređenju s preporukama prikazanim na slici 10b, čak i za najmanje vrednosti imperfekcija od 1,0 i 2,0 mm. Ovakav rezultat je očekivan, imajući u vidu to što su empirijske krive za redukcioni faktor, prikazane na slici 10, razvijene na osnovu rezultata eksperimentalnih ispitivanja cilindričnih ljuski od ugljeničnog čelika.

5 ZAKLJUČAK

Na osnovu rezultata numeričke analize, prikazanih u ovom radu, mogu se izvesti sledeći zaključci:

– Početne geometrijske imperfekcije imaju velik uticaj na nosivost tankih kružnih cilindričnih i konusnih ljuski. Imperfekcija od 1,0 mm za elemente debljine 10 mm izaziva redukciju nosivosti od 12% odnosno 17%, za ugljenični čelik i nerđajući čelik kod cilindričnih ljuski, respektivno. Povećanjem početnih geometrijskih imperfekcija do 5,0 mm, redukcija nosivosti dostiže i do 45% kod ugljeničnih čelika, odnosno 50% kod nerđajućeg čelika. Ova redukcija nosivosti kod konusnih ljuski od ugljeničnog čelika dostiže 30% u slučaju imperfekcija od 5 mm.

– Cilindrične ljuske od nerđajućeg čelika pokazuju progresivni pad tangentnog modula elastičnosti E_t, u oblasti napona između napona proporcionalnosti f_p i konvencionalne granice razvlačenja f_{02} u poređenju s ljuskama od ugljeničnog čelika, što utiče na znatno smanjenje krutosti.

– Povećanjem debljine zida cilindrične ljuske, smanjuje se uticaj početnih geometrijskih imperfekcija na nosivost elementa, a povećava uticaj materijalne nelinearnosti.

– Vrednost redukcionog (*KDF*) faktora za imperfekcije definisane u skladu s preporukama EN 1993-1-6 [9] za cilindrične ljuske od ugljeničnog čelika pokazale su dobro slaganje s preporukama za KDF faktora prema Wagner-u [18].

Kako je redukcija teorijske vrednosti kritičnog napona – koju propisuju standardi – velika, buduće unapređivanje proračunskih preporuka bazirano je na optimizaciji redukcionog faktora. Jedan od načina jeste formiranje i primena baze podataka o imperfekcijama kružnih cilindričnih ljuski, na osnovu sprovedenih eksperimentalnih ispitivanja (*Imperfection Data Bank*). Time bi se omogućila optimizacija proračuna i projektovanje za određeni tip cilindra, analizirajući samo rezultate dobijene na sličnim tipovima. Pored navedenog, nameće se pitanje može li se efekat početnih imperfekcija na smanjenje nosivosti izbeći odgovarajućim dizajnom, odnosno proizvodnim kvalitetom. Na ovom polju postoje mnogi radovi, a svi oni pokušavaju da dođu do ljuski neosetljivih na početne deformacije. Može se zaključiti da je polje analize ponašanja kružnih cilindričnih i konusnih ljuski i dalje umnogome otvoreno za istraživanje.

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the higher values of initial imperfections. The results of the numerical analysis for cylindrical shell of stainless steel are also conservative in respect to the recommendations displayed in figure 10b, even for the lowest values of imperfections of 1,0 and 2,0 mm. Such a result is expected regarding that the empirical curves for the knockdown factor given in figure 10 are developed based on the results of experimental tests of cylindrical shells made of carbon steel.

5 CONCLUSION

Based on the results of numerical analysis presented in this paper, the following conclusions can be drawn:

– Initial geometrical imperfections have a great impact on the buckling resistance of thin-walled cylindrical and conical shells. An imperfection of 1,0 mm causes the buckling resistance reduction of 12%, i.e. 17%, for carbon steel and stainless steel cylindrical shells, respectively. By increasing initial geometric imperfections up to 5.0 mm, the reduction of buckling resistance reaches up to 45% for carbon steel and 50% for stainless steel. This reduction in the case of conical, carbon steel shells reaches 30% for imperfection of 5mm.

– Cylindrical shells of stainless steel shows a progressive decline of tangent elastic modulus E_t, in the stress range between the proportionality limit stress f_p and the 0.2% proof stress f_{02} in comparison with carbon steel cylindrical shells, which causes the considerable decrease of stiffness.

– With the increase of the cylindrical shell thickness, the impact of initial geometrical imperfections on the buckling resistance is decreasing, and the impact of material nonlinearity is increasing.

– The value of KDF factor for geometrical imperfections determined according to the recommendations given in EN 1993-1-6 [9] for cylindrical shells made of carbon steel represent a good agreement with recommendations given by Wagner [18].

As the reduction of the theoretical value of critical stress, prescribed by the standards is high, the future improvement of design recommendations is based on the optimization of the knockdown factor. One way is forming and implementation of a data base on imperfections of cylindrical shells based on the conducted experimental tests (*Imperfection Data Bank*). This facilitates optimization of calculation and design of a specific type of cylinder, by analysing the result obtained on the similar types of cylinders. In addition to the previous, there is also a question whether the initial imperfections effect, resulting in reduced bearing capacity, can be avoided using appropriate design, i.e. production quality. There are many papers on this topic, and all of them are striving to attain the shells insensitive to initial deformations. Therefore, it can be concluded that the field of analysis of cylindrical and conical shells behaviour is still largely open to research.

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6 LITERATURA

[1] ABAQUS User Manual. Version 6.12. Providence, RI, USA: DS SIMULIA Corp, 2012.

[2] Batdorf, S. B.: A Simplified Method of Elastic-Stability Analysis for Thin Cylindrical Shells, NACA, Technical Report, No. 874, Washington, D.C., 1947.

[3] Chajes, A.: Principles of Structural Stability Theory, Prentice-Hall, Englewood Cliffs, New Jersey, 1974, pp. 303−330.

[4] De Vries, J.: The Imperfection Data Bank and its Applications, Giethoorn ten Brink, Netherlands, 2009, p.11.

[5] Dobrić, J., Buđevac, D., Marković, Z., Gluhović, N.: Behaviour of stainless steel press-braked channel sections under compression, Journal of Constructional Steel Research 139, 2017, 236-253.

[6] Donnell, L. H.: Stability of Thin-Walled Tubes Under Torsion, NACA, Technical Report, No. 479, Washington, D.C., 1933.

[7] Donnell, L. H.: A New Theory for the Buckling of Thin Cylinders under Axial Compression and Bending, Transactions, ASME, Vol. 56, 1934.

[8] Donnell, L. H., Wan, C. C.: Effect of Imperfections on Buckling of Thin Cylinders and Columns Under Axial Compression, Journal of Applied Mechanics, ASME, Vol. 17, No. 1, 1950.

[9] Eurocode 3 – Design of steel structures – Part 1 – 6: Strength and stability of shell structures, EN 1993-1-6, CEN, 2007.

[10] [Jones, R.: Buckling of Bars, Plates and Shells, Bull Ridge Publishing, Blacksburg, 2006, pp.702.

[11] Koiter, W. T.: The effects of axisymmetric imperfections on the buckling of cylindrical shells under axial compression, Proc. Royal Netherlands Academy of Sciences, Amsterdam, Series B, 66, 1963.

[12] Riks, E.: An Incremental Approach to the solution of Snapping and Buckling Problems, Int. J. Solid Structures, Vol.15, 1979.

[13] Seide, P.: Axisymmetric Buckling of Circular Cones Under Axial Compression, Journal of Applied Mechanics, vol. 23, no. 4, 1956.

[14] Seide, P., Weingarten, V.I., Peterson, J.P.: Buckling of thin-walled circular cylinders, Technical Report SP 8007, NASA Langley Research Center, Virginia, 1968.

[15] Seide, P., Weingarten, V.I.: Buckling of Thin-walled Truncated Cone, Technical Report SP 8019, NASA Langley Research Center, Washington D.C., 1968.

[16] Spremić M., Pavlović, M., Marković, Z., Veljković M., Buđevac D.: FE validation of the equivalent diameter calculation model for grouped headed studs, Steel and Composite Structures, 26 (3), 2018, 375-386.

[17] Weingarten, V.I., Morgan, E.J., and Seide, P.: Elastic stability of thin-walled cylindrical and conical shells under axial compression, AIAA Journal, 1965, 3(3):500–505.

[18] Wagner, H.N.R., Huhne, C., Niemann, S.: Robust knockdown factors for the design of the axially loaded cylindrical and conical composite shells – development and validation, Composite structures 173, 2017.

[19] Yoshimura, Y.: On the mechanism of buckling of a circular cylindrical shell under axial compression and bending, Reports of the Institute of Science and Technology of the University of Tokyo, vol. 5, 1951.

REZIME

POREĐENJE PONAŠANJA TANKIH CILINDRIČNIH I KONUSNIH LJUSKI OD UGLJENIČNOG I NERĐAJUĆEG ČELIKA

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Tanke kružne cilindrične i konusne ljuske predstavljaju jedan od složenijih konstruktivnih elemenata u pogledu ponašanja i osetljivosti na izbočavanje. U radu je dat kratak teorijski osvrt, s prikazom različitih, trenutno dostupnih, proračunskih preporuka. Prikazana je numerička analiza uticaja početnih imperfekcija na nelinearno ponašanje kružnih cilindričnih i konusnih ljuski. Analizirane su ljuske različite debljine zida, s konstantnim vrednostima dužine i prečnika ljuske, kao i s različitim vrednostima početnih imperfekcija. Analiza obuhvata uticaj materijalne i geometrijske nelinearnosti na ponašanje ljuski od ugljeničnog i nerđajućeg čelika, uključujući eksperimentalne krive napon-dilatacija. U radu je pokazano da materijalna nelineranost i početna geometrijska imperfekcija dovode do značajnog pada nosivosti na izbočavanje ljuski.

Ključne reči: ljuske srednje dužine, postkritično ponašanje, početne imperfekcije, izbočavanje, redukcioni faktor

SUMMARY

BEHAVIOR OF THIN-WALLED CYLINDRICAL AND CONICAL SHELLS - CARBON STEEL vs. STAINLESS STEEL

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Thin-walled cylindrical and conical shells represent one of the most complex structural elements considering their behaviour and susceptibility to buckling. A brief theoretical review including the presentation of different currently available design recommendations is given in this paper. Influence of initial imperfections on nonlinear behaviour of cylindrical and conical shells is also presented through numerical analysis. Shells with different wall thicknesses and different values of initial imperfections, but constant length and diameter of shell are analysed. Numerical analysis includes materially and geometrically nonlinear analysis of cylindrical and conical shells, using experimentally obtained stress-strain relation of carbon steel and stainless steel. Material nonlinearity and initial geometrical imperfections resulted in significantly lower buckling resistance of shells.

Key words: medium-length shells, post critical behaviour, initial imperfections, buckling, knockdown factor